THE ART OF LOGIC

An Anthology

Avi Sion, Ph.D.

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1. ABSTRACT

The Art of Logic by Avi Sion is a collection of recent essays on various topics in logic theory and in applied logic. The same faculty and art of logic is called for in formulating theoretical logic and in applying its findings to diverse fields. The essays here collected deal with some very important issues in logic, philosophy, and spirituality, which he had not previously treated in as much detail if at all.

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3. FOREWORD

The Art of Logic is a collection of my more recent essays on various topics in logic theory and in applied logic.

Over thirty years ago, in my first book, *Future Logic*, I stated that logic is both an art and a science. The art of logic is our ability to practice logic instinctively, our use of 'common sense', in our everyday thinking. The science of logic is the product of our detailed investigations into this practice to determine precisely how it proceeds, and when it is reliable and when it is not. But this scientific logic is not the product of some superior faculty or method – it too is developed *by means of* the selfsame art of logic, i.e. by ordinary common sense, though perhaps more carefully exercised. However, significantly, our practice of logic is greatly improved in direct proportion to our knowledge of theoretical logic. When we are acquainted with the logic theory, generally or in a specific field of logic, then our practice of logic can accurately be characterized as 'applied logic'.

Thus, in the present book, we are using the art of logic both in sections dealing with logic theory and in sections applying logic to other philosophical problems. I could have called this book 'The Science and Art of Logic', since both are involved; but this would have made it seem like a comprehensive work on logic theory as well as on applied logic, which it is not. Some of my past works have been exclusively devoted to theoretical logic, while others have included applications. The present work, though partly a study of some aspects of theoretical logic (notably the logic of analogy), is mainly aimed at logical philosophy, i.e. at applying logic to solve problems of philosophy (in a broad sense of the term); whence the preferred title of 'The Art of Logic'. That title brings to mind the famous 'The Art of War' - and indeed logic is a mighty weapon in the unending intellectual war against falsehood, foolishness and folly. Yes, logic is a martial art; but of course, it is not only that. Logic is the way of reason: combined with experience, it helps us both to attain understanding and to guide our actions.

The present collection of essays deals with some very important issues in logic, philosophy, and spirituality, which I had not previously treated in sufficient detail if at all. Some have previously been published separately or within thematic compilations. This volume begins with 'First Philosophy', which depicts my philosophy of mind, my phenomenology, and my theory of sense-perception, and their significance for biological science and ethics. Follows 'Causality Fundamentals', which summarizes my theory of causality. Next is 'The Logic of Analogy', which brings important innovations to this field. And so forth - the subjects vary, but their common factor is the logical treatment given them. All these essays bring truly new and valuable insights into the subjects they deal with.

1. FIRST PHILOSOPHY

1. Philosophy of mind

Clarifying the true 'philosophy of mind' is essential to understanding volition, and other forms of causality. David Hume's assault on volition and more broadly on all causality[1], was made possible by his erroneous general views on mind and cognition. It is amazing how naïve, inaccurate, and fallacious, much of what is nowadays made to pass as philosophy of mind is in fact.

For a start, what do we mean by 'mind'? With regard to etymology, briefly put, this word is thought to derive from '*men*-' a Proto-Indo-European root meaning 'to think', with derivatives referring to qualities and states of mind or thought[2]. The term 'mind' gives rise to numerous nouns, adjectives, verbs, adverbs, and phrases. **The mind** (n.) refers colloquially to a non-material space, playground or storehouse (as it were), for all the entities, states, qualities, actions, etc. which we would consider mental as against material (more on that distinction later). This would include primarily contents of sensory-perception, memories of such contents, imaginations derived from them, and intuitions. Such *concrete* contents and derivatives are, to be precise, all mental *qualia*; that is, they have the phenomenal qualities of what is seen, heard, smelled, tasted, touched, or felt (in the intuitive or emotional sense). At a later stage, the term mind includes a large number of more and more *abstract* contents, such as the concepts of sensation, of perception, of intuition, of memory, of imagination, of qualia, of concrete and abstract things, and a lot more. From the term mind, we derive a large number of others, such as **mental** (adj., pertaining to the mind), minding (v., paying attention to or caring about), not minding (ignoring or being indifferent to), reminding (calling to one's attention again), mindful (being fully aware), mindless (lacking awareness), and many more.

We are thought and said to have, minimally, a body (a physical entity we seem to inhabit and own) and a mind (a non-physical entity we seem to inhabit and own). In this perspective, the mind is often taken to include ourselves, i.e. our souls – that to which body and mind 'belong'. I personally prefer, for clarity's sake, to call this larger concept of mind the psyche, and to reserve the term mind for the more specific concrete and abstract contents (not including the soul itself, but of course including concrete intuitions and abstract thoughts of it and its three functions). Thus, for me, psyche includes soul (the self, the spiritual aspect of our existence) and mind (in the narrower sense). Both soul and mind are evidently closely related to the body in various ways.

Thus, we have three dimensions of being, as it were: our soul, our mind, and our body. These clarifications are not trivial, but a necessary beginning, if we aim for an intelligent philosophy of mind. The soul has three basic functions, three ways its existence is expressed and is made evident. These are: consciousness (the cognitive function), volition (the function of inner and outer action) and valuation (the function of choosing and deciding).

It must be stressed that I do not (or no longer do since I wrote the essay "Critique of the Buddhist Five Skandhas Doctrine") conceive of a 'soul' as a sort of 'substantial entity', a sort of 'body' composed of a 'spiritual' stuff (as distinct from one of 'material' stuff), something with an extension and a location in space, existing in some other dimension(s) of being, somehow

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1 In *An Enquiry Concerning Human Understanding* (1748).

2 See: www.etymonline.com.

intertwined with the material world. No, all these descriptive terms are *mere metaphors, not to be taken literally*. The spiritual 'realm' (again, just a simile) is so very different from the more manifest physical realm (or indeed the mental 'realm' of personal memories and imaginations, and feelings and emotions) that we have no words for it. For this reason, we are forced to think and speak of it by means of rough analogies. But the existence of this ineffable thing is, through personal intuitive experience, indubitable; and there is much reason for each of us to assume others have (are) the same.

The term **consciousness** refers primarily to *a cognitive act or relation* between a subject (the soul or self) and an object (whether the soul itself, or a mental content, or a bodily or wider physical phenomenon). Cognition is something *sui generis*; it cannot be 'reduced' to any other concept (like a physical process in the brain or a physical field of sorts). Consciousness is thus something essentially static; it is *not primarily an entity or a quality or a state or a process*. Consciousness means, primarily, *being aware of something*; not just 'aware', note well, but 'aware *of* something'. Usually, when we are conscious, our focus is on the object of consciousness; but it is also possible to focus on our consciousness, in which case it is then an additional object of consciousness[3].

Whether awake or asleep, we cannot be said to be conscious if we are literally not at all conscious of anything at the time. A person may be said to qualitatively 'be conscious', i.e. to have a 'state of consciousness', only derivatively – only if and when and while it is actively conscious *of* something, i.e. performing the cognitive *act* of consciousness which *relates* it in a special way to some concrete or abstract object(s), whether material (based on sense-perception), mental (memory, imagination, or dream) or spiritual (as in self-awareness and intuition of functions of self).

Similarly, the term 'stream of consciousness' is a misnomer – it is used to describe, more precisely, a stream *of thought*, i.e. a stream of successive mental contents such as intuitions, perceptions, memories, mental images and sounds, verbal comments, feelings, conceptualization, and so on. Thought is made possible by many and diverse acts of consciousness, but the terms cannot be equated. It follows that, though we can speak of *sub*conscious thoughts, by which we mean thoughts occurring with a very low intensity of attention, we cannot speak of literally *un*conscious thoughts – doing so would be self-contradictory.

It is likewise inaccurate and misleading to equate, as some people do (notably Buddhist philosophers), the terms 'mind' and 'consciousness'. It is also erroneous to describe the mind as an 'entity' made of consciousness. Consciousness is not the stuff of mind – mental contents might be regarded as made of sort of distinctly mental stuff, but consciousness is something else entirely (as above already defined). Most important, *consciousness is not conscious*; it is we (the self, the soul or spirit) who are conscious – conscious of other things, or even of oneself, or even of the fact of consciousness.

Consciousness is thus essentially a verb, not a noun or adjective or adverb. It *relates* the Subject (one's very self) to some object or other. Intuition, sense-perception, inner perception (of memories and imaginations), conception (of abstractions based on the preceding concretes), verbal

[3] Indeed, contemplation of one's consciousness while one is aware of whatever (else) one happens to be aware of here and now (whether inside or around one) – this is a powerful meditation technique (which I occasionally practice and recommend). In this meditation, note, the first object of consciousness is not ignored; but the fact of consciousness of it becomes in turn an object of consciousness, superposed on the first, and indeed henceforth the principal focus of our attention. The act and state of consciousness, the miracle, marvel, and mystery, of the cognitive relation we are able to have with things in and around us, all become objects of one's meditation.

thought – these are species or types of consciousness, all involving the marvelous phenomenon of cognition.

Consciousness of something (i.e. many things, internal and/or external, concrete and/or abstract) is behind and a necessary condition for every volition (act of will) and valuation (projection of value onto something). Consciousness may be switched on and intensified by an effort of volition, but its cognitive functioning is independent of volition. Likewise, consciousness can occur without valuation, whereas valuation cannot occur without consciousness.

Volition can reasonably be placed ahead of valuation; because volition can and does (though relatively rarely) occur without any goal in mind, as pure caprice (with not even the goal of being capricious); and also, because valuation itself involves an act of volition (that of choosing and deciding), as well as of cognition.

Thus, every act of volition is preceded by and accompanied by some degree of consciousness, however dim, and usually aimed at some value which we have imagined or reasoned as worth pursuing (and which duly influences our act). To speak of will as being literally unconscious (i.e. to say 'unconscious will') is inaccurate; all willing necessarily involves at least a bit of consciousness, and if the latter is truly absent the movement involved cannot be characterized as volitional (but only as mechanical or fortuitous).

Valuation refers to the soul's assignment of value (or disvalue) to something thought of or currently perceived. Obviously, this involves both consciousness and volition. Valuation is not to be confused with desire, because though one may experience a strong desire (e.g. a lust for sex or greed for food), one is still free to grant it value or not, and thus pursue it or not. We often try to evaluate things rationally, by going through a research and reasoning process of some sort, or by referring to an existing ethical doctrine; but often, too, we attach value (or disvalue) to things in a rather instinctive or arbitrary manner.

(The ethical concept of *wisdom* is, of course, relevant here. Wisdom is consciousness, volition, and valuation, in accord with objective ethical truth. Of course, such truth is not easy to determine, but there is surely such a thing.)

The three basic functions of soul thus form a nexus, even though they are conceptually distinguishable. The concepts of thought and belief, action and behavior, and motive and purpose, are less fundamental than the three basic concepts of consciousness, volition, and valuation – the former are bunches of the latter.

The epistemological basis of all we have said so far is *phenomenological*, note well. It is through careful first-person *introspection*, and discussion with other people, with an eye to logical consistency, and with ample use of intelligence and intellectual honesty, that we arrive at these insights fundamental to any philosophy of mind. Introspection is simply becoming aware of the phenomenological givens. This is as scientific as any other body of knowledge, provided it is well understood that the result is *inductive* and not dogmatic certainty. One can reasonably imagine that some of the appearances manifest to us might turn out to be wrong someday; but one cannot reasonably claim that such imagined theoretical possibility invalidates all our insights from the start. To propose an alternative theory, one would have to provide specific stronger evidence; it would not suffice to express a general skepticism about the reliability of introspection.

I am thinking here of the currently widespread prejudice, among alleged scientists of the mind, that what appears as 'mental' can be expected to one day be 'reduced' to purely physical phenomena, in the brain and nervous system. This claim is based on the prejudicial claim that the laws of physics are *universal*, although that claim is in fact merely based on *generalization* from observations in the physical world. These people argue that if the laws of physics did not ultimately

apply to mental phenomena (meaning mainly consciousness, volition, and valuation, but also all other mental experiences), then these laws *would not be* universal. This is true; but it is a mere circular argument, with no logical force whatsoever, to consider this as a good reason for denying the distinctive nature of mental phenomena.

The **reductionist thesis** is a mere hypothesis, which would only become credible if and when (if ever) the laws of physics could *predict, through exact mathematical formulae and theoretical explanations applied to precise empirical findings*, the relationship of mental phenomena to physical ones. Until then, all these people have to offer is *a statement of faith*, a materialist ideology that arbitrarily ignores and denies inconvenient phenomenological evidence, viz. the empirical givens of mental experience and life. Note well, the materialists *have not* (and I dare say never will) empirically or theoretically proved that mental phenomena are *kinds of* physical phenomena; they just presume they 'ultimately' are.

For example, Francis Crick[4] famously declared, in his *The Astonishing Hypothesis: The Scientific Search for the Soul* (1994), "you, your joys and sorrows, your memories and ambitions, your sense of personal identity and free will are, in fact, no more than the behaviour of a vast assembly of nerve cells." In this one short sentence, he at once denied self/soul, consciousness, volition, and valuation!

It should be clear to all honest thinkers that even if specific electrochemical and other physical processes in the brain and other parts of the nervous system are identified as always underlying specific mental events, that would not prove that the former are the causatives of the latter; it would only at best go to show initially that physical processes *accompany* mental ones, i.e. are parallel events; for it remains conceivable given such comparative data that, on the contrary, the latter are the causatives of the former. It is also possible, and (in my view) most probable, that sometimes the physical causes the mental and sometimes the mental causes the physical. It would certainly not constitute valid reductionism, unless and until (to repeat) mental phenomena as such were *themselves proven* to fall under the laws of physics, i.e. to be material phenomena.

What is sure is that the proponents of such materialist views of mind have nowhere near achieved this necessary scientific proof. Until they do (and they never will, I wager) we should assume, on the contrary, that physical events always accompanying mental ones are (sometimes if not always) effects, rather than causes, of them – in view of the arguable autonomy of volitional acts by people (and probably animals) as against the determinism of observable purely physical movements, and in view of the exceptional nature of consciousness (of objects by subjects).

Some people go so far as to deny outright the very existence of mental phenomena, claiming that what we mean by these words is nothing other than the neurological events that physical science can actually detect, describe, and quantify. This is known as 'identity theory' – because it does not merely claim that mental and physical processes are parallel, with the one causing the other or vice versa, but that they 'are' one and the same, and indistinguishable, and entirely physical. This is an extreme version of behaviorism. Effectively, these people are willing to deny the very existence of anything that they cannot wrap their little heads around!

According to them, there is no such thing as persons with powers of consciousness, volition, and valuation, experiencing distinctively mental phenomena. Humans and animals are just sophisticated inanimate machines, subject to blind determinism. If so, how could these people at

[4] The British physicist, neuroscientist, and molecular biologist (1916-2004) who discovered the double helix structure of DNA in 1953 together with James Watson and others. About his book, see: https://en.wikipedia.org/wiki/The_Astonishing_Hypothesis.

all formulate their self-denying theory? What would they be talking about if there were nothing mental to talk about? If they don't exist and they lack awareness, free will, and value judgment, how would they have come to see the mental domain as a problem needing their inane solution? These people are not scientists, because they discard the scientific method. They choose to adapt reality to their theory, rather than adapting their theory to reality.

Without normal scientific proof, i.e. without strict application of the scientific method, and blithely ignoring all the evidence of mental life and stupidly ignoring numerous logical considerations such as those above laid out, the materialist reductionists arbitrarily claim a thesis of 'matter over mind', in place of the commonsense thesis of 'mind over matter' (or more precisely put, of 'soul over mind and matter'). They want to simplify and unify knowledge at any cost. They think that what seems 'obvious' to them needs no further proof.

In truth, ***it is useless and self-contradictory to deny reality to consciousness and volition***, as these people are desperately attempting to do, ***because any such denial itself necessarily involves and logically implies acts of consciousness and volition*** (which they maliciously fail to take into account and explain). Only a person with consciousness and volition can engage in meaningful affirmation or denial of anything; and such judgments can only be understood by persons with the same powers. All they have to offer, to repeat, is a prejudice – a far from credible, indeed absurd, yet amazingly persistent prejudice.

Unfortunately, this materialist mindset is widespread today and has been for the last couple of centuries or more; so, it is hard to awaken people to its absurdity. It is effectively a faith, a sort of atheistic religion. Those who accept such nonsense are truly simpletons.

2. Phenomenology

The logical starting point of epistemology, the study of knowledge as such, i.e. the philosophical description, explanation, and justification, of knowledge acquisition by humans, cannot be sense-perception per se because sense-perception must itself be correctly described, explained, and justified, before it can be correctly placed in the course of our knowledge. The starting point has to be more fundamental, less specific – it has to be phenomenology, the study of appearance as such.

The primary givens of human knowledge are appearances; the contents of sense-perceptions must, to begin with, be taken as appearances. Doing that does not mean that we reject our common belief that certain appearances are due to sense-perception; it merely means that we are going deeper and looking for the true basis of that belief. The logical order of things from the beginning should be presented briefly as follows.

There are **appearances**, therefore **existents** exist; therefore, existence exists. An appearance is something that is an object of **consciousness**. An existent is something that is actually present in the world at a given time, whether it is an object of consciousness or not. All appearances are existents, but not all existents are appearances. Existence (or being – I consider these terms as identical) is what all existents have in common, and indeed all appearances as such have in common. The statement that ***existence exists*** is more than mere tautology; it is informative, the very first true proposition. **Non-existence** does not exist; it is not a subdomain of existence.

Appearances may be **real** or **illusory**. Reality is a more specific characterization than existence; it applies only to some appearances. The real can exist outside of consciousness; the illusory can only arise within consciousness. Normally, we assume that ***whatever appears is real, unless we have, or until we find, some logical reason to reclassify the appearance as illusory***. The illusory

also appears, note well; but it involves an imaginary distortion or misinterpretation of some preceding appearance. We know we are caught in an illusion when some contradiction is found in an appearance or set of appearances. The illusory is unreal; it is the antithesis of real.

Appearance, existence, consciousness, reality, illusion, are fundamental terms, which cannot be entirely defined (since they refer to things so basic that they cannot be reduced to other things, i.e. they are irreducible primaries), even if as just done we can to some extent correlate them. These things are understood by all of us, not through verbal definitions, but by direct insight. It cannot be said that there are no appearances or no existents or no consciousness, since in the very act of saying so we are in fact generating an appearance and claiming an existent and demonstrating consciousness. It cannot be said that nothing is real and everything is illusory, because by so doing we are claiming to know this as a reality and not an illusion, which is self-contradictory. So, all this is logically on firm ground, not open to any doubt whatsoever.

Everything that appears exists; and even many things that do not appear exist. We know the latter from the fact that over time new things appear to us which it is *inductively* reasonable to suppose were there even before they were objects of consciousness; and likewise, things may disappear yet be assumed to continue to exist irrespective. This is confirmed by our comparing the experiences of different people: I may see something before you get to do so, or after you cease to do so; and vice-versa. But not all that apparently exists is real; illusions exist even though they are not real. Whatever appears must be taken to exist, but may turn out to be unreal; it may prima facie be assumed to be real, but it must be kept in mind that subsequent appearance may reveal it to be illusory. Thus, we can say that everything that appears is real, *except* the relatively rare things that are later found to be illusory. By extension, we may assume that things may without ever appearing be not only existent but also real.

'Existence exists' means that something exists – or some things exist. That this is true is evident the moment it is said; and it is incontrovertible, since even discussion of the issue is an admission of existence. It is, moreover, evident that existence exists in our daily experience, whether this experience be considered to be of mere appearance; or of reality or illusion; or of matter, or mind, or spirit; or of concrete or abstract; or of whatever distinction.

The distinction between **matter, mind, and spirit** is one of domains (let's say). It can be and usually is characterized as a distinction of substance, but that is mere metaphor because the experience of substance is essentially physical: it might credibly be extended by analogy to concrete mental phenomena like memories, imaginations, and feelings, but is hardly applicable to the spiritual domain, i.e. to introspective experience of self, i.e. of soul and its functions (consciousness, volition, and valuation). Even the characterization of matter, mind and spirit as different domains is metaphorical, since it draws on the experience of space, which is applicable primarily to physical objects, and by extension eventually to mental ones, but cannot literally be assumed of spiritual ones.

The other distinction important to phenomenology and epistemology is that between **concretes and abstracts**. What we primarily experience, whether it is physical, mental or spiritual, is concrete; this is the empirical level of our intellectual life. We humans then mentally compare and contrast, group, measure, and sort, such empirical givens – this involves abstraction and concept-formation, and constitutes the theoretical level of our intellection. Abstracts are the expressions of our insights; consciousness is essential to them. *They have no existence apart from the human beings engaged in such intellectual activity. The world we face (including its physical, mental, and spiritual aspects) is wholly concrete; classes and principles are implied by it, but not concretely embedded in it.*

Abstracts per se are not themselves concrete things, existing, as it were, side by side with, or above or below, concrete things. Abstracts are intellectual constructs: they are conceptual products, aided by words and language, of volitional acts of abstraction, which mentally separate or conjoin various aspects of things. Nevertheless, abstracts are essential to the development and success of human knowledge. They help us to summarize and store information about known concretes and thereby, by various logical means, to predict to some extent information about unknown concretes.

Even though abstracts are intellectual constructs, the propositions composed of them may be true or false; and those that are true may be said to be objective, i.e. to reflect reality in significant ways. *This cannot be denied without self-contradiction*, note well; therefore, it is self-evidently true.

Clearly, the above primary insights have phenomenological, ontological, epistemological, and logical significance. None of these aspects can be developed independently of the others; they are deeply intertwined.

3. Theory of sense-perception

This brings us to ***the self-contradiction of current materialist theories of sense-perception***. The currently normative theory of sense-perception, which has its roots in modern 'Enlightenment' philosophy – notably the philosophies of René Descartes, Thomas Hobbes, John Locke, and most destructively that of David Hume – presents the sense organs as channels between the material world and our minds. For instance, it is claimed that light from some physical object produces an electrical signal in the eye, which then travels to the brain, where a mental image is produced, which is what we actually see rather than the physical object itself. The following description, by the American National Eye Institute, of the supposed process is one example among very many of this doctrine:

> "When light hits the retina (a light-sensitive layer of tissue at the back of the eye), special cells called photoreceptors turn the light into electrical signals. These electrical signals travel from the retina through the optic nerve to the brain. Then the brain turns the signals into the images you see."[5]

Consider the enormity of the claim made here, with zero awareness of its profound implications and utter absurdity. ***If what we actually see are mental images of the world out there, rather than aspects of the external world itself, then what reliable proof have we that there are any physical sources to our mental images? The sense-organs and brain we claim to have are known to us only through sense-perception, and therefore must themselves, under that theory, be mere mental images. In that case, there is no basis at all – other than mere imagination – for claiming that there exists around us a physical world producing mental images in us. In which case, the given theory of sensory perception is self-contradictory, <u>starting with a premise that it ends up effectively denying</u>!***

Think about that; reflect on it. Proponents of this 'representative' or 'correspondence' theory claim that external physical objects are known to us by way of internal mental images; yet they propose no way for us to know the external objects independently *and so be able to judge the existence and accuracy* of such representation[6]. No 'correspondence' between internal mental images and

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[5] https://www.nei.nih.gov/learn-about-eye-health/healthy-vision/how-eyes-work

[6] Note in passing that the Lockean distinction between primary and secondary qualities is, from my point of view, rather arbitrary. It presents some qualities as indeed in external objects (e.g. shape) and some as having mental

external physical objects can be established if we only have access to one side (the mental side) of the story. All our perceptions would then have to be considered 'true', or alternatively 'false'. There would be no way to verify, by means of comparison and contrast, some perceptions as true and others as false. Such verification is only possible if we are indeed able to distinguish between mental phenomena and physical ones.

Bishop Berkeley[7] was the first 'Enlightenment' philosopher to realize the implication of the above stated claim about the course of sense-perception. This led him, unfortunately, to deny the existence of matter and to claim that only mind exists. He called this viewpoint 'Immaterialism'; but a more positive name for it would be 'Mentalism'. Berkeley was intelligent enough to see the absurdity of the representative theory; but he was not intelligent enough, it seems, to draw a better conclusion from the conundrum than that of Subjectivism and Idealism. Moreover, he did not realize that the distinction between mentality and materiality, or dream and non-dream, is only possible by way of contrast, if both exist and both are accessible to us; if only one of them exists and is accessible to us, it cannot properly be characterized as either mind or matter, dream or non-dream.

Berkeley's insight effectively meant that all his experience was a massive 'dream'. Being a religious thinker, he chose to acknowledge the existence of other people and place his mind and theirs in the single mind of God. In this view, life was indeed a dream, but it was God's dream. But logically this was the wrong conclusion to draw. The right conclusion to draw was that of Solipsism, viz. that Berkeley was the only existing person, and his own mind was the sole mind in the (his) world, and all other supposed people, including a supposed God, and their minds, were mere contents of his dream. If you the reader exists, your conclusion should be that Berkeley and me, and everyone else around you, are mere figments of your imagination, mere contents of your dream, and that only you exist.

Although the inference of Solipsism strikes most of us as unbelievable, Berkeley's insight that the proposed theory of sense perception, as not involving direct perception of physical objects, but only their indirect perception by way of mental objects, logically implies that all a human perceiver ever perceives is mental images, and never their alleged physical source, remains valid.

To this day, I have not come across any modern philosophers or scientists who realize (even at least as much as Berkeley did) the crazy self-contradiction inherent in the dominant 'representative' theory of sense-perception. The world 'out there' is still dogmatically assumed to be physical, even while at the same time it is paradoxically taken for granted that we *only* have access to mental images of it, i.e. only to an 'inner' world.

The mind-body chasm inherent in that viewpoint *cannot*, unfortunately, be mitigated by adductive logic, i.e. by claiming that the putative physical world, though not inferable with certainty from our exclusively mental experiences, can still be assumed hypothetically and considered as at least inductively confirmed and made somewhat probable by those mental experiences. Such an approach would not solve the logical problem but constitute pure speculation, impossible to ever

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origin (e.g. color); but it is hard to see how this doctrine rhymes with the 'representative' theory – that is, how we could possibly make such a distinction under the regime of that theory. In my view, a quality like color can well, without logical difficulty, be claimed as a real property of the external object as a whole without denying that this is expressed by means of different electromagnetic rays. That is a reasonable phenomenological and positivistic outlook.

[7] George Berkeley (Ireland, 1685-1753), in *An Essay Towards a New Theory of Vision* (1709) and subsequent books. Long before Berkeley, perhaps some 1500 years before him, Mind-Only Buddhism (Yogachara) seems to have made somewhat similar claims, though in a much more complex manner.

verify scientifically; it would be methodologically dishonest, being effectively merely circular argument posing as credible reasoning.

The logical conundrum, the self-contradiction of claiming both that a physical world exists and that we cannot at all perceive it, but can only perceive mental images (allegedly of it or brought about by it), has to be addressed and solved first. We must find some way to acknowledge that we actually, directly, perceive (some of) the physical world. An inductive hypothesis is indeed needed; but only one leading to such conclusion would resolve the issue.

It should also be pointed out that those who advocate the said muddle-headed theory of sense-perception imagine that they thereby free themselves of the need to acknowledge and explicate the fact of *consciousness*, i.e. the sui generis relation between subject and object implied by every act of cognition (including their own). But this supposition is mere fantasy, because whether consciousness is of (i.e. related to) an external object or of an internal one, it is still the same event of consciousness – only its object changes. And therefore, still the same mysterious and (I daresay) nothing less than miraculous phenomenon, itself neither physical nor mental nor even spiritual, but something somehow linking the subject and any sort of object.

In other words, the said advocates think that, by claiming perception only occurs internally, exclusively during perception of mental objects (whether derived from alleged sensory processes or entirely formed in the mind), they simplify and solve the problem of cognition. But no, the cognitive fact of perception is as difficult to understand and explain in the mental domain as it is in the physical domain. Presenting 'sensory' perception as not perception of physical objects, but as a physical process resulting solely in mental perception, misses the point entirely, the central issue – which is the mystery of perception as such, whatever its object.

When one realizes the profound challenges that cognition (whether intuitive, perceptual, or conceptual), and indeed similarly volition and valuation, present science with, one clearly sees that all attempts to escape the problems by postulating simplistic solutions, either purely physical or purely mental scenarios, or inconsistent formulae involving both matter and mind, are bound to fail. It is important to realize that consciousness and will are irreducible primaries, whose scope includes both the physical and mental domains, and indeed the spiritual domain too.

Moreover, when we think of our 'mind's eye' watching mental images (or by analogy, our 'mind's ear' hearing mental sounds, and so on for other qualia), we are presenting mental images as projected on a screen in our heads (or by analogy, mental sounds as coming out of a loudspeaker in our heads, etc.). But this colloquial metaphor implies *some person* doing the watching or listening, etc. It would be absurd to speak of mental images or sounds without assuming a conscious Subject (a self, a soul) observing these mental objects. Similarly, imaginations, which are products of active reshuffling of memories, whether voluntary or involuntary, are only meaningful if someone is present as their spectator or auditor. We cannot speak of mental phenomena while denying *an experiencing Subject* (as some materialists try to do). Purely material things do not have the capacity to experience; at least, no such power of matter has been demonstrated to date.

Indeed, even the mere existence of images or sounds made of a 'mental' substance (which they must likewise admit, if only by implication) is a major challenge to purely materialist claims, albeit a lesser challenge than that posed by consciousness (and volition and valuation). The enlightenment notion of 'ideas', defined as mental contents or entities (and no doubt ultimately simplistically imagined as sorts of balls of mental fluff) is not compatible with the purely physicalist ontology currently regarded as exclusively scientific. These alleged entities (or events or whatever) are necessarily of a different substance from physical entities; or at least must be

assumed to be so, until and unless they are one day proven through empirical and mathematical methods to be themselves truly physical.

Until such equation is positively demonstrated, we are logically bound to regard matter and mind as distinct substances or domains, which interact and have some common features but are not the same. Moreover, since they evidently interact, all known physical laws must be formulated with the frank caveat that mental (and indeed spiritual) phenomena *have not yet been taken into consideration and included in these laws*.

What is the way out of the fundamental and supremely important conundrum pointed out here? The only way to retain our belief in matter, and not become exclusive 'mentalists' like Berkeley, or continue dishonestly turning a blind eye to the above-described self-contradiction like today's materialist philosophers and scientists, is to assume that what we really see (and more broadly, sense, and even what we eventually conceive), when we apparently see the material world, *is indeed (parts or aspects of) the material world itself and not mere (alleged) images thereof*, not necessarily all parts or aspects of it (of course) but at least some. Only this assumption can logically justify our belief in a material world, and at the same time in a mental world distinct from it.

We *are* able to judge our mental images (memories and imaginations) as realistic or not, *because* and only because we can compare and contrast them to actual physical objects of sense-perception. If we only had mental images, with nothing beyond them to compare or contrast them to, we *could not* evaluate their realism (as Berkeley well understood).

To achieve this positive doctrine, called objectivism or realism (which is quite distinct from and antithetical to Berkeley's subjectivism and idealism, note well), we need to creatively *rethink* the process of perception and the role played in it by our sense-organs and brain. Courage and honesty are needed to face this very difficult problem and try and solve it convincingly. Ingenious empirical research programs aimed at understanding what role the senses really play in perception are badly needed. Note well: I ask the question, but I do not claim to know the answer to it. That the answer is to date not known does not mean that there is no question or that the question can be ignored. What is sure is that if the question is not asked, the answer will never be found.

We must assume perception of matter is *direct*, i.e. that we perceive matter *itself* and not mere mental images of it. This seems a very reasonable assumption, considering *the immensity and richness* of the world we perceive as surrounding us – the Sun, the Moon, the oceans, the fields and forests, the cities, the people and animals – all this, with all its manifold details, cannot be produced and stored in our little brains. Admittedly, as Descartes pointed out, we sometimes have very clear and scenic dreams; but these are temporary flashes, not comparable to our much broader, clearer, multi-sensory, more logical, and persistent awake experiences. Our brains are very powerful, but not that powerful.

We must assume that the sense-organs and brain do indeed play an important role in perception of physical phenomena; but *their joint role is surely only to make possible somehow our direct perception of the external world*. They are the keys that open the doors for us (the souls that are perceiving), giving us access to (perception of) the physical world embracing us. In this new theory, the incoming signal from the retina to the brain is not useless, but it does not have the primary function of creating an inner image for us to perceive. It only serves to 'open the door' to direct perception, though it accessorily creates a mental memory of what we perceive through that door for future use.

What we are perceiving, in such case, is something physical outside us, i.e. material sights, sounds, odors, tastes, touch-sensations, and bodily feelings. There may well be errors in such perceptions, but they would be errors of appreciation and interpretation rather than errors of information. We

still need, in such context, to properly evaluate our direct perceptions through reason. This is routinely done by referring to other, earlier or later, perceptions; i.e. we do not judge our perceptions through *some other* information source, but through *the very same* information source over time. Then we use inductive logic to draw the most reliable conclusions possible from the totality of information at our disposal.

Science must of course find out *how exactly* this 'opening of the doors' to perception operates, in order to replace, finally and credibly, the current self-contradictory 'representative' theory of perception. Complex conclusive experiments need to be devised to settle the issue. Admittedly, the needed exact explanation of direct perception is not easy to conceive (at least, no one has to date conceived of one). It is a problem of the same order of difficulty as the constant velocity of light was at one time; and a genius like Einstein is needed to find a credible and convincing solution to it. But that direct perception is *a fact* is logically indubitable since the opposite hypothesis of indirect perception is (as already pointed out) self-contradictory, and Berkeley's mentalist option is contrary to the way we commonly regard and conduct our affairs.

The theory that we directly perceive external objects (even if, of course, only parts or facets of them), and not merely mental representations of them, is the theory adopted by people of commonsense everywhere; only some intellectuals cognitively divorced from reality, sundry mystics, and very mentally deranged persons, think otherwise (thus creating the 'mind-body dichotomy problem'). However, this commonsense theory is rightly characterized as '*naïve* realism' if it is not combined with an appropriate *phenomenological approach*. It becomes truly '*subtle* realism' only if and when we understand that it is a consistent construct based on, ordering, and thus explaining, the totality of phenomenal data (inner and outer) that we collectively face. It is through the methodology of adduction (formulating a theory and testing it, ensuring its compliance with logic and empirical appearances) that we validate the conversion of the phenomenological givens into a *direct realist* theory of perception. The result is inductively sure (better than all alternatives), even if not absolutely certain.

4. A soul is needed for cognition

I would like now to draw attention to another contradiction in today's widespread belief system.

Firstly, most of today's biologists, if pressed, would deny that humans or animals have souls; some might rather evade the issue. But this question is, in truth, not even posed in contemporary biology; let alone answered or avoided. It is long forgotten, ignored as a consequence of the alleged refutation by David Hume[8] of the soul or self. Biologists nowadays are thoroughgoing philosophical Materialists.

For them (very briefly and simply put), living organisms are composed of physical matter; which over time combined into more and more complex chemicals, first mineral ones, then organic ones; which eventually, naturally and inexorably, due to appropriate environmental conditions, combined into living cells; which in time joined together and formed multi-cellular organisms; which through further evolution gave rise to diverse forms of plants and animals, some of which survive today.

For current biological science, life is the end result of physical processes inscribed in matter from the beginning. It is itself a physical process and nothing more. 'Vitalism', the assumption that life

[8] Scotland, 1711-1776. In *A Treatise of Human Nature* (1739-40) and *An Enquiry Concerning Human Understanding* (1748).

has something non-physical in it, some substance or essence that distinguishes it from inanimate matter, has long been abandoned. What distinguishes living organisms from inanimate matter is only the complexity and variety of organization and process they involve. 'Spirituality', i.e. the possession of a soul, is likewise an unnecessary supposition, a superstition of no relevance to the scientific study of humans or any other species.

Secondly, most (probably all) biologists today sign on unquestioningly to the theory of sense-perception proposed by John Locke[9] which we delineated and criticized in the previous section. That is, the notion that when we perceive a physical object through the sense-organs in our body, we are perceiving a mental image of that object, not the object itself. According to this idea, we do not perceive the physical world of visual, auditory, and other phenomena, but only (at best) mental reproductions of these various qualia. Not physical sights and sounds, smells, tastes, touch-sensations, but mental equivalents of them, presumably caused by them, brought to our minds through the sense-organs. Most biologists go further and deny that mental images are reproductions of material phenomena, at all resembling them, even if caused by them.

As already argued above, that hypothesis is self-contradictory. Because it starts with the assumption that we have and know of *physical* bodies, graced with various sense-organs, and then ends with a claim that all that we have access to and can know of are *mental* images (using the term 'image' in a broad sense, including sights, sounds, etc.). The contradiction is even greater when the mental images are additionally claimed not to resemble the physical things they are supposed to represent, since one can wonder how they can know that if they only have access to internal things and none to external things.

The inconsistency in this 'correspondence' or 'representative' theory of sense-perception was immediately noticed by George Berkeley, who went on to deny the very existence of physical phenomena and acknowledged only mental ones. In his reading, we have no physical bodies, no physical sense-organs, no central nervous system – only mental images which mislead us into thinking that we have these things.

Since there is no way to formulate Locke's indirect perception thesis without self-contradiction, logic requires us to abandon it and replace it with some other thesis. But the problem is, of course, that we have not so far managed to even imagine a possible replacement theory. So, we turn our eyes away from the problem and pretend there is none. As I explained already, this contradiction can only be resolved by admitting that we are able to *directly* perceive physical matter itself somehow, even if the sense-organs evidently play some sort of significant role.

I now want to point out another contradiction – one between the no-soul theory referred to above and the theory of indirect perception of physical objects[10]. One cannot believe in and advocate both these theories at once: they are mutually exclusive. Why? Because ***if there is no soul – no Subject with the power of consciousness watching them – then the mental images allegedly displayed in the mind are of no use to anyone!*** If there is no one to see a visual display in the mind, of what use is it? If there is no one to hear a sound in the mind, of what use is it? Likewise

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[9] England, 1632-1704. In *An Essay Concerning Human Understanding* (1689/90). Note that Thomas Hobbes (England 1588-1679) preceded Locke in describing sense perception as a stimulation by external motions of motions in the sense organs resulting in allegedly 'mental' events, but it seems that for Hobbes the latter were also physical motions and not distinctively 'mental' events (*De Homine*, 1658). So, Hobbes was more materialistic than Locke.

[10] Locke of course still believed in the soul, so he was not guilty of this contradiction. But he was guilty of the contradiction inherent in his theory of sense-perception.

for smells, tastes and touch-sensations. 'Images' (sensations of any sort) cannot be said to 'appear' if there is no one there to experience them; they might 'occur' but not 'appear'.[11]

Think about that; digest it. If we suppose that we are like computers, physical entities with built-in cameras and microphones, and a CPU processing information coming from them: of what use to such an entity would a screen and a loudspeaker be? If there is no one to see the images on the screen or hear the sounds emitted by the loudspeaker, they are useless appendages. Sure, the computer might well usefully record visual and auditory images of itself and its surroundings; but that would be just to store the recordings in its memory bank for future use. It would never actually need to view or listen to the material recorded when the time came to use it, but could and would simply react directly and mechanically to the electronic data constituting it.

The only reason our existing computers have screens and loudspeakers is because *we* are there, sitting in front of them, viewing and listening. It is only the added presence of *a soul* with cognitive powers that makes such accessories necessary. Note well: computers may well need and have physical 'sensors' receiving information from their environment; but they would not have or need anything equivalent to *mental* 'sensations'. They would simply physically store and process the information received as physical signals. This means that the very fact we are presented with mental images (of sights, sound, feelings, etc.) is empirical *proof* that we are souls, entities with cognitive powers.

This is, of course, what René Descartes[12] realized when he declared: "*cogito, ergo sum*" – or "*I think, therefore I am*." ***Sense-organs without mental sensations are conceivable***: they would be possible in a soul-less entity. ***But mental sensations without a soul to watch them would simply be absurd.*** The moment you accept that images are produced in the mind, apparently through the operation of physical sense-organs, you must logically accept that there is 'someone' residing there, watching them. You cannot both affirm the Lockean scenario of sense-perception and deny the soul. Furthermore, the mental phenomena are merely information presented to the soul – the truly *cognitive* event occurs only when the soul is actually 'informed' by this information.

Descartes, of course, wrote some fifty years before Locke did, but he evidently had in mind a similar naive view of sense-perception. He also anticipated Berkeley's skepticism, some seventy years ahead of him, by clearly realizing that such a view of sense-perception signified an unbridgeable *mind-body gap*. This is why he sought a solution to that problem, and came to the realization that the very act of cognition, be it of mental instead of physical phenomena, was indicative of someone (oneself) doing the cognizing. The mere *occurrence* of phenomena, be they mental or physical, would not prove the existence of a self; but the *appearance* of phenomena certainly does, appearance being *cognized* occurrence.

Descartes' solution is not, however, complete, because although he managed to affirm the existence of self or soul, he did not tackle the issue of how the soul could access the body indirectly through the mind if the soul had no direct access to the body. He thought we could somehow infer body (and a larger physical world beyond it) from mind, but he did not clarify just how. This problem remains still unsolved today. Berkeley did not solve it, but merely reiterated it and regarded it as unsolvable. In fact, it seems that Berkeley's view of the self was (though earlier) closer to Hume's than to Descartes' or Locke's: he does not seem to have believed in a real, unitary,

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[11] The same is, of course, true regarding direct perception of external objects, be they physical or whatever. If they are perceived, they are necessarily perceived by someone, i.e. a soul. Perception without a perceiver is an oxymoron.

[12] France, 1596-1650. In *Meditations on First Philosophy* (1641).

spiritual self (i.e. a soul), but rather imagined the apparent self as a bundle of mental phenomena (i.e. a non-soul).[13]

Self-contradictory hypotheses must be rejected and replaced somehow; it is not scientific to ignore logical problems.

5. A biological science that acknowledges soul

Let us now consider **how the idea of 'soul' might be fitted in the narrative proposed by modern evolutionary biology**. This science teaches us that *sensory perception was a product of evolution*. Moreover, the sense of sight evolved relatively suddenly (at the beginning of the Cambrian Explosion, which it effectively caused), whereas the other senses (sound, smell, taste, touch) evolved more gradually. We have much to learn from these findings[14].

In early life forms, which were unicellular (i.e. consisted of a single cell), there might have been simple 'perception' of light, in the sense of an ability to 'detect' the presence or absence of light, or different intensities of light, or the direction of light; but there was no true 'sight' until the eye evolved as a distinct, specialized organ. Similarly for mechanical vibrations, temperature variations, chemicals, and so forth. I have put the terms 'perception' and 'detection' in inverted commas, because it is not sure that what occurred at that stage could be called cognition – it could well have simply consisted of mechanical responses to physical stimuli, which could only be likened to cognition metaphorically.

When life forms became multicellular, true perception could emerge, with some cells specializing as a nervous system and others as sense organs. But such more complex systems could also have involved mechanical responses to stimuli, similar to what a well-programmed complex robot might perform, without actual cognition taking place. It is only as of the moment that certain select organisms acquired a 'self', over and above the physical cells making up their bodies, that they acquired minds (i.e. mental experiences by the self, in addition to physical ones) and cognitive powers (and inevitably volitional and valuative powers too).

It may alternatively be that selfhood (i.e. soul), and minimal degrees of the powers associated with it, were present from the start, even in unicellular organisms. When unicellular organisms joined together to form more complex multicellular ones, in which individual cells became specialized, they collectively acquired a single self, with probably increased collective powers of cognition, volition, and valuation. If that was the case, the evolution of ever more complex multicellular organisms would have merely broadened and sharpened the cognitive and volitional and valuative powers of the self that were inherent in life as such. The eye made possible more precise vision, the ear more precise hearing, smell and taste more precise detection of chemical composition, the sense-organs of touch finer touch sensations, our feet better mobility, our hands better prehension, and the central nervous system provided more varied and efficient integrative potentials.

The self might perhaps be viewed as the unifying force of the body, that which literally integrates its material components (whether the molecules composing a cell, or the cells constituting a larger entity) and gives them a single and common identity; but it certainly cannot be viewed as itself a physical element of the body, or as a mental substance, or as an abstraction or essence, or as an idea or concept. It must be viewed as something concrete (i.e. not abstract) yet distinct from matter

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[13] Note the dates: 1) Descartes, 1641; 2) Locke, 1689-90; 3) Berkeley, 1709; 4) Hume, 1740.

[14] Read for instance the works of Andrew Parker, notably *In the Blink of an Eye* (Sydney, Australia, The Free Press, 2003).

or mind, because only thus could it have the exceptional powers attributed to it, namely consciousness, free will and value-judgment, and those relative to both matter and mind.

It is certainly not some physical stuff lodged somewhere in the body (e.g. the brain) that has these powers, nor any mental stuff; it must be something else entirely, capable of interaction with the phenomena of matter and mind, yet quite distinct from them. As I have argued elsewhere, we might metaphorically refer to the spiritual realm as a dimension perpendicular to the two dimensions of matter and mind. But this cannot be taken literally, because the very notion of dimensions is a spatial one and we have no basis for claiming that soul has spatial extension or location. All we can say for sure is that a soul is somehow associated with the body and mind of the living organism concerned for the duration of its life.

Note well that I am not advocating all living things as having a single kind of soul, with the same powers of cognition, volition, and valuation. What I have said is that each species of organism that has a soul, has one with a naturally determined range of cognitive, volitional and valuative, powers specific to it. And indeed, individuals within a given species are naturally endowed with different ranges of cognitive, volitional, and valuative, powers; and they may even lose some of these powers through some physical mishap in the course of their lives if at all born with them. Moreover, to say that an organism has a soul is not necessarily to claim that its soul can or does exist without its body, as a sort of ghost, before it is born or after its death. This issue is left open for lack of evidence one way or the other. All we need claim is that during their lifetime (certain or all) organisms have a single ongoing soul.

Having said that souls in different species (among those which have soul, if not all of them do), and in different individuals within them, have different scopes and degrees of cognitive, volitional, and valuative, powers, it occurs to me that there might be something, after all, in the Aristotelian idea, which was widely believed until the advent of modern biology, that there are three kinds of soul – the vegetative, the animal, and the human[15]. This doctrine claims a hierarchy of increasing powers (very roughly stated): the vegetative being limited to the powers of growth and reproduction, the animal additionally involving the powers of sense-perception and mobility, and the human adding the power of rationality. I do not here advocate it, but I can see that some such doctrine could reasonably be revived: considering that as organisms become more complex, they acquire souls with increasing powers, it is conceivable that three (or more or less) classes of soul might be defined (assuming such clear lines can be drawn and that all organisms can be fitted in these three classes).

In any case, it is certainly possible to formulate a narrative of life's evolution which includes the emergence (either from the start of life on earth and generally – or alternatively, eventually and particularly) of the soul and its essential powers (including mental experience). Contrary to what many people think and claim, the theory of evolution is not, logically, *inherently* materialist. It can and should be fully adapted to include mind and soul. The important point to note is that an account of this sort (whichever it is) *must* be true, since we (people) are here to prove the fact with our phenomenological experience of matter, mind, and self, implying powers of cognition, volition, and valuation. Introspective experiences cannot just be dismissed and ignored, as some people try to do; it is not scientific to do that. All evident facts must be acknowledged and seriously taken

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[15] Aristotle, I gather, did not view 'souls' as distinct substances inhabiting physical bodies (as Plato did), but rather as the organizing principles of living organisms, their physical structures and processes. Both these views are very different, note, from my notion of soul as self. Plato's soul is a concrete spiritual stuff; Aristotle's is an essence, an abstraction; whereas mine is something else again, undefinable even by analogy because it is so *sui generis*.

into consideration in scientific discourse; we cannot pick and choose, simplifying things as convenient. All complexities must be acknowledged and taken into consideration.

A pertinent question in this context is: as of when in its development does an individual organism's soul come into being? As regards humans, this question is politically charged nowadays, since the answer is relevant to the controversial issue of abortion. The logical answer would seem to me, in view of the above reflections, to be that a human being may be said to have a soul, before birth, at least as of when it evidently has some degree of consciousness and volition. This means: when a fetus has evidently developed sensory and motor faculties, since it responds to perceivable sensory stimuli with voluntary motor responses. After exactly how many days, weeks, or months, that behavior pattern first manifestly occurs is up to medical science to determine.[16]

It might be argued that such sensory-motor behavior is only indicative of an 'animal' soul, not a 'human' soul, since presumably a fetus does not yet show signs of rationality (conceptualization, classification, verbalization, logic, memory, imagination, anticipation, and the like) any more than a recently born baby does (correct me if I am wrong). But surely, the brain and nervous system of a human fetus (and all the more so that of a newly born baby) are at some point all set for rational functioning, so that we can as of that point at least say that the fetus (and a fortiori the subsequent baby) has rational potential. Perhaps this point, if it can be identified, should be viewed as the true threshold for specifically human identity.

6. Three principles for biologists to ponder

Allow me, as a result of the above reflections, to formulate **a number of basic principles for the advancement of the science of biology**, to help it evolve from its current narrow, materialist perspective to a more advanced, enlightened one. Modern biologists are scared to even confront these issues for fear of being regarded by their colleagues as unscientific; but a true scientist takes all available information into account and courageously follows reason.

1. There is no possibility of consciousness without selfhood.

Wherever consciousness is evident, selfhood must be present. That is, any living organism which displays evidence of any degree or extent of consciousness, in its structure (notably, sensory organs, a nervous system, a motor system) or its behavior patterns (notably, movement apparently of its own accord, without wind, water currents, or other mechanical forces impinging on it), must be assumed to have *a self – meaning a 'soul', a 'spiritual' presence, a 'personality'*. Consciousness is inconceivable without a cognizing Subject as well as a cognized Object. Consciousness necessarily implies someone being conscious, i.e. an entity doing the cognizing, and a content of

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[16] As of then, therefore, deliberately killing the fetus (voluntary abortion) could well be considered as murder of a defenseless creature, albeit one residing within the body of another (the mother) and still in need of the hospitality of that host to fully develop. Women with unwanted pregnancies argue that they have the right to freely choose to rid their sovereign body of the unwelcome guest. This is obviously a complex ethical issue, pitting the interests of two creatures against each other. It cannot be adequately dealt with in a mere footnote; but my opinion is, briefly put, as follows. How did the 'guest' get into the 'host'? If the woman is pregnant through *voluntary* sexual intercourse, she is morally responsible for the consequences; if she foolishly allows the fetus time enough to develop a soul, she cannot thereafter kill it at will on the ground that it is something found within her body against her will. However, if she was raped, she is of course free to rid herself of the foreign body *forcibly* injected into hers by some rapist, even if she discovers her pregnancy after the fetus has developed a soul. Abortion would also be quite ethical, if the mother is seriously physically endangered by the pregnancy or if the fetus is tragically very abnorma1 – though in such cases doctors and judges would have to be consulted to estimate whether interruption of pregnancy is reasonably necessary or at least recommended.

consciousness, i.e. something being cognized. These are not merely verbal, grammatical, necessities, but logical, conceptual, necessities.

2. Consciousness also necessarily implies volition and valuation.

There is no possibility of a living organism with evident powers and exercise of consciousness, to whatever degree and extent, that does not also have powers and exercise of *volition (that is, free will)*, to some degree and extent, and powers and exercise of *valuation (that is, value-judgement)*. Consciousness by itself, witnessing the world passively, would have no biological utility to a living organism, and would thus be rapidly eliminated by evolution if it perchance arose.

The biological utility of consciousness is that it makes possible the ability to actively respond, through some sort of self-generated action, to whatever is cognized in the environment of the self. If a living organism is found to move of its own accord, i.e. evidently not through the agency of an external mechanical force, it can and must be assumed to have volition to that degree and extent, especially if its physiology includes organs of motricity (nerves, muscles, arms, legs, etc.).

Moreover, consciousness and volition imply valuation: without some sort of value-judgment (opting for some things or courses of action as favorable to self in some way, and avoiding others as unfavorable to self), which need not be verbal and rational, but may be instinctive or emotional, the environmental information gathered by consciousness would be useless, and any action taken by the organism would be capricious, devoid of useful direction, and eventually surely self-destructive.

3. Therefore, selfhood has three intertwined powers: cognition, volition, and valuation.

Every living organism that displays any of these three powers necessarily has all three, and a self that exercises them. A ‘self’ means a ‘soul’, and it can mean nothing else. It is possible that there are primitive living organisms that do not have self and its three functions, but function on a purely mechanical action-and-reaction plane. But it is certain that, at the other end of the spectrum, there are living organisms – namely, at least ourselves and the higher animals – which *do* have the three said powers and the self that they logically imply. It is up to biological science to determine empirically precisely where the line, if any, must be drawn. What is sure, however, is that we, human beings, evidently have self, and the powers of consciousness, volition, and valuation; and evidently at least some other animal species do. Without admission of this truism, biologists cannot truly claim to be scientists because they function in an intellectual straitjacket divorced from some significant facts.

To be perfectly clear: anyone who thinks otherwise, who thinks there is no such thing as a self/soul, no consciousness, no volition, no valuation, is engaged in *self-contradiction*, since he or she is evidently *a person thinking* these claims, and probably also saying or writing them for other people to think about them. Self-contradiction is not ‘scientific’. As Descartes rightly reflected: My thinking is plentiful *proof* of my existence. Ignoring manifest information is not ‘scientific’. It is impossible to perceive or conceive anything, to formulate any thought about anything, without consciousness, the power and act of cognition necessary to all thinking; without volition, in this context the power and act of moving one’s attention through the necessary thinking processes; or without valuation, in this context the ability to estimate what is true or false, a thought to be retained or one to be dismissed.

So long as biologists do not accept, and take into full consideration in their research and theories, the facts of self, and its faculties of cognition, volition, and valuation, they are engaged, at best, in poor science. When they get the courage and integrity to finally do so in earnest, their science will surely be greatly enriched.

7. Selfhood is the basis of morality and society

In my book *Meditations* (2006), I argued that the self must be assumed to be a unity persisting during the whole life of an organism. Those reflections are very relevant to the present context.

> According to Buddhist theory, the self has no continuity, i.e. our self of today is not the same person as our self of yesterday or of tomorrow. In this perspective, they are causatively connected, in the sense that earlier conglomerations of phenomena constituting a self 'cause' later ones – but there is no thread of constancy that can be identified as the underlying one and the same entity. It is not a case of mere succession of totally discrete events; but there is no essential identity between the events, either.
>
> However, many (myself included) object to this theory on various grounds. While we may admit that one can logically regard selfhood (i.e. being a Subject and Agent) as punctual at every instant without having to assume its extension over a lifetime, we must realize that such an assumption removes all logical possibility of a concept of moral responsibility for past actions.
>
> If one is no longer ever the same person as the person committing a past virtuous or vicious act, then no good deed may be claimed by anyone or rewarded, and no crime may be blamed on anyone or punished. Ex post facto, strictly speaking, the doer of any deed no longer exists. Similarly, looking forward, there is nothing to be gained or lost by any Agent in doing anything, since by the time any consequences of action emerge the Agent has already disappeared.
>
> In such a framework, all personal morality and social harmony would be completely destroyed. There would be no justification for abstaining from vice or for pursuing virtue. Even the pursuit of spiritual realization would be absurd. Of course, some people do not mind such a prospect, which releases them from all moral obligations or responsibility and lets them go wild.
>
> It is very doubtful that Buddhism (given its overall concerns and aims) supports such a nihilist thesis. In any case, such a viewpoint cannot be considered credible, in the light of all the above observations and arguments.

If we deny the self or soul, we deny, not only the functions of cognition, volition, and valuation, but morality and society. Without the continuity provided by the self, there is no responsibility. Without responsibility, there are no values, no virtues, no disvalues, no vices, no moral imperatives or prohibitions; anything goes. Without morality, not only is the individual unable to function intelligently, but society, the peaceful coexistence and cooperation of people, cannot exist. Social cohesion depends on respect of the individual, respect of his or her person, body and mind, and material and intellectual property. Without such respect, anyone can kill or maim anyone, torture or enslave anyone, steal their property, at will. There would be no justification for any laws or ethics.

The very facts that we have individual moral consciences and that more or less organized societies exist are proofs of our awareness that we have abiding souls, powers of consciousness, free will, and rational valuation, and consequent responsibilities. Even if the consciences of individuals vary considerably, and our societies are diverse and imperfect, the very existence of these psychological and social phenomena demonstrate that the human race, fundamentally, believes in soul, its powers, and its responsibility. The people who deny these things are just pretending; they are not honest witnesses. Sure, there are among us psychopaths and sociopaths; but these are exceptions, not true representative of humanity. And anyway, one may well wonder whether the psychopaths

and sociopaths among us can be so characterized entirely – i.e. whether they are that sick or evil exclusively. More probably, just as they have human bodies and faculties, they have partly functioning souls.

'First philosophy'[17] is concerned not only with the relatively abstract disciplines of philosophy of mind, phenomenology, and the theory of sense perception, but more concretely with biology and evolution, and with anthropology and sociology.

[17] The term was, I believe, coined by Aristotle and revived by Descartes.

2. CAUSALITY FUNDAMENTALS

1. The four main concepts of causality

We all frequently engage in causal thinking, looking for or claiming to have found the causes or effects of something, whether for personal purposes or sundry scientific ones. Our life would be impossible without such thought processes; we would be left blind, unable to understand what goes on and unable to respond. However, most people are not able to say precisely what is meant by the terms 'causes', 'effects', and 'causality'.

These terms have, in fact, many meanings. Knowing their various meanings, and the methodology appropriate to each, can considerably improve our ability to find what we are looking for. The concepts of cause and effect were, to begin with, naturally, rather vague and intuitive. But gradually, thanks to the thoughtful work of many philosophers and logicians, many of whom were primarily natural scientists, these concepts have in time become clearer and more precise. In my original works on this subject, I push the effort of clarification and precision to new heights through systematic treatment.

My object in this essay is to present as briefly as possible some of the main concepts, formal analyses, and explications, concerning causality that I have proposed over the years in some of my major works, namely *Future Logic* (FL, 1990), *The Logic of Causation* (LC, 1999-2010), *Volition and Allied Causal Concepts* (VACC, 2004), *Logical and Spiritual Reflections* (LSR, 2008). This essay is intended as a short introduction to the subject of causality, and to the main tools of causal thinking, to encourage people interested in these issues to study my more detailed works. These works, though very innovative, have unfortunately not to date been as widely studied as they deserved to be.

Causality refers to the *abstract relation* existing between a cause and its effect(s), or an effect and its cause(s). It is difficult to define because the varieties are technically so different that we are hard put to say just what they have in common.

The study of causality is called ætiology[1]. This is a branch of philosophy. It includes both descriptions of the ways of causality (*ratio essendi*) and descriptions of the ways to our knowledge of it (*ratio cognoscenti*), i.e. both the ontology and the epistemology of causality.

The four main concepts of causality are: **causation (deterministic causality) and natural spontaneity (non-volitional indeterminism), and volition (free will by human or other souls) and influence (things intuited, perceived or conceived, making will easier or harder, without however annulling its freedom).** We use the term 'causality' to signify a genus for these four species of cause-effect relations.

[1] Many now spell this 'etiology', but I prefer to retain the older spelling which reflects the Greek etymology (*aitia*, meaning cause).

2. The four determinations of causation

Causation, which is deterministic causality, involves four generic *determinations*, two strong and two weak[2]:

- **Complete** causation (symbol **m**): If C, then E; if notC, not-then E; where: C is possible.
- **Necessary** causation (symbol **n**): If notC, then notE; if C, not-then notE; where: C is unnecessary.
- **Partial** causation (symbol **p**): If (C1 + C2), then E; if (notC1 + C2), not-then E; if (C1 + notC2), not-then E; where: (C1 + C2) is possible.
- **Contingent** causation (symbol **q**): If (notC1 + notC2), then notE; if (C1 + notC2), not-then notE; if (notC1 + C2), not-then notE; where: (notC1 + notC2) is possible.

Note the precision of these definitions and the symmetries in this list. Causation is defined with reference to positive and negative hypothetical (or conditional) propositions. A proposition of the positive form 'if X, *then* Y' means 'X without (or not followed by) Y is *impossible* (in the mode of modality concerned)'; a proposition of the negative form 'if X, *not-then* Y' means 'X without (or not followed by) Y is *possible* (in the mode of modality concerned)'.

Note that the two strong determinations are composed of two hypothetical propositions, one positive and one negative, plus a modal categorical; the two weak determinations involve one positive and two negative hypothetical propositions, plus a modal categorical[3]. All the stated elements are needed to define each of these compounds; no element is redundant. Since no other determinations of causation are logically conceivable, this list is exhaustive.

Further analysis proves that *only* four specific combinations of these four generic determinations are logically consistent, namely: **complete-necessary** causation (**mn**), **complete-contingent** causation (**mq**); **necessary-partial** causation (**np**), **partial-contingent** causation (**pq**). These **four joint determinations** are (to repeat) exhaustive, and (moreover) mutually exclusive, so that we may safely say that: *nothing can be said to be a cause or effect of something else, in the causative sense, if it is not related to it by way of one – and indeed, only one – of these four species of causation.*

Having thus arrived at an exhaustive listing of the forms that causation may logically take, we can gradually develop a thorough logic of causation. This must begin by interpreting the specific negations of the four forms of causation, and thence the general negation of causation. Note well that we cannot theoretically define **non-causation** except through *negation of all four forms of causation*, which have to be defined first"[4].

Next the oppositions and eductions of the four forms are investigated, and various issues such as parallelism of causes and causal and effectual chains can be discussed. Thereafter, *syllogistic*

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[2] The symbols **m**, **n**, **p**, **q**, are derived from the words co**M**plete, **N**ecessary, **P**artial, and **Q**ontigent. Note in passing that a complete cause is often, in common discourse, referred to as a sufficient cause; the two terms mean the same. The word 'cause' may, in the context of causation, be replaced by the more specific word 'causative'. Here, the symbols C and E refer, of course, to *putative* causes and effects until they fall within one of the valid definitions of causes and effects.

[3] The positive hypothetical alone cannot define causation, since formally it could be combined with another positive hypothetical with the same antecedent and the opposite consequent; the negative hypothetical(s) is (are) needed and used to eliminate this logical possibility. The modal categorical ensures that the antecedent C or notC, or (C1 + C2) or (notC1 + notC2), has a basis. In the event of partial or contingent causation, the terms C1 + C2, etc. refer to combination of only two causes; but when there are more than two they can gradually be redefined as two.

[4] The importance of this statement is that all past attempts to deny causation (such as Nagarjuna's or David Hume's) are worthless, since they do not even first accurately define what it is they are trying to deny.

reasoning involving causative propositions can be systematically investigated. This is to guide us in deduction and to ensure consistency in one's knowledge. The formal solution of all deductive problems is by no means easy, and requires successively more and more complex techniques[5].

As regards **the induction of causation**, this seemingly difficult problem is reduced to the easier problems of induction of the conditional and categorical propositions which together make up a causative proposition. To fully understand the issues involved in induction of causation, it is first necessary to realize that there are different *modes* of causation, corresponding to the different modes of modality. This means that *there are four full determinations of causation for each of the modes of modality*.

In the above definitions, the if-then statements are not intended as exclusively logical, but as generic, being applicable to logical conditionals, extensional conditionals, natural conditionals, and temporal (and similarly spatial) conditionals. The deductive and inductive formal logics of these various modes of conditioning are treated systematically and in great detail in FL. The work of clarification and validation of induction of causation is thus already formally done.

Briefly put, we can describe the induction of causation, i.e. of a proposition like 'A causes B', as follows. Consider, say, two events A and B. If A is always *in our experience so far* found to be followed by B, i.e. never followed by not-B, then we can by generalization induce (i.e. infer by inductive logic) that A implies B, *until if ever* we come across even a single case of A *not* followed by B, in which case we must particularize our previous proposition and conclude instead that A does not imply B[6]. The same process applies to the negations of these same terms, viz. to not-A and not-B, which also need to be considered before any sort of causation can be declared. If not-A is always *in our experience so far* found to be followed by not-B, i.e. never followed by B, then we can by generalization induce that not-A implies not-B, *until if ever* we come across even a single case of not-A *not* followed by not-B, in which case we must particularize our previous proposition and conclude instead that not-A does not imply not-B[7]. Once we have settled both these issues, one way or the other, and combined the results, we can formulate a causative proposition of whatever degree.

The induction of non-causation, i.e. of a proposition like 'A does *not* cause B', is much more difficult. Granting we have by the above said means arrived at the conclusion that 'A does not imply B and not-A does not imply not-B', we might well first conclude that 'A is a partial and contingent cause of B' before concluding that 'A does not at all cause B'. The former, more positive conclusion presupposes that there are some additional causal factors involved, which we have already identified or expect to readily identify; whereas the latter, negative conclusion presupposes that additional causal factors were diligently sought but couldn't be found. In either case, the conclusion can only be inductive, as it could easily be overturned at some later stage.[8]

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[5] This work takes up most of my work *The Logic of Causation* and need not concern us here.

[6] Needless to say, if we know from the start that A is *not* always followed by B, we do not bother generalizing and particularizing, but go straight to the negative conclusion.

[7] Again, if we know from the start that not-A is *not* always followed by not-B, we do not bother generalizing and particularizing, but go straight to the negative conclusion.

[8] I would like to one day develop software for causative logic, made available online, which could be used by scientists and everyone else to determine the best logical conclusions about causation given certain combinations of raw data, based on my detailed work in *The Logic of Causation*.
a) In any given knowledge context, the information currently available concerning three variables (P, Q, R) or four variables (P, Q, R, S), or more, would be introduced, and the program would determine *which* causative relation is the most fitting. Until new data is found, the chosen causative relation (**mn**, **mq**, **np**, or **pq**) would be declared the most probably true one.

We thus arrive at a full and indubitable development of the concept of causation. This permits us to examine and if necessary to criticize past and present discourse regarding causation with masterly finality. Some past and current works on this subject deserve attention and praise, even if they are not exhaustive. Some past and current works deserve criticism and can be rejected in a decisive manner once and for all. We thus move from mere opinion and controversy to genuine scientific discourse.

One thing that the above-described analysis proves authoritatively is that *causation exists and can be known*. It is not a figment of our imaginations, as some (notably David Hume, as we shall see) have contended. The forms of causation proceed *logically and inevitably* from modality and the categorical and hypothetical forms that naturally emerge from it which are the bases and means of all rational discussion. No one can attack these rational means without using them and thus putting his own discourse in doubt and indeed invalidating it. It may in the past have seemed possible to criticize causation when we had no precise idea of its exact nature; but now that we know precisely how it is constructed out of simpler discursive elements, it can no longer be doubted or rejected without self-contradiction. Skepticism regarding causation's very existence and knowability is no longer an option.

Furthermore, although causation is something *abstract*, it is still something *objective*, i.e. it exists independently of our knowledge (or not) of it. Immanuel Kant's attempt to respond to Hume's skepticism by proposing that causality is a 'category' imposed by us on empirical data implied causality to be something subjective. This was not a solution, but an additional layer of error. Both these philosophers have had a devastating effect on the theoretical understanding of causal thinking.

Another important finding is that the much touted 'law of causation', according to which everything must have a cause, in the deterministic sense of the term, is *not* formally guaranteed. It may be true; but then again it may not. We cannot, through our logically established definitions of causation and non-causation, prove that spontaneity is impossible; nor for that matter that it is possible. Thus, logic must make *formal allowance* for the idea of *spontaneity*, i.e. of natural spontaneity and/or human volition (freewill). Then, when we speak of causality in the widest sense, we should bear in mind not only causation (the deterministic variety of causality), but also natural spontaneity and volition (the indeterministic varieties)[9]. In that case, we can formulate ***a more open 'law of causality', such that everything must be caused in one or more of these three senses***.

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b) Then, given two or more such probable causations, arguments would be used by the program to draw conclusions. These conclusions would predict the probable causative relations between items for which we did not previously determine the relation in the manner (a).

c) At this point, we would want to empirically verify the latter findings. If they turned out to be wrong (i.e. if the outcome of an (a)-type proposition is not in accord with the outcome of a (b)-type inference) we would of course conclude that one or both premises giving rise to our conclusion in (b) needs revision.

d) In such event, we would be called upon to do further empirical research on the doubtful items. In this way, the program would generate consistency checks and when necessary further research.

Not being able myself to produce such software, I am hoping someone else will eventually do it. Or maybe the job will eventually be easily done using AI.

[9] Here regarding the causal relation of 'influence' as a concept within the domain of volition, because it only has significance in relation to volition.

3. The logical possibility of natural spontaneity

As it turns out, natural spontaneity is advocated by modern physicists in the field of quantum mechanics. So, the theoretical allowance for it by aetiology provides a rational basis and a language for the putative phenomenon. This concept has no positive definition, but is *defined by negation*, note well. Events subject to natural spontaneity would be events without any discernible complete causative (deterministic cause), events not subject to natural law – and, of course, not products of human (or more broadly animal) volition. Thus, natural spontaneity refers to *chance* – chance in a real and deep sense; it implies that we cannot make an exact prediction, even if we may be able to estimate probability, of occurrence.

Granting the existence of natural spontaneity, it should be regarded – together with causation – as a *mechanical* event; a sort of determinism, in an expanded sense of the term, because it too involves no will.

Logic cannot, of course, affirm the existence of such pure chance events *a priori*; but nor can it deny them offhand. It can only admit the possibility of *a posteriori* proof, directly or indirectly, by scientific observation/experiment and credible ad hoc theorizing efforts. The truth is that it is notoriously difficult to definitively empirically prove that an event is naturally spontaneous, for we may always imagine that perhaps it is due to some underlying deterministic cause(s) that we have not yet discovered[10]. Even if today we do not find a sufficient cause for the observed event, and must assume natural spontaneity, we may conceivably at some future time discover some new physical phenomena that allow us to find a sufficient cause for it. The assumption of natural spontaneity is always, necessarily, inductive.

In any case, to repeat, natural spontaneity would qualify as mechanistic, in the sense that we humans have no power over it and cannot control it, i.e. that it is not to any extent due to volition (see below). However, on the basis of the principle of induction, if we have diligently searched, in a presumably mechanistic domain, for a deterministic cause (to wit, a complete/sufficient causative) and not found one, we can reasonably say that there may well be natural spontaneity in this specific context. This does not establish the possibility of natural spontaneity with deductive force, but it does establish it inductively (*until, if ever,* physicists change their minds concerning it).

This seems to be the methodological argument behind the general acceptance of the Heisenberg Principle in quantum mechanics. This Uncertainty Principle, as it is also called, may be briefly formulated as follows:

> "The more precisely the position of a particle is known, the less precisely its momentum can be known, and vice versa. To measure position, we need to interact with the particle in some way. This interaction will inevitably disturb the particle's momentum, making it more uncertain. Similarly, to measure momentum, we need to interact with the particle in a way that will disturb its position."[11]

Some argue that because the observer's attempt to find out the physical fact may, at those levels, affect that fact, it is appropriate to speak of uncertainty. However, I would reply that the researcher's possible effect on the fact is still of a physical nature and is not about consciousness.

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[10] This argument was already put forward long ago by the Stoic **Chrysippus** of Soli (Greece, 279-206 BCE), who "taught that apparent causelessness in a particular event could mean only a kind of causation hidden from human insight" (Windelband, p. 181). Chrysippus was (like Democritus, but unlike Aristotle) a thoroughgoing determinist, who made no allowance for chance or voluntary events.

[11] Quoting Gemini. This suffices for our purpose here.

The phenomenon itself is quite physical, even if it results in unpredictability. The principle concerns physical processes, and not primarily the mind's relation to the physical world. Thus, the principle is not an epistemic one, as the word uncertainty suggests, but more precisely put an ontical one, so that the name Indeterminacy Principle would seem preferable.

The Heisenberg Principle is due to the wave-particle duality of matter. Quantum physicists have discovered that all particles have both wave-like and particle-like properties. In certain physical circumstances an electron has the detected characteristics of a particle, and in other physical circumstances it has the detected characteristics of a wave. The fact that these physical circumstances may happen to have been generated by a human being in the context of a scientific experiment, and that due to such unexpected results, the researcher is unable to determine whether the electron is a particle or a wave, does not make this experiment anything to do with "the observer" (as it is usually presented).

The act of observation in such experiment is quite incidental to the unpredictability of the electron's physical behavior. The mind is no more involved here than in any ordinary physical activities, such as a man driving a car or playing tennis; once put in motion by human will, the car and the tennis racquet are entirely subject to physical laws. Similarly, in the above-mentioned quantum physics experiment, the experimenter drives the machinery that sends out an electron, and reads the machinery that tracks the electron's behavior, but the mind is not especially involved, i.e. the process observed is entirely physical.

I am not a physicist and have no axe to grind in that field: I always accept whatever physicists consider the best hypothesis at any given time. My concern here is only with the *philosophical spin* applied to, and the language consequently used for, certain physical discoveries. There is a perverse desire, in some circles, to deny objectivity and affirm subjectivity. This fashionable epistemology projects a wholly fallacious interpretation on the physical events observed, concluding that in quantum mechanics experiments the observer *as such* somehow affects the result. This is a false, tendentious conclusion from the given data, a misuse of the facts of the case to support an anti-rational philosophical posture.[12]

We can well admit natural spontaneity, without taking it to have irrational implications.

4. Volition and influence

Volition, or freewill, is causality by an *unmoved mover*[13], meaning a self-moving entity, something moving literally by itself, without being immediately forced or indirectly programmed to do so either by a complete causative or by natural spontaneity (or by another unmoved mover). An event occurring through volition has a cause – namely, *the agent* of that volition, i.e. the entity that freely willed the event. The cause of an act of will (or volitional act) is not any thought or property of the agent; but the agent personally. Moreover, *it is not 'the will' that wills: it is the agent that wills*. The term 'willing' (or volition) refers to the causal *relation* between the agent and the internal or external product (effect) of his act of will; it does not refer to the cause or to the effect being thus

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[12] Consider for instance the following citations: "The stuff of the world is mind stuff" (Sir Arthur Eddington, in 'The Nature of the Physical World'). "For what quantum mechanics says is that nothing is real and we cannot say anything about what things are doing when we are not looking at them" (John Gribbin, in 'In Search of Schrödinger's Cat'). Both proposed by Wong Kiew Kit, in *The Complete Book of Zen* (UK: Vermilion, 2001), p. 104, in support of his belief that modern science agrees with Buddhist mind-only claims.

[13] 'Unmoved mover' is, of course, a phrase drawn from Aristotle (e.g. in *Physics*, 8:5).

related[14]. Note that the term 'spontaneity', when used in the context of volition, refers to willing without purpose (i.e. without any influence for or against – see more on that below).

Under a mechanical regime, of causation and/or (if that exists) of natural spontaneity, a static thing can completely cause another static thing or partially cause a dynamic thing, and a dynamic effect can only emerge from a complete or partial dynamic cause, or from no cause at all. Not so under the non-mechanistic regime of volition; here, a relatively static source (the willing agent) produces a dynamic result (certain willed movements or changes and static phenomena emerging from such movements or changes). This is one important differentia of volition from causation and natural spontaneity, although the agent can well also be said to be in part, but not only, a necessary causative of its volitions (since without that particular agent those particular volitions would not occur), and human spontaneity does have some resemblance to natural spontaneity (although the causelessness involved is of a different sort).

Another important differentia is that the agent of volition is free of determination in its (or his or her) act of will: it is an unmoved mover *within the limits of the specific powers of self-movement granted by its nature*. Volition means freewill; there is no such thing as volition without freedom of the will. To be precise, in volitional acts, it is not the will that is free, but the agent of the will who is free. But this should not be taken to imply that the agent's freedom is unlimited; volition (at least that of humans and animals) is always circumscribed by some natural and circumstantial bounds.

Freedom of the will means that nothing can directly or indirectly force the agent to perform or not-perform its will (given that it has the requisite power in the specific case at hand); willing is thus markedly different from deterministic causality (causation). Willing also differs from natural spontaneity in that the former involves an agent whereas the latter does not ('Nature' is not literally an agent, even though we often speak of it as if it were, thus reifying it). This does not mean that the agent is never subject to overwhelming force, whether through causation or natural spontaneity; it means that when and where the agent is evidently literally *overpowered* to do or not-do something, what results is not volition but mechanical action.

To be sure, volition does not imply *unlimited* freedom. The agent of volition is usually if not always subject to varying degrees of influence. An **influence** is something *whose cognition by the agent* makes the agent's act of will *easier or harder* at the time concerned; that is my definition of influence. Consciousness is central to it, note well. Without this concept of influence, we cannot understand and accept the concept of freedom of the will, in view of the evidence that we are indeed 'affected' by things in us and around us when we will. Together with causation, natural spontaneity and volition, influence is one of the four distinct, irreducible, and complementary types of causality.

There are many sorts of influence: all sorts of external objects; bodily events, states or sensations; mental events, states or emotions; memories, imaginations, abstract thoughts, rational decisions, ethical doctrines, and so forth. Note that the concept of influence includes, but is wider than, that of purpose. The thought of a certain goal *influences* the agent to perform a certain action by means of will – in that sense, Aristotle's 'final cause' is actually a sort of 'efficient cause', though more specifically (note well) in the sense of influence. But objects of consciousness other than goals can also influence volitions.[15]

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[14] Contrary to the impression given by the ways we often word things in everyday thought and speech.

[15] For references to Aristotle in the present volume, see *The Works of Aristotle* (2 vols. Ed. Jonathan Barnes. Princeton, N.J.: Princeton UP, 1984).

An agent can also, it seems, engage in acts of will that are devoid of teleology, i.e. devoid of the influence that a goal or motive or desire or aversion constitutes; this refers to will that is not only free, but also spontaneous. We call this impulsiveness or caprice. Although this is conceivable, and apparently occurs occasionally, most volition occurs under many influences, pro and con.

An influence is *never* determining, note well; but it can make things so extremely easy or hard that it *seems almost* determining. Influence implies a *probability* of occurrence or non-occurrence. For this reason, we can somewhat predict human behavior, with reference to group statistics. Such predictions cannot be certain, but they can be indicative. However, if the putative influence is found to be literally 100% facilitating or impeding, it *cannot* be called an influence, but must be regarded as a complete causative, a determining cause. Influences are necessarily, by definition, less than 100% effective.

Influence, also note well, occurs through *consciousness* of an object (of any sort) by the subject (agent). If consciousness is *not* involved, i.e. if the object referred to as an influence affects the observed result without having been cognized to some degree by the subject, the putative influence *cannot* be so called, and the observed result cannot be said to have occurred through volition – it was deterministically caused.

Many people have doubts concerning the possibility of volition because they have not understood the nature of influence and so *tend to confuse it* with determinism. They think that the will cannot be free if it is influenced; they think influences are determining, i.e. that they are causatives. Though an influence might be viewed as a partial cause of a willed effect, next to the agent, the power of the influence is necessarily through the consciousness of the agent, taking the influencing object into consideration (wanting it, not wanting it, fearing it, hoping for it, being informed by it, being misled by it, etc.).

Influences motivate, by subjective attraction or repulsion, but the agent remains free to choose in spite of them. The influence does not literally drive the agent to act or not-act in a certain way, but only affects the agent's decision-making by making this choice easier or that one more difficult, i.e. by modifying the subjective *effort* involved in the act of will. The influence merely increases or diminishes the effort that a choice involves, for this agent, at this time. If the agent musters the effort needed to overcome all contrary influence, he exercises volition, whether successfully producing the intended result or not.

But the agent can also remain will-less in the face of contrary influences; in such an event we must conclude that he did not have the willpower needed to go this way or that, effectively letting things happen however they fell without his interference. Such *inertia* based on indecisiveness may be characterized as weakness of will, because the inaction is effectively acceptance of the easiest course of events. Passivity is a choice of sorts, one that many of us often make. Note that wishing is not willing – one may want something, wish for it to happen, but do nothing to make it happen.

There are many categories of influence. An important one of these is *instinct*. An instinct is a genetically programmed preference, presumably resulting from evolution, which in principle favors the survival and health of the individual organism or of its species, though in particular cases or circumstances it might have contrary effects. Instinct is not antithetical to volition: it does not literally force a certain behavior on the organism, but strongly influences the direction of its choices, providing it with a 'default' almost automatic first choice, without however making other choices impossible if they are willed. *Emotions* (be they gut feelings or more subtle mental phenomena) have instinctive influences; for example, fear may generate fight or flight or impotent freezing in place. Personal *habits*, on the other hand, are acquired influences, not instinctive ones;

still, they affect behavior strongly and are difficult though not impossible to overcome like instincts.

For human beings, another important category of influence is *reason*. This refers to thoughtful deliberation on courses of action, with consideration of ways and means and their possible consequences, with some appeal to experience and some use of logic, but also some influence by emotion, usually with attention to certain principles personally or socially taken for granted. Context of knowledge – intuitive, perceptual, and conceptual – has a strong influence on rational decision making. Social influences play a big role in the behavior of people (especially the young), of course. There are the influences of the family and immediate entourage, but also nowadays strong influences from society at large through schooling and the various media (written or visual and auditive), which are often politically motivated.

Evidently, different individuals respond to these various influences to different degrees, and the same individual may respond differently at different times. Some people are more instinctive, some more emotional, some more subject to habit, some more rational, some more scientific, some more mystical, some more poetic, some more influenced by family or some other segment of society, and so forth. Some, less. Men and women, adults and youths, seniors and children, people of different educational levels, cultural frameworks, or periods of history, and so forth, display different behavior patterns, broadly speaking. Much depends on each person's natural endowments, his or her innate physical and psychological facilities and constraints, notably the degree of intelligence. Also, of course, different animal species respond to influential stimuli differently. Obviously, humans are more rationally inclined, other animal species more instinctive. All such differences can be established through empirical statistical studies.

In any case, to repeat, instinct is not contradictory to free will; it influences choices but does not determine them. Humans, though largely moved by free will, also have instincts. Most mothers and fathers have parental instincts, though some occasionally willfully hurt or kill their children. Our instinct is to preserve our health, but some people choose to smoke tobacco or drink much alcohol or use psychotropic drugs. And so on.

I believe that all animals, including even insects, have both free will and instincts, though to vastly different degrees than we do. They are not robots, they have souls of sorts, since they evidently have cognitive and volitional powers, and appetites and aversions. Anyone who has owned a dog or cat, or has had extended contact with any other species of animal, knows that to be obvious. Of course, animals are much more instinctive and less free than we are, but they also display some ability to move with other motivations. A cat can befriend a pet bird in the same household, for instance. At the lower end of the animal spectrum, while a fly is undoubtedly mostly moving in accord with its instinctive behavior patterns, and has very little free will, almost none, nevertheless, it can be assumed to have just enough to choose to move this way or that for no particular reason (it has to move in some direction eventually, but which direction it moves in is not given it).

What sort of thing is an *agent* of will? We know of the existence of will through personal introspective observation, i.e. intuition; and also through things relatively external to the agent, namely perception of mental or physical events and by subsequent conceptual means. **I know I will; I know when I will and when I don't will; I know what I will or don't will; I very often, but not always, know why I will or why I don't; and I usually know whether, or to what extent, my will was successful or not.** It is a fact that we are able to and routinely do make these distinctions through self-awareness and introspection. If anyone truly lacks this self-awareness (which I doubt), they surely have a serious psychological problem.

Who am 'I'? I am not primarily *the physical body* I apparently reside in; nor am I *the mental phenomena* that I perceive and conceive inwardly (my experiences, memories, imaginings, thoughts, dreams). I am **the soul**, the spiritual entity, at the 'center' of my being; the soul is myself. It is the soul that wills; the soul is the free agent of all volitions emanating from it. The soul is fully responsible for all its volitions, no matter how much influence has impinged on it, because it has the final say, and can choose to do or not-do what is in its power to do or not-do, however easy or difficult it gets, and however successful or unsuccessful the result.

Volition is one of the three distinctive powers of the soul – the other two being *cognition* (consciousness of anything by whatever means) and *valuation* (ascribing value or disvalue to things cognized). Volition cannot be understood without reference to cognition and valuation. A soul has these three means of relating to the world: cognition, volition and valuation; and these three functions of the soul, as well as the soul itself, are known to it primarily through (I use this term here in a restricted sense) *intuition*; such intuition (or apperception) is *direct self-knowledge*.

It is impossible to fully define any of these four spiritual items (soul and its three said functions), because each of them is metaphysically *sui generis*. Soul is nothing like any mental or physical phenomenon that we routinely perceive; we cannot honestly say what it is made of or how it is constructed or how it operates or even where it is, and any attempt to do so can only be *rough analogy* for pragmatic purposes. Taking such analogies too literally can only result in error.

Similarly, cognition and volition cannot be reduced to anything else, because they are so ontologically unique. We can, however, observe that nothing (no living organism, no animal) has cognition without volition, or volition without cognition. Valuation likewise implies both cognition and volition, before, during and after its occurrence. In our experience, there is no soul without cognition and volition and valuation, and these three functions are only possible to and necessarily imply (are inconceivable without assuming) a soul.

When we consider what entities have soul and its three functions, this observation is seen as logical. In the world of our experience, all humans, and (to a lesser extent) higher animals and perhaps lower animals too, though to a much lesser extent,[16] seem to have cognition (since they manifestly have *means of* cognition such as sensory equipment and a nervous system), volition (since they can move without being moved, unexpectedly and in inexplicable directions) and valuation (desires and aversions of various sorts, some of which reflect real biological needs).

In this material context, the *biological* significance of these functions is obvious. Each animal organism needs to protect and further its life, and does so by taking stock of its physical environment, including its own body, as much as it can (by means of cognition), and by acting on its physical environment as well as it can (using volition), in pursuit of goals it estimates as conducive to its physical life (valuation).

Humans, and perhaps higher animals too, have additionally to take into account the mental environment of their existence, i.e. they additionally have *psychological* values to watch out for and take care of (e.g. their sanity or their character or their emotional states). Moreover, as many spiritual traditions suggest, it is likely that our lives extend beyond our mere physical and mental existences, into some purely spiritual realm, of which our souls are a small fraction, or where our souls permanently reside or come from and return to; in which case, we have to look out for, and behave in accord with, certain purely spiritual values too.

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[16] Plants seem to lack these powers; or maybe they have very primitive equivalents of them (being, after all, also living beings).

All the above considerations are crucial to any science of **ethics**. If there is no freewill, there is no ethics; anything goes, because we have no choice anyway. But, granting freewill, a scientific ethics can and must be conceived, provided the (human) subject has the conceptual power needed for such abstract study. Denial in principle of ethics is logically absurd, since it implies an imperative not to follow ethics. Ethical thinking is directed as determining what is good and what is bad, and what is indifferent, *for the organism* (person) concerned; ethics prescribes acts of will appropriate for the life (physical, mental, spiritual) of that organism. Ethical investigation is not for mere theoretical knowledge; its purpose is *to positively influence* (human) behavior in a wise direction in actual practice. Ready-made ethical guidelines, inasmuch as the individual is aware of them, and understands them, and wishes to follow them, function as influential factors, among other influential factors, in human volition. Ethics provides a rational counterweight to the forces of passion in people, including the foolishness of inertia. We could objectively propose ethics for all living beings, but of course only those organisms who can think such theoretical thoughts (certainly humans, and perhaps some other animal species if only preverbally) have any interest in it in practice.

Ethics differs from individual ad hoc reflections on particular ends and means, be they perfectly rational or partly or wholly based on emotional considerations and forces. Ethics is an attempt by interested individuals[17] to determine theoretically general principles for the practical guidance of all persons under a broad variety of life circumstances. Inevitably, there are competing ethical doctrines in circulation, some intelligent and some stupid, some very wise and some very foolish, and the individual must still use his (or her) head and heart to personally decide which to choose or which parts of each to choose. By 'use his (or her) head and heart' I mean judge matters rationally with due regard to human feelings. The ultimate standard of moral judgment has logically to be 'Life', because life is the end and means of all human existence.

As regards **theology**, we assume God's existence and powers by extreme extrapolation from our existence and powers. That is, while we evidently have only limited cognition and volition, and very often confused valuation, we assume that God is omniscient, omnipotent, and perfectly wise and good. Our assumption that God is the highest and greatest Soul allows us to conceive our souls as small parts of a larger entity, *and so to explain people's commonalities and community with reference to a common Source*. This can affect our ethical thinking. This is all still speculation, of course, but certainly a legitimate and valuable part of human thought. Admittedly, we do not empirically and logically know that God exists, nor any of his features, but it is useful to think about it and consider the possible implications of it.

In any case, it should be stressed that the idea of volition is contrary to the Cartesian doctrine of an unbridgeable dichotomy between **'mind' and matter**. The concept of volition is precisely that a 'spiritual' entity (the agent of an act of will) impinges on certain mental and physical phenomena, modifying (however minutely) the course of things in the material world. This is a statement that the 'laws of nature' that scientists consider unbreakable (even though they are only known *by generalization*) can be breached in very specific circumstances, notably within the context of a human or higher-animal existence. For instance, while the law of conservation of matter and energy states that the quantity of matter and energy in the material universe is constant, the freewill doctrine says that even so, a material event can occur (if only within the confines of a human nervous system) through the agency of a spiritual entity (the person's soul). Human beings are not sophisticated machines (and I believe, in view of our many biological similarities, contrary to René Descartes's supposition, neither are animals automata, at least not the higher ones).

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[17] Including religious, social, and political leaders, as well as philosophers.

A robot, even one with the most advanced **'artificial intelligence' (AI)** capabilities, is a machine because whatever it does, including its apparently autonomous decisions and movements, is the result of prior programming by people (or by machines programmed by people). Human beings differ radically, in that they have a soul, which has consciousness, volition, and values, which make it possible for them to choose freely no matter what influences they are under. Machines have no soul, no consciousness, no volition, and no values. Some are said to have 'artificial intelligence'; but it is doubtful the term intelligence is appropriate in their case, because intelligence is an attribute and measure of consciousness, and not per se of physical processes. Even so, machines can be useful or dangerous, according to how well or badly we program them. In that perspective, machine are mere extensions of human thoughts, will, and valuations, whether intelligent or stupid.

And yes, AI could conceivably destroy humanity and all life on earth. Some machine could well 'think' that this would be the most efficient way for it to perform some difficult function we asked of it – so, we must make sure such extreme solutions to problems by machines are made impossible by right programming. But what of malicious programming? A malevolent human individual or group with the required skills could well deliberately program an AI system to destroy humanity. Such crazy, humanity-hating people unfortunately exist – see how frequently some go out suddenly and kill as many people as they can for no apparent reason. Clearly, humanity is now, or will soon be, in great danger from AI; and it is doubtful that any legal restrictions can prevent such a historic disaster. Hopefully, benevolent programmers will come to our collective defense and stay one step ahead of the bad guys in an unending war of wits (just as today the internet is constantly under attack from numberless evil hackers and armies of good guys are busy fighting them).

Volition is a **spiritual** phenomenon. The concept of volition is definitely not a materialist-determinist one: it implies transcendence of sorts, a being existing over and above matter. It would be more difficult to claim that it is the physical body that has volition than to propose, as here done, that it is something which is not itself subject to physical laws (viz. the soul, our 'spiritual side') that has volition. Volition, like natural spontaneity, may be viewed as *a circumscribed exception* within a largely (but not exclusively) determinist universe.

The mechanical realm is, in this perspective, not fully predetermined; it involves, in many cases, mere *natural inertia* that can in principle be overcome by volition. Some natural events are beyond the power of anyone's volition to produce, prevent, or modify in any way; but some are subject to volitional interference. If we don't interfere, the 'natural course' of events proceeds; but if we do interfere, we 'change the course' of events. We can affirm that within a field largely occupied by mechanistic events, there may be scattered small pockets of freely willed events. It is not against nature; it is a special, highly developed power endowed by our natures, perhaps the peak of material evolution. Nature includes not only physical phenomena, but also mental phenomena and spiritual ones (i.e. at least individual souls and their three functions). There is no reason to shut our minds prejudicially against this thesis.

Many 'scientists' today prefer to reject this narrative at the outset, although they have no scientific basis for such outright denial; their *exclusive materialism and determinism* is not a result of valid induction, but rather a blind-faith ideological stance which makes them refuse *dogmatically* to take empirical and rational arguments into account. They 'postulate' that all the phenomena of life (including consciousness and volition) *will one day somehow* be reduced to material laws (i.e. be exactly predictable from purely material factors), but they have so far *not in fact* managed to propose any mathematical equations that could give credence to this wild speculation. It is just a

figment of their imaginations, a hope of theirs, or an intellectual fashion they cravenly follow. It is a closed-minded prejudice; it is not science.

Much more scientific, i.e. strictly in accord with logic and epistemology, is to assume the correctness of *our common inner experience* of self, cognition, volition, and valuation, and to theorize accordingly. We cannot just ignore such evidence, simply because it is too subtle and difficult to grasp precisely, so very different from the much more (sensorially and imaginatively) impressive physical and mental phenomena we also routinely experience.

5. Origins and evolution of the idea of causality

I believe the origins and evolution of the idea of causality to be as follows. This may be viewed as a hypothesis to be confirmed (in whole or in part) through empirical (psychological, sociological, and historical) studies.

On the *individual* level, the ultimate origin of our notion of causality is introspection: simple and direct awareness of oneself deciding, anywhere from very consciously to almost unconsciously, to do something – move a hand or foot, and so forth – and then actually doing it. This awareness is something all human beings have, and is the source of the notion of *volition*, which eventually (after more complex philosophical reflection) becomes the concept of freewill. Underlying this notion is the sense of self, so that the causality is spiritually based, i.e. logically requires a soul.

The *self or soul or spirit* is something non-phenomenal; that is, it has none of the phenomenal qualities (or qualia) which we apprehend through the senses of physical seeing, hearing, smelling, tasting, or touching, or any comparable mental projections derived from memories of such sensory experiences. We know of the soul through self-consciousness, i.e. directly and not though consciousness of something else. Such consciousness turned in the direction of the subject engaged in it, i.e. himself/herself, may be referred to as intuition or apperception. Just as we cannot hear with our eyes or see with our ears, and so on, but must use the appropriate sense organ for each phenomenal quality, so the self cannot be found in any sensory or imaginative experience, and yet we are capable of apperceiving it. Possibly, self-awareness is a power granted only to humans, not to animals; but maybe some have it, too.

The idea of *influence* is closely allied with that of volition – that is, we are all somewhat aware, though to very different degrees, that the thought of something may make our volition easier or more difficult. People are mostly motivated by objects of desire or restrained by objects of aversion, and act on the basis of their ideas as to how to attain or avoid them. The behavior of people is also strongly influenced by belief in the values and disvalues, and virtues and vices, identified by sundry ethical doctrines. Volition does not function out of context but depends on our perceptions, be they rational or irrational. Influences are not determining; we are able to will as we wish in the absence of perceived incentives and in the presence of perceived disincentives. It is after such more advanced reflections that the concept of freedom of the will is formed.

Next comes our personal experience of interaction with physical objects beyond our own bodies. This includes experiences of pushing or grabbing and pulling some external physical body away from or toward one's own. Such experiences are the source of our idea of causality in the sense of *causation*, as distinct from volition, because they occur beyond one's direct agency. Here, the focus is not on our causing the external body's movement, but on the interaction between our own body (put in motion by our willing) and the body external to it; so, it is a more materialist form of causality, in which we play no part other than having initially set the chain reaction in motion.

At a third stage, we look further out and assign causality to perceived events, such as the actions of other people, eventually of other animals (apparently also self-moving living things), regarding which we assume by analogy from ourselves volition (and consciousness and valuation) to be applicable; and the interactions of inanimate bodies, which we regard as subject to causation, and/or possibly to mere chance (natural spontaneity).

A later development is the application of the notion of volition to *purely mental* activity such as remembering, imagining, thinking, and so forth; this comes later because it requires a more advanced degree of introspection and reflection than awareness of one's powers of physical movement. We also discover, upon reflection, that the notion of causation is applicable to the interactions between distinct mental events (for example, a memory of pain might cause a fear) or between mental and physical events (for example, a fear might generate certain unpleasant sensations in one's stomach), because such events are usually not willed by us and in many cases evidently not under our power of will to produce or to prevent.

The important thing to note in view of the above sequence of events is that the notion of volition very probably historically *preceded* that of causation. I draw your attention to David Hume's questioning of the existence of causation by arguing that we cannot discern any empirical 'connection' between cause and effect. What he had in mind, when he sought for such an underlying bond, though he would not have admitted it, was the sensations we commonly experience when we push or pull external bodies – movements which in fact proceed from volition. This shows that, unconsciously, Hume was placing our experience of volition as *the model* for empirical evidence of causation. But of course, both volition and causation are in fact abstractions – neither requires obvious material, or even mental, concrete tokens.

Modern philosophy, strongly influenced by the scientific study of physical processes, tends to regard causation as the primary form of causality, and volition as a derived idea, one which is (it is claimed) much more elusive and open to doubt, if not entirely delusive. What is true is that the idea of causation is easier to formally define than that of volition, and for that reason seems simpler and more certain, and logically prior. But they are very different forms of causality, and we cannot logically reduce either one to the other, but must treat them separately.

In truth, we will probably never be able to define volition with as much precision as we can define causation because it is something *so fundamental and distinct* that it is not reducible to anything else. Similarly, we will probably never be able to define consciousness or soul or even existence – this is not a problem that only concerns volition. In truth, even physical concepts fundamental to the physical sciences, such as (for example, at the present stage of physical science) the 'forces' of attraction or repulsion in gravitational or electromagnetic fields, must be taken as primary givens, for they are not reducible to something else. When dealing with concepts so basic, the very building blocks of our world, we can only offer roughly descriptive discourse dependent on analogy. Certainly, infinite regression of reduction is logically impossible; the buck must stop somewhere.

Now, looking more broadly at the probable *collective* historical development of the idea of causality, I would suggest the following. Primitive people were probably *animists*, believing that even inanimate material bodies were possessed of spirit – that was their explanation of causation among such bodies: they were conceiving causation as essentially volition by invisible spirits. Later, this notion was sublimated by regarding different classes of material bodies as moved by different gods – this was the *polytheist* stage. Eventually, the *monotheist* belief dominated that a single God actively controlled the whole world.

Later still, following philosophical reflection, this doctrine was mitigated, so as to claim that God *could* control everything *but* preferably chose (in principle, unless otherwise decided by Him exceptionally[18]) to allow human beings and animals to control some things (through volition) and enacted 'laws of nature' ensuring the orderly (or even spontaneous) mechanical process of inanimate objects. After that came the *atheistic* doctrine of volition and causation without God, and then the purely *materialist* doctrine of a world of causation without possibility of volition, though perhaps some natural spontaneity, and beyond that a doctrine *denying all* causality (causation as well as volition) and accepting only chance as conceivable.

It must however be pointed out that denying all causality is not possible without first defining what is meant by causality; such denial is meaningless if no meaning is granted to the concept of causality. The same remark applies to the more specific concepts of volition and causation, and even to that of natural spontaneity. The serious study of causality cannot be avoided by rejecting causality off the cuff.

I do not, of course, claim that this proposed sequence of events was exactly followed in history and everywhere. It is schematic, drawing broad lines of probable evolution in most cases. Looking at actual historical accounts, it appears that some stages were skipped here and there, and the order of things was not always as logical. But the above theoretical prediction is still helpful for evaluating historical stages.

Let us now consider how we may **validate our concepts of causality**. It should be said that, whereas in the individual's and in humanity's history, the notion of volition probably precedes and is used to understand the dawning notion of causation (and by denial of both of these, that of causeless happenstance), in a more sophisticated philosophical context, the concept of causation seems more basic than that of volition. That is because *the idea of causation is logically inevitable, whereas that of volition is not so (at least, not at first)*.

Why is causation logical undeniable? Clearly, in a world of multiplicity such as ours, there *cannot but be* some regularity. A world where there is no similarity at all between any pair of things, is unthinkable. Indeed, it is *self-contradictory*, since any such claim is rich in assumptions of similarity. Without prejudice regarding what things are alike and which are not, we can say for sure that some fixed patterns are bound to be displayed in the diverse world we face. That is, some item X is bound to be invariably present in the presence of some other item Y and invariably absent in its absence; or the constancy may be more complicated, involving more than two items. The moment such holding patterns are found to occur, if only inductively by generalization, causative relations can be constructed and assumed.

To be sure, if this or that assumed causative relation is found upon further scrutiny of the facts not to hold in fact (in a specific case under consideration), then some other causative relation is logically bound to be found to hold – so, *the existence of the causative relation as such is never in doubt*. It cannot be claimed that the empirically evident regularities are mere happenstance, mere chance. They can credibly be so viewed at one level, avoiding intellectual reflection; but this view leaves open the issue of why this order of things occurred rather than any other. Whereas, once the idea of causation is introduced in our thinking, the empirically evident order of things is thereby explained as being 'forced' into being by the very nature of things, their de facto identities – it is

[18] Whence the expression 'God willing' – meaning that volition or causation (or even spontaneity) may be expected to run their usual course, unless God chooses to intervene exceptionally in the specific case under consideration.

acknowledged as the given order of the world[19]. So, the latter is a rationally more convincing approach.

As regards volition, it is (to repeat) not as inevitable as that of causation. But this is only true looking outward toward the material world without awareness of the fact that someone (oneself) is at that very moment looking out. For, as of when one becomes self-conscious, and seeks to include oneself, the subject of cognition, in the world, aiming at a fuller conception of the world which includes apparent mental and spiritual objects, one is inevitably pushed by inner experience to consider and acknowledge the fact of volition. Because it is quickly evident, if one is perceptive, logically-minded, and scrupulously honest, that our respective selves are, distinctively from mere material entities, *'souls'* ('spiritual' entities) with three essential powers – namely, *cognition, volition and valuation*. These three core functions of the soul are intimately related in fact, so that one cannot have any one of them without the others.

One could imagine a soul statically cognizing the world (having consciousness), quite passively, without getting involved in it in any way (lacking volition) and without having any preference in relation to it in any way (lacking valuation). But this is evidently not the way things in fact are for us, i.e. for our souls. We are constantly interacting with the world beyond our self, and indeed active in our inner life. Such interactions require exercise of will and necessitate values. The will must be free to a certain extent, for it to indeed be an expression of the soul's own desires and aversions. External and internal things might well affect the will, either by deterministically delimiting its scope, or by facilitating it or making it more difficult through the influence of various inner or outer objects of consciousness; but some willing must ultimately be free of any sort of coercion or it does not logically qualify as willing.

The concept of non-causality, i.e. that of natural spontaneity, is not excluded in principle by those of causation or volition. In a narrow sense of non-causality, this term has particular applications. Just as we can say that A causes B (whether through causation or volition), we *must* be able to say (when appropriate) that A does not cause B. It is unthinkable that everything is causally related to everything else in an identical way. For each causal relation that we do identify, there are a multitude of causal relations we have no basis for claiming and must therefore accept to deny. If everything was equally related to everything else, there would be no logical need for or thought of any concept of causality. Such concepts are based on the diversity in being and becoming evident in our world.

The wider and deeper concept of non-causality, i.e. that of natural spontaneity of a single event, is still conceivable. We can imagine something as having no causal relation, not just to *some* other things, but to *all* other things. Note well that our earlier argument that causation is an inevitable consequence of diversity is *not* affected by the assumption that some non-causality exists; our argument was that there had to be some instances of causality, not that it had to have universal scope. I see no logical basis for the traditionally formulated 'law of causality' in the sense of a total and a priori exclusion of the possibility of natural spontaneity. This is similar to pointing out that though volition is usually aimed at some goal, it can occasionally be entirely capricious.

Empirically, the existence of natural spontaneity seems suggested by certain events observed through various experiments by modern physicists at the deepest levels of matter (consider, notably, the 'Copenhagen Interpretation') – and therefore, we must needs include this concept in our ætiological arsenal, whether it ultimately proves applicable or not. In that case, we might as

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[19] There is no intent to appeal here to a thought, like that of Leibnitz, of a 'pre-established harmony'. We just accept things as they are, no matter how easy it is for our minds to imagine how different they might have been.

well for brevity's sake expand the term 'causality' to include not only causation and volition (and influences on volition), but also the absence of both of these, i.e. natural spontaneity. That is, even though natural spontaneity is strictly speaking non-causality, we can include it as a subject-matter in the expanded sense of the term causality referring to the conceptual domain of causal issues.

Surely, natural spontaneity can be viewed as a sort of *singular* and unpredictable causation, in addition to the more commonplace *plural* and law-abiding form of causation. It is similar to volition in being singular and unpredictable; but it is distinguishable from volition in that there is no soul behind it, no purposeful will (except possibly God and His will). It is an aspect of nature – though one for which no 'law' of nature can be discerned or declared by us; it is, metaphorically, nature in its capricious moods. This is here discussed as a theoretical possibility, without denying that the claim might eventually one day be found empirically – and perhaps even logically – untenable.

6. Hume's many problems with causation

David Hume denied the reality and knowability of causation due to numerous conceptual confusions and much fallacious thinking; and unfortunately, many philosophers have foolishly followed and continue to follow his lead to this day. There are two kinds of philosophers, the malevolent, destructive ones, who look for problems they hope unsolvable; and the benevolent, constructive ones, who look for solutions; Hume was definitely of the former variety. The following is an example of Hume's assertions concerning causation:

> "We cannot discover any necessary connexion between the cause and the effect, even though we may observe their constant conjunction, in all past instances."[20]

Hume's problems with causation are rooted in problems with *modality*. He did not sufficiently analyze the concepts of modality before making claims about causation. His main thesis that man cannot know necessity, and more deeply that there is no necessity, is self-contradictory. If something is *impossible*, then logically this implies that there is *necessity* for that thing not to exist. Moreover, it is logically impossible to claim the *impossibility* for man to know necessity, because that is itself a claim to the *necessity* of man's ignorance about necessity. Impossibility and necessity are concepts that logically go hand in hand; impossibility is necessity of a negative and necessity is impossibility of a positive. One cannot accept one concept without the other: their forms do not differ, only their contents do.

Similarly, one cannot logically claim the negation of necessity without accepting the negation of impossibility, and vice versa. If everything is unnecessary, then nothing is necessary, in which case no necessity whatsoever can be claimed; likewise, if everything is possible, then nothing is impossible, in which case no impossibility whatsoever can be claimed. If nothing is necessary, then nothing is impossible; and if nothing is impossible, then nothing is necessary. These are not word games, but logical imperatives. Differences in polarity do not affect the concepts of modality. Hume ignored the square of opposition between the four concepts of necessity, impossibility (necessity-not), possibility (non-impossibility), and unnecessity (possibility-not). He thought he could pick and choose and accept one concept of modality and reject another. But no! – these four concepts are logically inextricably tied together; they cannot be taken up separately.

Furthermore, Hume confused different modes (or types) of modality, notably logical modality and natural modality. Logical necessity (the necessity of conceptual implications) is a broader concept

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[20] *An Enquiry Concerning Human Understanding* (1748), Section IV, Part I.

than natural necessity (the necessity of natural law). *Logical necessity logically implies natural necessity; but natural necessity does not imply logical necessity*. Natural necessity is logically compatible with logical unnecessity. This means that, while we can indeed, using our imagination, conceive that natural laws (apparent natural necessities) could have been otherwise than they in fact are (i.e. that they are not logical necessities), that does not change the fact that they are natural necessities and we can know them by induction (instead of deduction), i.e. by generalization from empirical observations (until and unless further observations *belie* such generalization and impose particularization). Hume, through stupidity or dishonesty, conflated these two concepts of logical and natural necessity when he denied causation.

Hume was evidently also ignorant of the differences between the dynamic forms of causation based on 'natural' (and temporal) modality and the static forms of causation based on 'extensional' (and spatial) modality. These modes involve different forms of conditional propositions. In the natural mode (and likewise the lesser temporal mode), a dynamic event is followed or not-followed by another dynamic event, immediately or over time; here, causation refers to succession of events (changes) in time. The extensional mode (and likewise the lesser spatial mode), on the other hand, concerns the conjunction of static things (concrete entities or qualities or even actions, or abstract categories) irrespective of time. For examples: when a gas is heated, it expands, when cooled, it contracts (dynamic causation); whatever has legs, can walk, whatever hasn't, can't (static causation).[21]

Additionally, Hume vaguely claimed that causation refers to 'constant conjunction'; but this is formally inaccurate, it is only half the story. Knowing that something A is always followed by or conjoined with another thing B (and therefore is never followed or conjoined with the negation of B) does not suffice to prove that B depends on A; for it is possible that the negation of A is also always followed by or conjoined with B, in which case B would surely be independent of A. Hume's definition of causation as 'constant conjunction' refers to the positive relation of A and B, but fails to mention the negative relation of not-A to B. To claim causation between two items, one must always specify both the positive side and the negative side of their relation. Only when *both* these sides of the question are settled can we claim that there is, or may be, causation. The positive side is not enough by itself to do that.

In truth, to establish the strongest form of causation we must demonstrate both that A is constantly conjoined with B and that not-A is constantly conjoined with not-B; this is complete-necessary causation. If we find that A is constantly conjoined with B, while not-A is not constantly conjoined with not-B, that is still causation, though of a weaker sort, namely complete-contingent causation. Moreover, contrary to Hume's naïve definition, there can also be causation *without* A being constantly conjoined with B. If A is not constantly conjoined with B, but not-A is constantly conjoined with not-B, that is partial-necessary causation. And if A is not constantly conjoined with B *and* not-A is not constantly conjoined with not-B, that is consistent with partial-contingent causation, the weakest sort of causation.

Thus, *irregularity*, whether on the positive or the negative side, is *not* per se proof of non-causation, because *partial* and *contingent* causation are logical possibilities. If a thing is a *partial* causative of another, they will not always be conjoined; and if a thing is a *contingent* causative of another, their negations will not always be conjoined. Complete (or sufficient) and necessary (or sine qua non) causation is only the strongest determination of causation (whence its paradigmatic role) – it is not the only possible form. In other words, causation may itself be conditional (whether on the

[21] See my work *Future Logic* (1990) for a full analysis of the different modes of conditioning.

positive or negative side); it does not *only* exist in unconditional form[22]. Hume did not take these crucial formal facts into consideration, but offered a very simplistic definition of causation, No wonder, then, that causation seemed to him something feeble.

Hume denied causation by referring to its lacking some vague, undefined, metaphysical "connexion" (suggesting a bond or cement or glue) between the putative cause and effect. His definition of causation as constant conjunction being inaccurate, he could well deny that this bond existed in the midst of what we call causation. But the truth is, when causation is properly defined with reference both to positive *and negative* regularities, as above pointed out, so that there is *no way out of it* with reference to the invisibility of a presumed natural glue between the terms – the problem that Hume imagined simply disappears. If both positive and negative sides are accounted for, no distinction can be drawn between regularity *with* glue and regularity *without* glue! Note this well. What is indeed regular (to any degree) is glued (to that degree); and only what is truly irregular truly lacks glue.

Causation is a complex abstract concept, not a simple concrete percept; so, to deny causation because one does not directly perceive it is as such absurd. The truth is that causation is so complex a relation, with four alternative full determinations possible, that it cannot just be denied point-blank, as Hume tried, without awareness of all its possibilities and thence exhaustive denial *of them all*, and that needs to be done for each and every pair of items *in the whole universe*. Before causation can be denied to exist, it must be shown empirically, for any pair of items, that none of the four determinations of causation are applicable to them. This is no easy task, because *even if no regularity is immediately spotted in their relation, they may still be related by partial-contingent causation at a deeper level*. Hume denied the reality and knowability of causation because he was unaware of the variety of its determinations and of *the great difficulties in establishing non-causation*. It is much easier to establish causation than non-causation.

Hume's weak notion of causation may be construed *post factum* as being due to his mind's tendency to resort to 'material' (Philonian) conditioning rather than 'strict' conditioning. This is evident in his speaking of causation in terms of 'constant conjunction' instead of 'necessary conjunction'. In material implication, 'A implies B' means only that the conjunction of A and not-B is *not actual*; whereas in strict implication it means that such conjunction is *not possible*. In the former case, the relation is tenuous, mere happenstance at a given time; in the latter, the relation is utterly firm, one of actuality in all circumstances and throughout time and space, i.e. one of necessity. Behind this error of appreciation by Hume was his failure to understand inductive reasoning; that is, to grasp our ability to infer general actuality (necessity) from current actuality (happenstance), by means of generalization *subject to* corrective particularization in the event that evidence to the contrary is encountered at some later time.

Furthermore, Hume denied our knowledge of causation by claiming that it could only be strictly established through a putative '*uniformity principle*' (used as a grand major premise); and then arguing that such principle cannot be relied on in practice since it is itself, at best, a mere product of generalization. But this argument, as will now easily be demonstrated, is absurd. The alleged uniformity principle is a red herring, in that there is no way to formulate such a principle so that it predicts uniformity of nature *in specific cases*. All we can say about the uniformity of nature is a vague statement that uniformity *exists* in nature, but we cannot in advance say *where or when* it does so. In other words, there is no *principle* of uniformity, only a *fact* of uniformity. And this fact

[22] For a more detailed analysis, see my earlier listing of the four determinations of causation, drawn from my work *The Logic of Causation*.

is axiomatic and self-evident, because to claim otherwise is self-contradictory since any such explicit claim is based on concepts and all concepts presuppose similarity (uniformity of appearance) between two or more things (and incidentally, since we live in a diverse world, their dissimilarity from other things).

If we try to adopt a Heraclitean premise that no two things are the same, anywhere, at any time, we are using concepts – namely, 'no', 'two', 'things', and so on – concepts which all depend for their comprehensibility and realism on there being things indeed similar enough to be included in the same classes and denoted by the same terms. A world of utter chaos, for which the dictum that 'you cannot step into the same river twice' is literally and universally true, is simply unthinkable. Any world containing two or more things is logically bound to involve some uniformity, if only the shared fact of existing in the same world. Heraclitus' philosophy was sophistry, in that he did not apply his insight of multiplicity to his own discourse[23]. All we need to acknowledge, in order to form concepts, is that *there are* apparent similarities in characteristics between diverse things; we do not need to enumerate or prove these similarities a priori. Likewise, all we need to acknowledge to form causative propositions is that *there are* apparent similarities in behavior between diverse things; we do not need to enumerate or prove these similarities a priori. To repeat, this is not an appeal to a principle of uniformity, but only to a fact of uniformity.

After introducing the uniformity principle, Hume rightly rejected it as involving a circular argument (the principle being itself a product of generalization could not be used to logically justify generalization). But he did not realize that such a general principle was logically *impossible to even formulate*; i.e. that there is no way to state a priori *how far* uniformities can and must go for each class of things – we can only discover this a posteriori, by means of pedestrian *ad hoc* empirical and theoretical considerations. Also, for any class of things, we know for sure that there will at some point be an end to uniformity – but we cannot foretell *where* that point lies. Nevertheless, having rejected what he wrongly thought to be the theoretical basis of our knowledge of causation (and other concepts), namely generalizations from observations, Hume concluded that causation could not be proven and therefore was in reality non-existent. But since his major premise was wrong, namely that knowledge of causation depends on an alleged uniformity principle, it is not surprising that his conclusion was wrong.

The true basis of generalization is simply that when we observe (or less directly induce from observations) a set of things to have some (positive or negative) property or behavior, **we are *logically bound to assume* that things of the same kind which we have not yet encountered *will* indeed be found to have *the same* (positive or negative) property or behavior, *until and unless* we happen to come across one or more things of that same kind which *do not* in fact have that property or behavior (in which event we would adjust our judgment by means of particularization).** This is not arbitrary inference, but is firmly based on the fact that the property or behavior of the known cases is *empirically based*, whereas (for the time being, at least) any expectation of *opposite* property or behavior for the yet unknown cases is based on *nothing but imagination*[24]. We rightly prefer to base our knowledge on experience rather than on imagination.

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[23] For the same reason, of course, Parmenides was a sophist, since he formulated a philosophy of absolute, universal, and exclusive monism by means of discourse involving multiple concepts. The concepts of similarity and dissimilarity are inextricably tied together.

[24] That is, if the initial experience (or more broadly, induction) is positive, any expectation of a negative case is merely imaginary until and unless it is actually experienced (or induced). Inversely, if the initial experience (or induction) is negative, any expectation of a positive case is merely imaginary until and unless it is actually experienced (or induced).

This is the true validation of generalization – it is the genuine verdict of intelligent reason and the very basis of induction.

Sure, we too can well, as Hume did, *imagine*, in a general way, that cases we have not observed (or induced) *might* turn out to be different from cases we have already observed, since *such changes of polarity* have indeed in the past been observed ex post facto *in other specific contexts*. But notice the absurdity of this alleged critique: it is itself an attempt at generalization, moving from past observations of changes in polarity (in other contexts) to prediction of future changes in polarity (in the present context)! Note this well: Hume denied generalization from same polarity to same polarity *by* implicitly arbitrarily admitting generalization from polarity change to polarity change. This was logical inconsistency on his part; and **the fact that his attempted refutation of causation is self-contradictory proves irrefutably that *causation cannot logically be denied to exist***. The truth is that until and unless we *actually do* observe (or induce) a change of polarity in the present specific context, we cannot logically assume or claim that it will ever happen. Because observation is epistemologically more reliable than imagination, we are logically bound to prefer the former to the latter, and generalize accordingly, *until if ever* some new observation shifts the dividing line.[25]

Hume, having rejected the validity of generalization (due to his unawareness of the processes of rectification of overgeneralization by particularization, and due to his resort to sophistical rationalizations, among other things), sought to explain our belief in causation with reference to mere habit or custom. He sought to deny causation by claiming that what we perceive as 'regularity' is really mere *association of ideas* rather than *constant conjunction of facts*. He claimed our belief in causation is not based on reason but on imagination; so that there is no objective connection between cause and effect, but merely a subjective feeling of connection (that elusive 'glue'). This pseudo-explanation of our belief in causation was also fallacious. We do not assume causation by introspective observation of our mental processes, but from awareness of the behavior of the external (or, where applicable, internal) phenomena that these mental processes are about. Looking inward is not our usual way; it involves a different effort from our more common outward orientation of awareness.

Hume distinguished between 'impressions' and 'ideas', the former term apparently including concrete mental phenomena and the latter more abstract ones. But the expression 'association of ideas' may reasonably be taken as intended to cover all mental phenomena, i.e. both the primary concrete data and the derived abstract data. That is, here the term 'ideas' has a broad connotation, including sensations, intuitions, percepts, and imaginations, and also concepts of these, propositions with the latter, and complex thoughts built up from those. Additionally, feelings, emotions, moods, attitudes, and even habits and behavior patterns, and more complex psychological elements, could here be considered as 'ideas'. However, Hume, when speaking of association of ideas in the context of his theory of causation probably had in mind mainly the cognitive aspect of ideas, since in his view they replaced objective 'facts'. Thus, for him, here, the ideas associated were principally *memories of* sensations, intuitions, perceptions, imaginations, concepts, and discourses. He seems to have imagined these mental entities (as he evidently saw them in his mind's eye) as succeeding each other like billiard balls in motion.

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[25] See my work *Future Logic* for more detailed treatment of inductive reasoning, and further reflections in subsequent works of mine. If Hume had reflected more carefully on Francis Bacon's "negative instance" (in *Novum Organum*. First published 1843. Tr. R. Ellis & James Spedding. London: Routledge, year not specified,), he might have realized its significance and saved the world centuries of intellectual imbecility and perversion.

In any case, it is naïve to view 'ideas' (here including 'impressions') as literally mental *entities*, they are rather the ideating subject's more or less conscious *intentions* of various sorts – they have 'intentionality', they point to external and/or internal objects in various ways (with or without word labels). They do not succeed each other completely independently of the subject's consciousness and volition, like mere billiard balls, but do so through much more complex and varied processes. The way Hume effectively pictured ideas was ultimately very simplistic and visual. He lacked the perspicacity necessary for accurate introspection. His observations of his own inner life were very superficial and approximate. He was satisfied with very brief incursions into his own mind, not investing enough effort in his meditations, not looking inward very carefully and deeply, and not persevering. And he evidently lacked the intelligence needed to properly evaluate what he did observe. He also lacked logical skill, and the self-criticism required for that, i.e. the ability to direct his critical eye on his own discourse.

In truth, observed facts can only *incidentally* be called ideas (mental objects), insofar as once observed they usually or often remain in the mind as memories (which are then used in further imagination and thought). Observed facts, be they external or internal, are *phenomenologically given* – in that sense, they are all necessarily *objective* and in no way subjective. Although objects of consciousness (and thereby new contents of the mind), they are *facts*, not fictions. One cannot claim that consciousness is necessarily distortive without putting one's own claim in doubt and disqualifying it; that is, skepticism is logically self-contradictory and therefore self-defeating. Moreover, to refer to the constant conjunction (and thence causation) of ideas instead of facts is to willfully ignore that ideas are also, *in themselves*, facts. From which it logically follows that ***if constant conjunction of ideas is admitted as possible, then there is no reason to deny that constant conjunction (and thence causation, if the negative side is taken into consideration) of other facts is equally possible*****,** whether these are facts to which ideas refer or facts that have not yet produced ideas in the mind.

To admit the one and deny the other would be baseless and arbitrary, because there is no *formal* difference between these two sorts of constant conjunction, only a difference *of content*. The propositions 'X causes Y' and 'The thought of X causes the thought of Y' are formally the same because they concern the same relation ('causes'), even though they differ in content (the terms 'X' and 'Y', and the terms 'the thought of X' and 'the thought of Y', respectively); a partly symbolic term like 'the thought of X' is *just an instance* of the generic, fully symbolic term 'X'. If generalizations of observations regarding subjective ideas are acceptable (as inductive truths), then the same form of reasoning can inevitably and undoubtedly be applied to objective observed facts.

Moreover, note well, the concept of 'association of ideas' is wider than that of causation; Hume stole this wider concept when he limited it to apparent causation[26]. For example, I may remember that time I sat having coffee with my friend, and then go off mentally thinking about someone we talked about among other things, and then from there go off thinking about that other person's deeds or character or whatever. The ideas associated in this example would not be considered as causatively related; they would be viewed as accidentally succeeding each other, because they do not invariably follow each other[27]. We do *induce by generalization* from repetitive successions of

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[26] He also referred to association of ideas to explain resemblance and contiguity in time and place. But the refutation of the three claims (which are intertwined, anyway) is the same.

[27] Note in passing that two or more people having a conversation may jointly engage in association of ideas. One says something, this brings to mind something else to another, which in turn reminds the first of something else

observed facts (or, more broadly, using adduction, theory formulation and confirmation) that these facts are causatively related (until and unless exceptions are found). But this does not mean that *all* ideas which happen to succeed each other *occasionally* are causatively related; in truth, very few *ideas*, if any, are constantly conjoined[28]. Yet Hume relied on the absence of generality and fortuitousness in the latter case to throw doubt on the firmness of the causative link.

Furthermore, if we look more closely at the said example, we see that it consists of a random leap *of attention* from some detail of one topic to a related detail of another topic which is not substantially or causally related to the former. The two (or more) topics are very incidentally related, by means of *a volitional shift of attention*. It cannot be said that there is causation between the 'ideas' involved – rather, they involve some common detail which *influences* the subject *to will* a change in direction of his attention. The common detail may be viewed as a thread traversing both topics; but the association between them is an isolated event, not a case of constant conjunction. There are perhaps, said offhand, 'ideas' that are constantly conjoined (though much depends on what we admit as constituting an 'idea'). But even if so, such causative links in the mental sphere are not to be confused with the notion of association of ideas. Perhaps the constant conjunction between the emotion of fear and that of hate, or the constant conjunction between the attitude and practice of careful scrutiny of information and the personality trait of perspicacity, would be fitting examples.

Finally, I would like to draw attention to a further self-contradiction in Hume's skeptical thesis. By categorically denying the existence of volition, he was effectively claiming, a priori, the logical necessity of determinism and/or natural spontaneity, *as there are no other conceptual possibilities besides these three*. But what is logically necessary is necessarily naturally necessary (though not vice versa). Logical necessity logically implies (though is not implied by) natural necessity (i.e. physical necessity in the material realm and mental necessity in the realm of mind)! Yet Hume denies the knowability, and indeed the very existence, of natural necessity (i.e. natural law); and as for natural spontaneity (i.e. natural happenstance not subject to law), if this was his preference, this cannot logically be upheld as *universally* applicable, because that would make it a natural *law* (a generality which Hume denies). So, he was tacitly contradicting himself; and his skeptical thesis was (and is) untenable.

From all the above arguments, we see that Hume's denial and critique of causation was utter nonsense, the product of a confused or malignant mind. The same, of course, can be said of his denial and critique of soul and volition (free will), and thence of ethics; I have done this elsewhere in considerable detail[29]. His ideas were not the product of a superior intellect, but the result of a poisonous mélange of lack of reflection, ignorance, stupidity, and bad will. It is amazing that there were, and still are, people so naïve and unintelligent as to be convinced by his absurd arguments.

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entirely, and so forth. The same people having a conversation the next day, from the same starting point, would likely proceed quite differently (although, to be sure, some people tend to repeat themselves, as if they have one track minds).

28 Even if, for me, these days, a certain tune awakens in me fond memories (e.g. of a woman I once loved), it is very improbable that this tune will do that again at some future time, say at a time I happen to have other concerns or when I am no longer interested in the object of those memories. Furthermore, even if this sequence of events remains with me all my life, it does not follow that other people experience the exact same sequence (with the same tune and fond memories). *Constant* conjunction of ideas is a very rare phenomenon, assuming it ever at all occurs. Hume's claim that such mental 'habits' are the source of our belief in causation is not based on any common experience, but is rather a figment of his undisciplined imagination.

29 In my book *Hume's Problems With Induction* (2008). Regarding the no-soul claim, see my essay "Critique of the Buddhist Five Skandhas Doctrine."

3. THE LOGIC OF ANALOGY

I analyzed in some detail the basic formalities of the argument by analogy close to ten years ago in my book *A Fortiori Logic*. I there showed in what ways it resembles and differs from a fortiori argument. However, I left the matter at that, and did not consider the inconsistencies one can easily come across in the use of analogical argument. I also did not sufficiently investigate, as I should have, the use of such argument in scientific and legal (and in particular in Talmudic) discourse. In the present essay I try to broaden and deepen my investigation. The material presented below is original; no one has, to my knowledge, surprisingly, ever investigated the formal logic of analogy in such detail.

1. Qualitative analogy

To begin with, let us review some of the main findings of my past research regarding analogical argument and see where we can improve upon them. The following text is mostly drawn from my book *A Fortiori Logic* (Chapter 5.1), but with some significant editing.

Qualitative analogical argument consists of four terms, which we may label P, Q, R, S, and refer to as the *major, minor, middle and subsidiary terms*, respectively (remember the nomenclature). The major premise contains the terms P, Q, and R; the term S appears in both the minor premise and conclusion. The names major term (P) and minor term (Q), here, unlike in a fortiori argument, do not imply that P is greater in magnitude or degree than Q. For this reason, we can *conventionally* decide that the minor term will always be in the minor premise, and the major term will always be in the conclusion; meaning that all moods will have the form of so-called 'from minor to major' arguments.[1] This means that any valid 'minor to major' mood could, in principle, be reformulated as a valid 'major to minor' mood.[2]

The argument by analogy may then take the following four *copulative* forms (with a positive major premise, to start with).

a. The **positive subjectal** mood. *Given that subject P is similar to subject Q with respect to predicate R, and that Q is S, it follows that P is S*. We may analyze this argument step by step as follows:

> Major premise: P and Q are alike in that both have R. Note that this premise is fully convertible; it has no direction.

This implies both 'P is R' and 'Q is R', and is implied by them together.

> Minor premise: Q is S.

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[1] In my book *A Fortiori Logic*, where my treatment of analogical argument was aimed at comparison with a fortiori argument, I had to impose the same forms as in the latter to the former. That is, positive subjectal and negative predicatal moods were 'minor to major', and negative subjectal and positive predicatal moods were 'major to minor'. Here, where my treatment of analogical argument is independent, such distinctions are irrelevant; and it is wiser to make all moods 'minor to major' or all moods 'major to minor', and the former choice (with the minor term always placed in the minor premise) is easier to remember.

[2] But when dealing with *quantitative* analogy (see further on) we must tread carefully, and distinguish between superior, inferior and equal terms.

The term S may of course be any predicate; although in legalistic reasoning, it is usually a legal predicate, like 'imperative', 'forbidden', 'permitted', or 'exempted'.

Intermediate conclusion and further premise: All R are S.

This proposition is obtained from the preceding two as follows. Given that Q is S and Q is R, it follows by a substitutive third figure syllogism that there is an R which is S, i.e. that 'some R are S'. This particular conclusion is then *generalized* to 'all R are S', provided of course we have no counterevidence. If we can, from whatever source, adduce evidence that some R (other than Q) are *not* S, then of course we cannot logically claim that all R are S. Thus, this stage of the argument by analogy is partly deductive and partly inductive.

Final conclusion: P is S.

This conclusion is derived syllogistically from All R are S and P is R.

If the middle term R is known and specified, the analogy between P and Q will be characterized as 'complex'; if R is unknown, or vaguely known but unspecified, the analogy between P and Q will be characterized as 'simple'. In **complex analogy**, the middle term R is explicit and clearly present; but in **simple analogy**, it is left tacit. In complex analogy, the similarity between P and Q is indirectly established, being manifestly due to their having some known feature R in common; whereas in simple analogy, the similarity between them is effectively directly intuited, and R is merely some indefinite thing assumed to underlie it, so that in the absence of additional information we are content define it as 'whatever it is that P and Q have in common'.

Quantification of terms. Let us next consider the issue of quantity of the terms, which is not dealt with in the above prototype.

In the singular version of this argument, the major premise is 'This P is R and this Q is R', where 'this' refers to two different individuals. The minor premise is 'This Q is S', where 'this Q' refers to the same individual as 'this Q' in the major premise does. From the minor premise and part of the major premise we infer (by syllogism 3/RRI[3]) that there is an R which is S, i.e. that some R are S – and this is generalized to all R are S, assuming (unless or until evidence to the contrary is found) there is no R which is not S. From the generality thus obtained and the rest of the major premise, viz. this P is R, we infer (by syllogism 1/ARR) the conclusion 'This P is S', where 'this P' refers to the same individual as 'this P' in the major premise does.

In the corresponding general version of the argument, the major premise is 'all P are R and all Q are R' and the minor premise is 'all Q are S'. From the minor premise and part of the major premise we infer (by syllogism 3/AAI) that some R are S – and this is generalized to all R are S, assuming (unless or until evidence to the contrary is found) there is no R which is not S. From the generality thus obtained and the rest of the major premise, viz. all P are R, we infer (by syllogism 1/AAA) the conclusion 'all P are S'. Note that the minor premise *must* here be general, because if only some Q are S, i.e. if some Q are not S, then, if all Q are R, it follows that some R are not S (by 3/OAO), and we cannot generalize to all R are S; and if only some Q are R, we have no valid syllogism to infer even that some R are S.

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[3] Here, the symbol R refers to a singular affiRmative proposition, as against G for a singular neGative one. I introduced these symbols in my book *Future Logic*, but singular syllogism is not something new. The Kneales (p. 67) point out that Aristotle gives an example of syllogism with a singular premise in his *Prior Analytics*, 2:27. The example they mean is supposedly: "Pittacus is generous, since ambitious men are generous, and Pittacus is ambitious" (1/ARR). Actually, there is another example in the same passage, viz.: "wise men [i.e. at least some of them] are good, since Pittacus is not only good but wise" (3/RRI). Note that the reason I did not choose the symbol F for aFfirmative was probably simply to avoid confusion with the symbol F for False. In any case, some symbols were clearly needed for singular propositions, since the traditional symbols A, E, I, O only concern plural propositions.

As regards the quantity of P and Q, there is much leeway. It suffices for the major premise to specify only that some Q are R; because, even if some Q are not R, we can still with all Q are S infer that some R are S (3/AII), and proceed with the same generalization and conclusion. Likewise, the major premise may be particular with respect to P, provided the conclusion follows suit; for, even if some P are not R, we can still from some P are R and all R are S conclude with some P are S (1/AII). Needless to say, we can substitute negative terms (e.g. not-S for S) throughout the argument, without affecting its validity.

It is inductive argument. Thus, more briefly put, the said analogical argument has the following form: Given that P and Q are alike in having R, and that Q is S, it follows that P is S. The validation of this argument is given in our above analysis of it. What we see there is that the argument as a whole is *not entirely deductive, but partly inductive*, since the general proposition 'All R are S' that it depends on is obtained by generalization.

Thus, it may well happen that, given the same major premise, we find (empirically or through some other reasoning process) that Q is S but P is not S. This just tells us that the generalization to 'All R are S' was in this case not appropriate – it does not put analogical argument as such in doubt. Such cases might be characterized as 'denials of analogy' or 'non-analogies'. Note also that if 'All R are S' is already given, so that the said generalization is not needed, then the argument as a whole is not analogical, but entirely syllogistic; i.e. it is: All R are S and P is R, therefore P is S. Thus, *analogy as such is inherently inductive*. And obviously, simple analogy is more inductive than complex analogy, since less is clearly known and sure in the former than in the latter.

Note well: inductive does not mean arbitrary. Induction is a logical process with its rules, even if it is more indulgent than deduction. One cannot just make a claim or mere speculation and give it credibility by characterizing it as 'inductive'. Its logical possibility and consistency must first be considered, and then ways of validating it found. Any 'analogical' argument not here specifically formally justified may be considered as invalid, until and unless some precise formal justification for them is put forward.

Other moods. The above, prototypical mood was positive subjectal. Let us now consider the other possible forms of analogical argument.

b. The **negative subjectal** mood. *Given that subject P is similar to subject Q with respect to predicate R, and that Q is not S, it follows that P is not S*. This mood follows from the positive mood simply by obversion of the minor premise and conclusion, i.e. changing them to 'Q is non-S' and 'P is non-S' (since the negative term 'non-S' is included in the positive symbol S of the positive mood). This argument is of course just as inductive as the one it is derived from; it is not deductive.

c. The **positive predicatal** mood. *Given that predicate P is similar to predicate Q in relation to subject R, and that S is Q, it follows that S is P*. We may analyze this argument step by step as follows:

> Major premise: P and Q are alike in that R has both. Note that this premise is fully convertible; it has no direction.

This implies both 'R is P' and 'R is Q', and is implied by them together.

> Minor premise: S is Q.
>
> Intermediate conclusion and further premise: S is R.

This proposition is obtained from the preceding two as follows. Given that R is Q, it follows by conversion that there is a Q which is R, i.e. that 'some Q are R', which is then *generalized* to 'all Q are R', provided of course we have no counterevidence. If we can, from whatever source, adduce

evidence that some Q are *not* R, then of course we cannot logically claim that all Q are R. Next, using this generality, i.e. 'all Q are R', coupled with the minor premise 'S is Q', we infer through first figure syllogism that 'S is R'. Clearly, here again, this stage of the argument by analogy is partly deductive and partly inductive.

Final conclusion: S is P.

This conclusion is derived syllogistically from R is P and S is R.

Note that the generalized proposition here (viz. all Q are R) concerns the minor and middle terms, whereas in positive subjectal argument it (i.e. all R are S) concerned the middle and subsidiary terms.

Let us now quantify the argument. In the singular version, the major premise is: this R is both P and Q, and in the general version it is: all R are both P and Q. The accompanying minor premise and conclusion are, in either case: and a certain S is Q (or some or all S are Q, for that matter); therefore, that S is P (or some or all S are P, as the case may be). We could also validate the argument if the major premise is all R are P and some R are Q; but if only some R are P, i.e. if some R are not P, we cannot do so for then the final syllogistic inference would be made impossible[4]. Such argument is clearly inductive, since it relies on generalization. No need for us to further belabor this topic.

d. The **negative predicatal** mood. *Given that predicate P is similar to predicate Q in relation to subject R, and that S is not Q, it follows that S is not P*. This mood follows from the positive mood by reductio ad absurdum (we cannot here use mere obversion as with subjectal argument): given the major premise, if S were P, then S would be Q (since analogical argument is non-directional, P and Q are interchangeable in it); but S is not Q is a given; therefore, S is not P may be inferred. This argument is of course just as inductive as the one it is derived from; it is not deductive.

Moods **with a negative major premise**. All the above-mentioned moods could equally well have a negative major premise (expressing non-similarity or dissimilarity, which mean the same), and yield a corresponding valid conclusion – one, as we shall now show, of opposite polarity to the preceding. We may refer to such movements of thought as **disanalogy**.

The **positive subjectal** mood would be: *Given that subject P is not similar (i.e. is dissimilar) to subject Q with respect to predicate R, and that Q is S, it follows that P is not S*. Here, the major premise means either (a) P is R but Q is not R; or (b) P is not R but Q is R. The minor premise is given as Q is S, and the conclusion is the negative P is not S. This can be validated as follows: (a) given Q is S and Q is not R, it follows that there is a S which is not R; this may (in the absence of counterevidence) be generalized to 'no S is R'; whence, given P is R, we infer that P is not S. Alternatively, (b) given Q is S and Q is R, it follows that there is a S which is R, i.e. some S are R; this may (in the absence of counterevidence) be generalized to 'all S are R'; whence, given P is not R, we infer that P is not S. The **negative subjectal** mood follows by obversion, and has as its minor premise that Q is not S and as its as its conclusion that P is S.

The **positive predicatal** mood would be: *Given that predicate P is not similar (i.e. is dissimilar) to predicate Q in relation to subject R, and that S is Q, it follows that S is not P.* Here, the major premise means either (a) R is not P but R is Q; or (b) R is P but R is not Q. The minor premise is given as S is Q, and the conclusion is the negative S is not P. This can be validated as follows: (a)

[4] However, if we know that some R are P, and do *not* know that some R are not P, we can generalize the positive particular to obtain the 'all R are P' proposition needed to infer the final conclusion. In that case, the argument as a whole would be doubly inductive, since involving two generalizations.

given R is Q and S is Q, it follows that there is a S which is R; and given R is not P, we may (in the absence of counterevidence) generalize to 'no R is P'; whence we infer that S is not P. Alternatively, (b) given R is not Q and S is Q, it follows that there is a S which is not R; given R is P, we may (in the absence of counterevidence) generalize to 'all P are R'; whence we infer that S is not P. The **negative predicatal** mood follows by reductio ad absurdum, and has as its minor premise that S is not Q and as its conclusion that S is P.

We can call analogical argument with a positive major premise (expressing similarity) **comparison**, and that with a negative major premise (expressing dissimilarity) **contrast**. As we shall see further on, such arguments may result in conflicting conclusions, when they are compounded with different middle terms.[5]

We can similarly develop an equal number of **implicational** moods of analogical argument, where P, Q, R, S, symbolize *theses* instead of terms and they are related through implications rather than through the copula 'is'. The positive antecedental would read: *Given that antecedent P is similar to antecedent Q with respect to consequent R, and that Q implies S, it follows that P implies S*. The negative antecedental would read: *Given the same major premise, and that Q does not imply S, it follows that P does not imply S*. The positive consequental mood would read: *Given that consequent P is similar to consequent Q in relation to antecedent R, and that S implies Q, it follows that S implies P*. The negative consequental mood would read: *Given the same major premise, and that S does not imply Q, it follows that S does not imply P*. Moods with negative major premises can similarly be formulated; but the minor premise and conclusion will have opposite polarity, i.e. if the minor premise is positive, the conclusion will be negative, and vice versa. All implicational moods are, of course, partly inductive arguments since they involve generalizations. Validations of the implicational moods should proceed in much the same way as those of the copulative moods.

2. Quantitative analogy

Analogy may be qualitative or quantitative. The various moods of analogical argument above described are the qualitative. In special cases, *given the appropriate additional information*, they become quantitative. For quantitative analogy, as for qualitative analogy, since the major and minor terms (P and Q) are functionally interchangeable, we may conventionally consider all moods as 'minor to major'. However, in the context of quantitative analogy, where there are underlying quantities, we must nevertheless distinguish between 'inferior to superior', 'superior to inferior', and 'equal to equal' inferences.[6]

The **positive subjectal** moods of quantitative analogy would read:

- Given that subject P is *greater* than subject Q with respect to predicate R, and that Q is S (Sq), it follows that P is *proportionately more* S (Sp) (argument from inferior to superior).
- Given that subject P is *lesser* than subject Q with respect to predicate R, and that Q is S (Sq), it follows that P is *proportionately less* S (Sp) (argument from superior to inferior).
- Given that subject P is *equal* to subject Q with respect to predicate R, and that Q is S (Sq), it follows that P is *proportionately as much* S (Sp) (argument from equal to equal).

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[5] I briefly mentioned moods with a negative major premise in my past treatment of the topic; but I did not fully analyze them. I now view them as more important than I realized at the time, having lately become aware of the issue of compounding comparison and contrast.

[6] My treatment here of quantitative analogy differs somewhat from that in my book *A Fortiori Logic*. The present treatment should be regarded as more accurate.

Note that each of these quantitative major premises implies the qualitative major premise 'subject P is similar to subject Q with respect to predicate R'; for this reason, we already know by qualitative analogy that, in conclusion, P is S; what the quantitative analogical argument does is provide an additional quantitative specification in the conclusion, telling us whether P is proportionately (compared to Q, with respect to R) more, less or as much S.

The **negative subjectal** mood of quantitative analogy is then simply:

> Whether it is given that subject P is greater or lesser or equal to subject Q with respect to predicate R, and it is given that Q is *not* S, it follows that P is *not* S.

Note that this has here been expressed as one mood, but it could equally be presented as three moods by repeating it for each of the three major premises. The proposed conclusion here is not quantitative; it does not merely deny that P is proportionately more, less or equally S – it denies that P is S to any degree, just as the minor premise denies that Q is S to any degree. This means that this mood is essentially qualitative, and not quantitative. Its operative major premise is 'subject P is similar to subject Q with respect to predicate R'. The validity of this negative mood is thus established, as previously, by mere obversion of the negative subsidiary term.

The **positive predicatal** moods of quantitative analogy would read:

- Given that predicate P is *greater* than predicate Q in relation to subject R, and that S (Sq) is Q, it follows that *proportionately more* S (Sp) is P (argument from inferior to superior).
- Given that predicate P is *lesser* than predicate Q in relation to subject R, and that S (Sq) is Q, it follows that *proportionately less* S (Sp) is P (argument from superior to inferior).
- Given that predicate P is *equal* to predicate Q in relation to subject R, and that S (Sq) is Q, it follows that *proportionately as much* S (Sp) is P (argument from equal to equal).

Note that each of these quantitative major premises implies the qualitative major premise 'predicate P is similar to predicate Q in relation to subject R'; for this reason, we already know by qualitative analogy that, in conclusion, S is P; what the quantitative analogical argument does is provide an additional quantitative specification in the conclusion, telling us whether S is proportionately (in relation to R) more, less or as much P (compared to Q).

The **negative predicatal** mood of quantitative analogy is then simply:

> Whether it is given that predicate P is greater or lesser or equal to predicate Q with respect to subject R, and it is given that S is *not* Q, it follows that S is *not* P.

Note that this has here been expressed as one mood, but it could equally be presented as three moods by repeating it for each of the three major premises. The proposed conclusion here is not quantitative; it does not merely deny that S is proportionately more, less or equally P – it denies that S is P to any degree, just as the minor premise denies that S is Q to any degree. This means that this mood is essentially qualitative, and not quantitative. Its operative major premise is 'predicate P is similar to predicate Q in relation to subject R'. The validity of this negative mood is thus established, as previously, by reductio ad absurdum.

Obviously, for the positive moods of both subjectal and predicatal analogy, the reasoning depends (though often tacitly) on an *additional premise* that *the ratio of Sp to Sq is the same as the ratio of P to Q (relative to R)*. Very often in practice, the ratios are not exactly the same, but only roughly the same (this of course affects the argument's validity strictly speaking, though we often let it pass). Also, the reference to the ratio of P to Q (relative to R) should perhaps be more precisely expressed as the ratio of Rp to Rq. Note that this argument effectively has five terms instead of only four (since the subsidiary term S effectively splits off into two terms, Sp and Sq). Of course, the additional premise about proportionality is usually known by inductive means. It might initially

be assumed, and thereafter found to be untrue or open to doubt. In such event, the argument would cease to be quantitative analogy and would revert to being merely qualitative analogy. Thus, quantitative analogy is inherently even more inductive than qualitative analogy.

Note that the arguments here are, briefly put: (i) just as P > Q, so Sp > Sq; (ii) just as P < Q, so Sp < Sq', (iii) just as P = Q, so Sp = Sq. In other words, positive quantitative analogy may as well be from the inferior to the superior, from the superior to the inferior, or from equal to equal; it is not restrictive regarding direction. In this respect, we may note in passing, it differs radically from a fortiori argument. In the latter case, the positive subjectal mood only allows for inference from the inferior to the superior, or from equal to equal, and excludes inference from the superior to the inferior; and the positive predicatal mood only allows for inference from the superior to the inferior, or from equal to equal, and excludes inference from the inferior to the superior. All this seems obvious intuitively; having validated the qualitative analogy as already shown, all we have left to validate here is the idea of ratios, and that is a function of simple mathematics.

We can similarly develop the corresponding forms **with a negative major premise** (i.e. the 'contrast' or 'disanalogy' forms) as follows.

Regarding **subjectal** argument. (a) In cases where it is known that qualitatively 'subject P is similar to subject Q with respect to predicate R', then the quantitatively negative major premise 'P is *not* greater than Q with respect to R' can be restated positively as 'P is either lesser than or equal to Q with respect to R'; 'P is *not* lesser than Q with respect to R' can be restated positively as 'P is either greater than or equal to Q with respect to R'; and likewise, 'P is *not* equal to Q with respect to R' can be restated positively as 'P is either greater or lesser than Q with respect to R'. The conclusions follow as already above detailed. That is, with a positive minor premise, not-greater implies a proportionately less or equal conclusion; not-lesser implies a proportionately more or equal conclusion; and not-equal implies a proportionately more or less conclusion. With a negative minor premise, the conclusion is simply negative. But (b) in cases where it is known that qualitatively 'subject P is *not* similar to subject Q with respect to predicate R', then the three quantitatively negative major premises are irrelevant, and the minor premise 'Q is S' yields the conclusion 'P is not S', or alternatively 'Q is not S' yields the conclusion 'P is S' (as earlier seen). Therefore, (c) in cases where it is not known whether the underlying relation of P and Q relative to R is positive or negative, the conclusion is moot.

Regarding **predicatal** argument. (a) In cases where it is known that qualitatively 'predicate P is similar to predicate Q in relation to subject R', then the quantitatively negative major premise 'P is *not* greater than Q in relation to R' can be restated positively as 'P is either lesser than or equal to Q with respect to R'; 'P is *not* lesser than Q with respect to R' can be restated positively as 'P is either greater than or equal to Q with respect to R'; and likewise, 'P is *not* equal to Q with respect to R' can be restated positively as 'P is either greater or lesser than Q with respect to R'. The conclusions follow as already above detailed. That is, with a positive minor premise, not-greater implies a proportionately less or equal conclusion; not-lesser implies a proportionately more or equal conclusion; and not-equal implies a proportionately more or less conclusion. With a negative minor premise, the conclusion is simply negative. But (b) in cases where it is known that qualitatively 'predicate P is *not* similar to predicate Q in relation to subject R', then the three quantitatively negative major premises are irrelevant, and the minor premise 'S is Q' yields the conclusion 'S is not P', or alternatively 'S is not Q' yields the conclusion 'S is P' (as earlier seen). Therefore, (c) in cases where it is *not* known whether the underlying qualitative relation of P and Q relative to R is positive or negative, the conclusion is moot.

We can similarly develop the various corresponding **implicational** moods of quantitative analogy. Thus, all moods of qualitative analogical argument can be turned into quantitative ones, provided we add additional information attesting to 'proportionality'.

3. Terms of unequal breadth

The issue of quantitative analogy brings to mind the issue of analogies involving terms which are not co-extensive, but one is broader than and includes the other, as a more generic term includes a more specific term or as an unconditional term includes a conditional one[7]. This is still qualitative analogy, note well. It concerns the scope of terms, not their magnitude or degree as subjects or predicates.

Consider, for a start, **positive subjectal** analogy such that the middle predicate R is not identical for the major subject P and the minor subject Q. We are given that 'P is Rp' and 'Q is Rq', but we do not yet have a comparative major premise with which to construct an analogical argument. To obtain one, we have to find the operative common property of P and Q. Clearly, it is *the more inclusive* (*or less conditional*) predicate of the two we were given (viz. Rp and Rq).

That is to say: (a) if Rp includes Rq, so that Rq is Rp (but not vice versa), then the effective middle term is the *broader* one, Rp, and the major premise is 'subject P is similar to subject Q with respect to predicate Rp', from which, given that Q is S, it follows that P is S. Note well that we cannot in such case build an analogical argument (of minor to major form) from the narrower middle term Rq.

On the other hand: (b) if Rq includes Rp, so that Rp is Rq (but not vice versa), then the effective middle term is the broader one, Rq, and the major premise is 'subject P is similar to subject Q with respect to predicate Rq', from which, given that Q is S, it follows that P is S. Note well that we cannot in such case build an analogical argument (of minor to major form) from the narrower middle term Rp.

It might seem paradoxical to say in (a) that we can infer from Rp but cannot infer from Rq, and in (b) that we can infer from Rq but cannot infer from Rp, and yet with the same minor premise 'Q is S' obtain the same conclusion 'P is S'. But we should keep in mind that the basis of analogy, the middle term Rq or Rp used in the major premise, is different in each case, so that arguments (a) and (b) are quite distinct claims; and anyway, we are here dealing with inductive argument.[8]

The corresponding **negative subjectal** moods have the same major premises, and both infer from the minor premise 'Q is not S' the conclusion 'P is not S'.

With regard to **positive predicatal** analogy, where the middle term is a subject and the major and minor terms are predicates, we begin with two propositions 'Rp is P' and 'Rq is Q', from which we need to build a comparative major premise. Here, the basis of analogy is the subject for which both P and Q can be predicated. Clearly, it is *the less inclusive* (*or more conditional*) subject of the two we were given (viz. Rp and Rq).

That is: (a) if Rp includes Rq, so that Rq is Rp (but not vice versa), then the effective middle term is the *narrower* one, Rq, and the major premise is 'predicate P is similar to predicate Q with respect

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[7] Note that in some cases, though the two terms compared are specific/conditional, they may still resemble each other sufficiently to be considered as one and the same term for the purposes of analogical argument. It is only when the terms are not so identified, but must be differentiated, that the issue of unequal scope arises.

[8] Of course, we can draw a conclusion based on the narrower, less inclusive, middle term, Rq in case (a), and Rp in case (b), by proceeding from major to minor. But our standard form, conventionally, is minor to major.

to subject Rq', from which, given that S is Q, it follows that S is P. Note well that we cannot in such case build an analogical argument (of minor to major form) from the broader middle term Rp.

But: (b) if Rq includes Rp, so that Rp is Rq (but not vice versa), then the effective middle term is the narrower one, Rp, and the major premise is 'predicate P is similar to predicate Q with respect to subject Rp', from which, given that S is Q, it follows that S is P. Note well that we cannot in such case build an analogical argument (of minor to major form) from the broader middle term Rq.

Again, it might seem paradoxical to say in (a) that we can infer from Rq but cannot infer from Rp, and in (b) that we can infer from Rp but cannot infer from Rq, and yet with the same minor premise 'S is Q' obtain the same conclusion 'S is P'. But we should keep in mind that the basis of analogy, the middle term Rp or Rq used in the major premise, is different in each case, so that arguments (a) and (b) are quite distinct claims; and anyway, we are here dealing with inductive argument.[9]

The corresponding **negative predicatal** moods have the same major premises, and both infer from the minor premise 'S is not Q' the conclusion 'S is not P'.

The same principles apply to analogical arguments **with a negative major premise**, even though they involve major and minor terms that are dissimilar, rather than similar as above. This is because the contrasting major premise must be a negative mirror image of the comparative major premise, with the same middle term. Thus, all the moods here resemble those above, except that their major premises will be negative (indicating disanalogy) and their conclusions will be contradictory to the foregoing (granting that the minor premises remain the same). There is no need for us to belabor this issue further.

Likewise, **quantitative** analogies involving middle terms of unequal breadth follow the rules already established once we have determined the operative middle term in each case.

What about cases where the two middle terms Rp and Rq are not equal and neither fully overlaps the other, i.e. where they merely *intersect*. In such cases, we have the conjunction 'Rp and Rq' as our operative middle term, R. Given a major premise with this compound middle term, we can use it in any kind of analogical argument already established as valid. Remember that analogical argument is inductive, so there is no restriction on the scope of the middle term; any middle term which happens to be true is valid.

However, while this seems simple enough at first sight, the plot thickens when we consider the other terms in such analogical arguments and quantify them. Thus, in subjectal argument, if all P are Rp and all Q are Rq, only some P and only some Q are *both* Rp and Rq, whence the minor premise and conclusion must be formulated as concerning 'certain Q' and 'certain P' respectively; which makes it practically useless. Again, in predicatal argument, while we can say of the compound R that it is all both P and Q, we cannot in the validation process generalize from 'some Q are R' to 'all Q are R', as we need to do if we wish to infer from the minor premise 'S is Q' that 'S is R', and thence (via 'R is P') the conclusion 'S is P'; so, here analogy is effectively invalid. Thus, we can say without going into more detail that argument by analogy is not applicable in cases involving intersection.

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[9] Of course, we can draw a conclusion based on the broader, more inclusive, middle term, Rp in case (a), and Rq in case (b), by proceeding from major to minor. But our standard form, conventionally, is minor to major.

We have thus far dealt with middle terms of different scope, but what about **subsidiary terms** of different breadth?[10]

In positive subjectal argument, the operative subsidiary term is the predicate in the minor premise (Sq, say); the subsidiary term in the conclusion may be different (Sp, say), if and only if the primary conclusion 'P is Sq' implies the further conclusion 'P is Sp'; and this is possible only provided that 'Sq is Sp', meaning that Sp must be *broader* than Sq.[11] Likewise, in the corresponding negative mood, if 'P is not Sq' is to imply 'P is not Sp', Sp must be *narrower* than Sq.[12]

In positive predicatal argument, on the contrary, the operative subsidiary term is the subject in the minor premise (Sq, say); the subsidiary term in the conclusion may be different (Sp, say), if and only if the primary conclusion 'Sq is P' implies the further conclusion 'Sp is P', and this is possible only provided that 'Sp is Sq', meaning that Sp is *narrower* than Sq.[13] Likewise, in the corresponding negative mood, if 'Sq is not P' is to imply 'Sp is not P', Sp must be *narrower* than Sq.[14]

4. Face-off with a fortiori argument

It is worth comparing and contrasting analogical argument with a fortiori argument, because these argument forms are often confused by people. For this purpose, I will simply here reproduce verbatim what my reflections on this topic in my *A Fortiori Logic*, Chapter 5.1:

Clearly, while qualitative analogy is somewhat comparable to purely a fortiori argument, quantitative analogy is somewhat comparable to a crescendo argument; but they are still far from the same. Let us first compare and contrast *qualitative* analogical argument to *pure* a fortiori argument. For this purpose, let us first focus on the positive subjectal mood, viz.:

> P is more R than (or as much R as) Q,
>
> and Q is R enough to be S;
>
> therefore, P is R enough to be S.

Here, as in analogy, the major premise implies that both P and Q are R, but unlike in analogy, it additionally implies that $Rp \geq Rq$, i.e. that the quantity of R in P is greater than (or equal to) that in Q. Thus, though we can deduce the major premise of analogical argument from that of a fortiori argument, we cannot reconstruct the major premise of a fortiori argument only from that of analogical argument. Similarly, though the minor premise of a fortiori argument implies that Q is S, and therefore implies the minor premise of analogical argument, the reverse is not true. The difference between the two minor premises is that in a fortiori argument there is the element of *sufficiency of R to be* S, which is clearly lacking in argument by analogy. For the same reason, although the conclusion of a fortiori argument implies that of analogy, the latter does not by itself enable us to reconstruct the former.

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[10] I must say, I am surprised by the results shown here for subsidiary terms, because they lack symmetry. We have here one mood requiring that Sp be broader than Sq, and three moods where Sp must be narrower than Sq. This, in my experience, is unusual. It seems to me that either all four moods should be the same, or two moods one way and two the other way. But try as I might I do not see any error in my treatment here; so, I must accept this finding.

[11] Syllogism: all P are Sq, all Sq are Sp, so all P are Sp.

[12] Syllogism: all P are nonSq, all nonSq are nonSp (= all Sp are Sq), so all P are nonSp.

[13] Syllogism: all Sq are P, all Sp are Sq, so all Sp are P.

[14] Syllogism: all Sq are nonP, all Sp are Sq, so all Sp are nonP.

Moreover, even though each of the propositions (the major and minor premises and the conclusion) involved in a fortiori argument implies the corresponding proposition of analogical argument, this does not mean that an a fortiori argument implies an analogical one. For, the a fortiori argument is deductive, i.e. its conclusion follow necessarily from its two premises; whereas, as we have just shown, the argument by analogy, even in its complex form, is inherently inductive, i.e. it requires a generalization of its minor premise to enable us to draw its conclusion. Therefore, even if both arguments may be said to yield a common conclusion, namely 'P is S', that conclusion has a very different logical status in the one and in the other.

It follows that we can neither reduce a fortiori argument to argument by analogy, since the latter's conclusion does not imply the former's (even though the premises of the former do imply those of the latter), nor can we do the reverse, since the premises of the latter do not imply those of the former (even though the conclusion of the former does imply that of the latter). It does happen that we know enough to form the major premise needed for a fortiori argument, but we do not know enough for its minor premise; or we know enough to form the minor premise needed for a fortiori argument, but we do not know enough for its major premise – in such cases we might have enough information to at least formulate an analogical argument. Thus, sometimes we have more information than we need for an analogy, but not enough for an a fortiori argument – in such cases we can only formulate an analogy.

Therefore, though we can say that a fortiori argument and argument by analogy have some features in common, we must admit that they are logically very distinct forms of argument. This is a formal and undeniable demonstration, once and for all. To repeat: neither argument can be *reduced* to the other. However, every valid a fortiori argument *implies* a corresponding argument by analogy involving less information and certainty. The premises of the latter, as we have just seen, lose the quantitative and/or sufficiency factors involved in the former; and the conclusion of the analogical argument is, as a result, both less informative and less sure (being now inductive instead of deductive). But of course, except for the present theoretical clarification, there is in practice no point in resorting to such implication, since the given a fortiori argument is better in all respects.

As regards the opposite direction, it cannot be said that every analogical argument implies a corresponding a fortiori argument. All we can say is that we can, sometimes, when the facts of the case permit it, construct an a fortiori argument which implies the given analogical argument. This is possible if the latter argument has a middle term (R), or an appropriate middle term can be found for it, which can both be used as a continuum of comparison (which, I think, is always possible in practice, although we *cannot* tell a priori which term is greater than the other) and at the same time serve as the sufficient condition for the subject (Q) to access the predicate (S) in the minor premise (and this is, of course, *not* always possible in practice). Thus, the construction of a corresponding a fortiori argument from a given analogical argument is not a mechanical matter and cannot always be performed. In effect, when it is found possible, it just means that we should in the first place have resorted to the stronger a fortiori argument yet foolishly opted for the weaker analogical argument.

All that we have said here applies equally well, *mutatis mutandis*, to the negative subjectal forms of these arguments, and to positive and negative predicatal forms, and again to the four implicational forms. These jobs are left to interested readers. For reminder, the form of positive predicatal a fortiori argument is as follows:

More R (or as much R) is required to be P than to be Q,

and S is R enough to be P;

therefore, S is R enough to be Q.

As regards comparison and contrast between *quantitative* analogy and *a crescendo* argument, i.e. 'proportional' a fortiori argument, the following need be said. The major premises are the same in both. But the minor premises and conclusions obviously differ, insofar as in quantitative analogy there is no idea of a threshold value of the middle term as there is in a fortiori argument. This explains why the 'proportionality' is essentially non-directional in quantitative analogical argument (inference is always possible both from minor to major and from major to minor); whereas it is clearly directional in a fortiori argument (inference is only possible from minor to major in positive subjectal and negative predicatal argument, and from major to minor in negative subjectal and positive predicatal argument).

Note in passing that although quantitative analogy and mere pro rata argument (i.e. used alone, outside of a crescendo argument) are not formally identical the two are effectively the same. Compare for example the following two formulas; clearly, the provisos in them are essentially the same (a concomitant variation between the values of S and the values of R) even if the terms are differently laid out.

Given that P is greater than Q with respect to R, and that Q is S (Sq), it follows that P is proportionately more S (Sp), *provided that* the ratio of Sp to Sq is the same as the ratio of P to Q (quantitative analogy).

Given that if R has value Rq then S has value Sq, it follows that if R has value more than Rq (say Rp), then S has value more than Sq (say Sp), *provided that* the values of S vary in proportion to the values of R (pro rata argument).

To conclude: there is, to be sure, an element of 'analogy' in all human thinking, including in syllogism and in a fortiori argument, since all abstraction is based on mental acts of comparison and contrast; but to say this loosely is not the same as equating syllogism or a fortiori argument to argument by analogy. When we look into the exact forms of these arguments, we clearly see their significant differences.

5. Conflicting analogies

We have above seen that analogical argument has numerous moods, which are formally expressible and capable of validation. We shall now consider the issue of *conflicting analogies*, by considering two or more middle terms, i.e. R1, R2, etc., which yield different or conflicting conclusions. One analogy may be more credible or weighty than another. This refers to compound analogical argument *comprising both comparison and contrast* (instead of each in isolation from the other). We must here focus our attention on four compounds, which combine two like forms (not just any pair of forms, note). We may call either argument (the comparison or the contrast) 'the argument' and the other 'the counterargument' (although I here place the comparison before the contrast, the opposite order would do just as well of course).

First compound: positive subjectal moods.

Comparison: given that subject P is similar to subject Q with respect to predicate R1, and that Q is S, it follows that P is S.

Contrast: given that subject P is *dis*similar to subject Q with respect to predicate R2, and that Q is S, it follows that P is not S.

Second compound: negative subjectal moods.

> Comparison: given that subject P is similar to subject Q with respect to predicate R1, and that Q is not S, it follows that P is not S.
>
> Contrast: given that subject P is *dis*similar to subject Q with respect to predicate R2, and that Q is not S, it follows that P is S.

Third compound: positive predicatal moods.

> Comparison: given that predicate P is similar to predicate Q in relation to subject R1, and that S is Q, it follows that S is P.
>
> Contrast: given that predicate P *dis*similar to predicate Q in relation to subject R2, and that S is Q, it follows that S is not P.

Fourth compound: negative predicatal moods.

> Comparison: given that predicate P is similar to predicate Q in relation to subject R1, and that S is not Q, it follows that S is not P.
>
> Contrast: given that predicate P is *dis*similar to predicate Q in relation to subject R2, and that S is not Q, it follows that S is P.

Here we see that by referring to different aspects of P and Q, namely R1 and R2, we may obtain conflicting conclusions, and therefore finally no conclusion. Note that the minor premise is made identical in both cases, and the two major premises are not formally in conflict (since their middle terms differ), and the two argument forms are equally valid. Yet the conclusions are contradictory! Such conundrum is, of course, made possible by the fact that analogical argument is not purely deductive, but in part inductive. Its conclusions are suggestive, not decisive.

To be sure, in some cases we may be able to resolve the contradiction by refuting the analogy (i.e. the similarity or dissimilarity) claimed in the argument or the counterargument, or both; but this is of course not always possible. In some cases, even after an analogy relative to some middle term is found weak, we may still be able to posit the same analogy relative to another middle term which more strongly supports the putative conclusion; in which case, the conundrum remains.

Obviously, as when faced with any contradiction, we are called upon to carefully check our premises and ensure their credibility. And clearly, while some analogies may not resist criticism, and finally fall, or at least remain inconclusive, others may stand with relative ease, being objectively credible. So, it is inevitable for us, in the pursuit of knowledge, to be faced with such conundrums.

A special case of conflicting analogy is when R1=R2, i.e. when there is only one middle term R. In such cases, the two major premises in the four above compounds are contradictory, and the comparison and contrast arguments cannot both be valid. Also, if either of R1 and R2 implies the other, but not vice versa, then the two major premises are contrary[15], and the conflicting arguments cannot be both valid.

In any case, it should be emphasized that *no two things are the same in all respects*, or they would not be two but one; and *no two things are different in all respects*, or they would not be in the same universe. This means that the above-listed compound arguments are applicable to all things, and the problem of distinguishing significant similarities and dissimilarities from less significant ones

[15] This is easily proven. If R1 implies R2, then the two major premises are incompatible through R2; if R2 implies R1, then they are incompatible through R1. But since either case is possible, neither is necessary; so, the two major premises are merely contrary, not contradictory.

is unavoidable. It follows that we constantly estimate by some means or other, in each context, which similarities and dissimilarities are the most significant.

This thought suggests that we should, ideally, for any two items (the subjects or predicates labeled P and Q), systematically find and list all the ways (i.e. the middle terms R1, R2, etc.) in relation to which they are similar or dissimilar. We would then verify, for each middle term considered, how the minor and major terms (P and Q) relate to the subsidiary term S. Where the relation of S to Q is known and to P is not, we would infer the latter from the former as shown above. Where the relation of S to P is known and to Q is not, we would infer the latter from the former in the same way.

Then, at the end of this systematic research process we would have some idea as to how often the conclusion is positive rather than negative, or negative rather than positive. But of course, such *complete enumeration*, though ideal and theoretically conceivable, is usually not possible in practice. There is just too much similarity and difference between any pair of things. In practice, we investigate and refer to the relations between things as and when they happen to come to our attention. Our knowledge evolves gradually as our experience (whether obtained by passive observation or active experiment) grows and our theoretical insights concerning it become more complex and accurate. Over time, then, our views may change regarding which conclusion is the most significant.

6. Statistics-based analogical arguments

The difficult question we need to try and answer here is: how to decide which of the two opposed arguments is the most convincing? I suspect that in everyday practice *intuition* plays a large role in most cases – our perceptions of which common factor, R1 or R2 (or others still), is the most 'significant' in the context concerned. A more formally expressible way to answer our question may, however, be to multiply the number of comparisons and contrasts (not limiting ourselves to two middle terms), and then base our final conclusion on the more numerically weighty side. This is a statistical method.

The principle would here be: If two things (P and Q) are alike in numerous ways (collectively, R1) and differ in numerous ways (collectively, R2), and they are alike more often than they differ, then we may assume that a subject or predicate (S) found to relate to the one (say, Q) probably also relates to the other (P) – the degree of probability being determined by the ratio of similarity to dissimilarity. If the major premise is that they are different more often than they are alike, then the probability is instead in favor of the conclusion being opposite to the minor premise.

The justification for such statistical argument is generalization: a relation that we found to hold in a majority of *known* cases may, by extrapolation, be assumed to hold in most *unknown* cases; inversely, if the relation holds only in a minority of known cases, there is no reason to expect it to hold in subsequent unknown cases. There is admittedly no certainty here, only probable expectation; but there is some justification: the conclusion is more likely to be thus than otherwise. The greater the probability the more trustworthy our conclusion.

We can thus propose the following four moods of what we may characterize as *statistics-based* analogical argument. Such forms of argument are clearly logically fuller than the forms initially proposed, because they consciously deal with the issue of conflicting analogies. Note that I have conventionally put the minor term in the minor premise and the major term in the conclusion in every case, although I could equally well have opted for the opposite ordering; this was done just to facilitate remembrance. In subjectal argument, the major term P is subject of the conclusion and

the subsidiary term S is predicate; whereas in predicatal argument, S is subject of the conclusion and P is predicate. In positive argument, the conclusion has the same polarity as the minor premise; while in negative argument, the conclusion has the opposite polarity to the minor premise.

Positive subjectal analogical argument:

> Given that subject P is like subject Q with respect to *considerably many* predicates (collectively, R1), and that Q is S (some new predicate), it follows that P is probably S too. For, given that subject P is *un*like subject Q with respect to *relatively few* predicates (collectively, R2), and that Q is S, it does *not* follow that P is probably not S. Conclusion: P is probably S.

Negative subjectal analogical argument:

> Given that subject P is *un*like subject Q with respect to *considerably many* predicates (collectively, R1), and that Q is S (some new predicate), it follows that P is probably not S. For, given that subject P is like subject Q with respect to *relatively few* predicates (collectively, R2), and that Q is S, it does *not* follow that P is probably S. Conclusion: P is probably not S.

Positive predicatal analogical argument:

> Given that predicate P is like predicate Q in relation to *considerably many* subjects (collectively, R1), and that (some new subject) S is Q, it follows that S is probably P too. For, given that predicate P is *un*like predicate Q in relation to *relatively few* subjects (collectively, R2), and that S is Q, it does *not* follow that S is probably not P. Conclusion: S is probably P.

Negative predicatal analogical argument:

> Given that predicate P is *un*like predicate Q in relation to *considerably many* subjects (collectively, R1), and that (some new subject) S is Q, it follows that S is probably not P. For, given that predicate P is like predicate Q in relation to *relatively few* subjects (collectively, R2), and that S is Q, it does *not* follow that S is probably P. Conclusion: S is probably not P.

The middle terms R1 and R2 are here referred to as ‘collective’ with the intent that each of them represents numerous unspecified middle terms for which the stated proposition applies. In subjectal moods, the middle terms are predicates of the major premises; while in predicatal moods, they are subjects. Obviously, if the expressions “considerably many” and “relatively few”, applied to the middle subjects or predicates (the Rs), can be more precisely quantified, and the bigger number grows and the smaller number shrinks, the probabilities of the conclusions increase.

Needless to say, all problematic conclusions arrived at here are inductive, meaning that they are valid only until and unless new empirical findings or deductions or stronger probabilities override them. They are not fixed, final results, but the best available results in the given context.

7. A scientific illustration

Needless to say, analogy is very often used in everyday thought, and therefore (though perhaps, ideally, more rigorously) in scientific thinking. All conceptualization (and therefore all knowledge, ultimately) is, of course, based on analogy, since we need to become aware of the apparent similarities and differences of things in order to decide whether to classify them together or apart.

I found a scientific illustration of analogical thinking in a recently published book on paleontology[16], which I happened to have purchased and started reading (with no purpose other than pleasure) just as I was developing the above thoughts on analogy. It is worth examining this illustration in some detail (without delving very deeply into the paleontological details) to see what logic can be learned from it.

There we are told that the hunting behavior of dinosaurs, for instance, is induced from other known features of dinosaurs with reference to "modern analogues" chosen, not randomly by referring to just any other predators, such as wolves or sharks, but by means of "**bracketing**." This consists in comparing dinosaurs more specifically to extant *close relatives* of theirs in the evolutionary tree, namely birds and crocodiles. The basis for analogy between 'close relatives' is, clearly, that they are already known (or even merely believed at that stage) to share many *distinctive* characteristics. The author explains:

> "If crocodiles *and* birds share some detail… then dinosaurs had it too. We can't say dinosaurs had feathers simply because birds have feathers – crocodiles do not have feathers, so dinosaurs are not bracketed as far as that character is concerned."[17]

Putting this argument in more formal terms we obtain the following:

> Subject A (dinosaurs) is known to have many characteristics (middle terms, left tacit here, e.g. genetic or morphological traits) in common with subjects B (birds) and C (crocodiles), therefore, with regard to some feature D (say, an anatomical detail or a behavior pattern): if both B and C have D, then A probably has D too, or if both B and C lack D, then A probably lacks D too; but if B has feature D whereas C lacks it, or if B lacks feature D whereas C has it, we cannot (with equal certainty) predict whether A has or lacks D.

This is, of course, merely probable reasoning – for it remains conceivable, and may well happen on occasion, that A differs as regards D from the indications *suggested by* B and C. It remains true that A may have some unique, novel trait D while B and C both lack it; or A may distinctively lack D while B and C both have it; or A and B may both have D while C lacks it; or A and B may both lack D while C has it; and so forth. Nevertheless, the proposed method of bracketing provides us with some direction, due to the major premise that A is already established as having many distinctive features (which are left tacit here, but together constitute the logically operative middle term) in common with both B and C.

Note that the form of this argument is positive subjectal, with A as the major term, B and C as two minor terms, the unspecified properties they all share as middle terms, and D as the subsidiary term. What is not mentioned here is the mass of differences between A on the one hand, and B & C on the other, although being non-identical they are bound to have many differences. This can be seen if we cast the argument more explicitly in the form of a standard statistics-based analogy:

> Given that subject P (A, dinosaurs) is like subject Q (comprising both B and C, birds and crocodiles) with respect to *considerably many* predicates (collectively, R1 – here unspecified), and that Q is S (some predicate D), it follows that P is probably S too. For, given that subject P is *un*like subject Q with respect to *relatively few* predicates (collectively, R2 – here unspecified), and that Q is S, it does *not* follow that P is probably not S. Conclusion: P is probably S (i.e., in our example, A is probably D).

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[16] *The Dinosaurs Rediscovered: How a scientific revolution is rewriting history.* By Michael J. Benton (London: Thames & Hudson, 2019-20).

[17] See pp. 16-17. Reasoning by bracketing was first proposed by Larry Witmer in 1995. The resort to 'analogy with living forms' (p. 189) to interpret aspects of fossil forms was an established method long before that, of course.

Clearly, the second part of the compound shown above (i.e. the negative counterargument) was *left tacit* in the above example, it being presumed that the differences between A and B & C, with respect to another set of middle terms (unspecified), which could point us to an opposite conclusion, were *insufficiently frequent to stand out and matter*. The counterargument is, no doubt, at least subconsciously considered by scientists in practice, drawing on their vast stores of individual and collective knowledge. But to be on the safe side, in practice scientists should always consciously consider and determine the relative likelihood of the counterargument. Because in fact, both sides of the full argument are logically relevant.

It should be obvious that the use of two minor terms (B and C), in preference to only one (either B or C alone), is that this increases the probability of the conclusion about A, which effectively is impressed on us convergently, twice instead of only once. Moreover, if the analogues B and C point to divergent conclusions (both D and not D), we are left with doubts concerning A. As already suggested, the terms A, B, and C should preferably be closely related, as this increases the probability of the result. If they have some characteristic(s) in common, that is good; but if they have some *distinctive* characteristic(s) in common, that is much better, for that fact ties them more closely together, and increases the chances (though of course, still does not ensure) that they will also share the concluded characteristic (D).

Obviously, too, this kind of compound reasoning can be pushed further, by involving more than two modern analogues. *The more analogues the merrier*, since this (to repeat) increases the probability of the conclusion. That is, if subject A is correlated with several more analogues (instead of just B and C) and they are also found to have D, the probability grows that A is also D. This, then, is one important lesson we can learn from the technique of bracketing – viz. that the probability of the conclusion can be increased by referring, not just to more numerous middle terms (as earlier remarked), but also to more numerous minor terms.

As regards probability ratings, that is not just talk here. It is true that in ordinary discourse, probabilities are very roughly 'estimated' based on personal experience and memory, and even bias, and people may well disagree as to their directions and magnitudes. But in scientific discourse, the issue is taken much more seriously, and great effort and expense are invested to determine probabilities as accurately as possible. Contemporary scientists[18] use a wide array of more and more sophisticated observational and experimental techniques, marvelous technological tools and measuring instruments, ingenious mathematical and computational methods, and extremely powerful computers, to obtain the data they seek. Their professional credibility and reputation depend on their rigor. The consequence is certainty increasing over time, sometimes at an exponential rate.

Modern researchers are admirable in the amount of care and effort they put in to arrive at their conclusions. This is well illustrated in the book on dinosaurs we have here mentioned[19]. By the year 2000, some 500 species of dinosaur had been discovered and named in the world. Scientists wished to classify them relative to each other, in a complete evolutionary tree, as accurately as possible. They collected, merged, and tabulated all known information from hundreds of published papers; and using complex software and powerful computers managed to find the statistically most likely classifications for hundreds of known species. More recently, they have started to reexamine specimens stored in museums and universities across the world, looking for the presence or

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[18] Such as the paleontologists in the referenced book.

[19] Pp. 76-77, 82-83.

absence of 457 anatomical characters in each case, to obtain a still more complete and more accurate tree.

Obviously, such a tree facilitates bracketing, among other things. It is a brief, visual repository of large numbers of comparisons and contrasts.

I should also mention, as illustrations of the use of analogy in scientific contexts, **medical applications of analogical argument**. When we visit a doctor for a checkup, he examines us in a variety of ways, and may diagnose some unhealthy condition, and prescribe some possibly preventive or curative measures, and venture a prognosis. All this involves reasoning of various kinds, especially causal and analogical. The following is the kind of thinking the doctor may engage in:

> Diagnosis: The symptoms a, b, c, observed in this patient resemble those of other people who were found to have disease so-and-so; therefore, my patient is likely to get probably or has the same disease.
>
> Prevention/cure: Clinical tests and widespread practice have shown that such-and-such medical treatment is efficacious in preempting or combating disease so-and-so; therefore, my patient will likely benefit from similar treatment.
>
> Prognosis: If the patient does not follow such treatment, his condition will likely deteriorate in this way and that; but if he takes proper care of his medical needs, as I prescribe, his condition will likely improve in various respects.

Diagnostic reasoning largely consists of causal logic; but it also manifestly involves some analogical logic, insofar as one compares and contrasts symptoms observed in the patient to a list of symptoms known to medical professionals to date through extensive research and experience. Symptoms may be observed with the naked eye by the doctor, or felt and reported by the patient, or identified by various medical tests using simple tools or complex technologies.

Obviously, different diseases may display some similar symptoms. One further confirms the identification of the disease by referring to more and more symptoms, ideally to an exhaustive list of symptoms. To be able to zero-in to and pinpoint the applicable disease with certainty, one needs to find an exclusive set of symptoms. If some essential (i.e. necessary, *sine qua non*) symptom(s) of a disease is/are lacking, then that disease can be eliminated from the list of possible diseases for the observed symptom or set of symptoms.

Similar reasoning is used in deciding preventive and curative measures, which may include not only medicines and surgical operations, but lifestyle and environmental recommendations, nutrition, exercise, physiotherapy, and so forth; and to prognosis, which may be positive or negative, and have various degrees of probability.

8. Use of analogy in making and applying law

Analogical argument is common not only in everyday thought and discourse by everyone, and in more scientific contexts, but it is also quite widespread in legal contexts. It is an instrument of law development and application used in all legal systems. Examples are easily found in ancient systems (like the Greek, the Roman or the Talmudic), in medieval systems (like the Christian, the Islamic or the Rabbinic), and in modern systems (like the British, the American or the French). It does not matter whether the political system involved is essentially dictatorial (as, say, in Russia

or China today[20]) or essentially free and democratic (as in Western countries today) – reasoning by analogy by legislators or judges is widespread.

Legislators aim to enact new laws, producing 'statutory law', while judges aim in principle to apply the laws the latter hand down to them, although, by establishing binding precedents, courts effectively amplify the law, producing 'case law', and moreover some supreme courts take this interpretative power far beyond the manifest original intent of legislation and get quite 'creative'.

Analogical argument helps maintain some degree of consistency and uniformity in the law. If analogies and disanalogies were ignored, a law system might include a smorgasbord of relatively contradictory laws, which could be used to arbitrarily form lenient or stern judgments, as judges please, depending on their political or other personal prejudices or even just their current moods. Such *à la carte* legislation is obviously contrary to justice.

The argument by analogy may be used in legal contexts in several ways: (a) we may formulate new laws on the basis of general ethical or political principles[21]; (b) we can derive specific laws from constitutional guidelines; (c) we can make new laws by imitation of existing laws for comparable situations; (d) we can argue for the application of an existing law to a particular case under consideration; (e) we can make use of legal precedents, examining past cases resembling the present case, and proposing a like judgment for it; or (f) we can resort to some combination of these ways. For each of these ways, or a combination of them, an argument by analogy can be constructed, provided we perceive (and preferably make explicit) some significant commonality between the source and target situations. The argument would look something like the following (positive subjectal, comparing):

> Since [major premise] the situation under consideration (= major term, P) *resembles* the situation envisioned by such and such general ethical or political principles (a), or constitutional guidelines (b), or existing laws (c, d), or legal precedents (e) (= minor term, Q), with respect to this and that (= middle term, R),
>
> and [minor premise] this source (Q) prescribes some legal course of action[22] (= subsidiary term, S),
>
> it follows by analogy that [conclusion] for the target situation (P) we ought to establish or apply a like legal course of action (S).

Needless to say, while the analogy may be *prima facie* quite convincing, it might eventually be credibly contested; because such argument is never logically decisive, but at best indicative. It might be argued that P does not resemble Q sufficiently or in significant respects R, or that while it is comparable with respect to R, it is rather different with respect to certain other factors (another middle term), and therefore that the formulation for P of a law or judgment S similar to that previously settled for Q is not wise. Such counterargument can also be formulated in standard form, as follows (negative subjectal, contrasting):

> Since the situation under consideration (P) *does not* resemble the situation envisioned by such and such general ethical principles (a), or constitutional guidelines (b), or existing

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[20] To give an example, Richard Pipes in *Russia Under the Bolshevik Regime* (Knopf, New York, 1993) writes (p. 402) concerning Art. 10 of the Criminal Code: "The principle of 'analogy' made it possible to charge citizens for crimes not directly defined but similar in nature."

[21] For instance, arguing that since a man has a natural right to life and liberty, he cannot be executed or imprisoned at will (but only eventually under specific conditions, i.e. following demonstrated criminal behavior punishable by law, and after due process). The legislation is intended to give concrete, practical expression to the abstract, philosophical principle.

[22] Such as an appropriate verdict or penalty.

laws (c, d), or legal precedents (e) (Q), with respect to this and that (R), or with respect to certain other factors, and this source (Q) prescribes some legal course of action (S), it follows by disanalogy that for the target situation (P) we ought *not* establish or apply a like legal course of action (S).

Analogical argument should not be confused with a fortiori argument, which is more complex (see my work *A Fortiori Logic* for a thorough treatment of such argument[23]). At this point, we should of course propose numerous examples from various historically and geographically different legal systems[24]. I shall, however, be content with the presentation of one Talmudic example, which I find intellectually interesting and challenging because of the convoluted thinking it involves. The reader would do well to read it carefully, even if indifferent to Talmudic content, as there is much to gain in logical acuity and skill from this demanding exercise.

9. A Talmudic illustration

We shall now examine a Talmudic illustration of the sort of more complex analogical reasoning we introduced earlier, with reference to a discussion found in the Babylonian Talmud, tractate *Baba Kama*, pp. 20a-21a. My attention was drawn to this long *sugya* (pericope) by R. Louis Jacobs, who presents a detailed literary analysis of it in one of his works[25]. I here only present a small of part of the discussion, and that as briefly as possible, because I am not really interested in the specific legal issue under discussion, but merely wish to illustrate and evaluate the use of analogy in the halachic discourse of the Talmud. My account is based on the Soncino English translation of the Talmud[26] as well as on Jacobs' reading; but all logical analyses and eventual critical comments are entirely my own.

It is evident from this lengthy example that analogical argument plays a large role in Talmudic (and later, rabbinic) reasoning. We learn from it that when the rabbis wish to establish a new legal ruling, they resort to various analogies found in Mishnaic (or, in other contexts, in Biblical or otherwise traditional proof-texts, or even as a last resort in authoritative statements by rabbinic deciders[27]), as the possible basis of that proposition – and this is where the issue of differing or even conflicting analogies comes into play. The issue being: which of a set of proposed analogies is the most apt, the one to prefer? The problem here, as against in more scientific contexts, is the difficulty in evaluating the relative relevance of conflicting analogies.

The central question posed by our *sygya* is the following. A certain rabbi, R. Hisda, wonders whether "*one who occupied his neighbour's premises unbeknown to him would have to pay rent*

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23 See section 4 of the present work for comparisons and contrasts between the two forms of argument.

24 The reader can, I assume, readily find many such examples through legal websites or in libraries.

25 *Structure and Form in the Babylonian Talmud* (Cambridge: Cambridge UP, 1991). See Chapter 5 (pp. 56-64). Indeed, it is through reading that essay that I realized that my presentation of analogical argument in *A Fortiori Logic* was far from complete, and I was moved to write the present more thorough essay. The aim of Jacobs' analysis is to show how the Talmud collects and orders information and arguments from different sources and times to form an instructive literary unit; it does not randomly or chronologically report discussions but organizes them purposely in a seemingly logical progression. My aim here is very different: it is to study the logical discourse used.

26 *Soncino Babylonian Talmud*. Ed. Rabbi Dr. I. Epstein. London: Soncino, 1952. The full text can be found in Halakhah.com. The explanatory comments in square brackets are given there, too.

27 In some cases, even within this *sugya*, they just seem to rely on the greater authority of some exponent. This is, of course, *ad hominem* argument, although its intent is positive. The authorities referred to are so considered because they are viewed as bearers of the oral traditions handed down since the time of Moses. However, there is no denying that they are in fact often at odds. Traditional commentary on this fact asserts that they are nevertheless (somehow) all right.

or not." I shall here call, for the sake brevity and clarity, the occupier 'the squatter' and the owner of the premises 'the landlord'. The Gemara[28] offers the following clarification of the issue:

> "But under what circumstances? It could hardly be supposed that the premises were not for hire [and would in any case have remained vacant], and he [the one who occupied them] was similarly a man who was not in the habit of hiring any [as he had friends who were willing to accommodate him without any pay], for [what liability could there be attached to a case where] the defendant derived no benefit and the plaintiff sustained no loss? If on the other hand the premises were for hire and he was a man whose wont it was to hire premises, [why should no liability be attached since] the defendant derived a benefit and the plaintiff sustained a loss? — No; the problem arises in a case where the premises were not for hire, but his wont was to hire premises."

From which we know that in the case under consideration the squatter benefits (since he lacked somewhere to stay free of rent), but the landlord does not suffer a loss (since he allowed the place to remain empty at that time, even if he usually sought to rent it) – in Hebrew this case is referred to as *zeh neheneh ve-zeh lo-haser* (= this one benefits and that one does not suffer loss). After the fact, the landlord might say to the squatter "Since you have derived a benefit [as otherwise you would have had to hire premises], you must pay rent accordingly;" while the squatter might refuse to pay rent to the landlord, arguing "What loss have I caused to you [since your premises were in any case not for hire]?"

The answer to the question is sought through consideration of the legal rulings made in other contexts involving a protagonist/defendant (like the squatter) who benefits from something and an antagonist/plaintiff (like the landlord) who does not suffer a loss, i.e. having the same *zeh neheneh ve-zeh lo-haser* scenario. If in such comparable situation the ruling was that the protagonist is liable to pay something to the antagonist, it is assumed that the same ruling of liability can be applied to 'our' case (i.e. the above-mentioned case of landlord versus squatter). If in such comparable situation the ruling was non-liability, then in our case that will be assumed to be the applicable ruling. The analogical argument pursued here is thus the following:

> Just as, in the proof-text, where the protagonist benefits and the antagonist does not suffer loss, the law was that the former is obligated (or not obligated, as the case may be) to pay some compensation to the latter;
>
> likewise, in our case, where the protagonist benefits and the antagonist does not suffer loss, the law must be that the former is obligated (or not obligated, as the case may be) to pay some compensation to the latter.

Call these two sentences the source of analogy and the target of analogy. Note well that both cases involve the scenario *zeh neheneh ve-zeh lo-haser*; this is what binds them together, their common ground. The first paragraph provides a hypothetical proposition (the source) that in a previous case involving this scenario (the antecedent) the ruling was so and so (the consequent); the second paragraph formulates a like if-then statement (the target) for the new case, arguing that since it has the same antecedent, it may be assumed to have the same consequent. In this way, a ruling is proposed for the new case. It must be stressed, however, that this inference is inductive, not

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[28] The Talmud includes Mishna and Gemara. Each Mishna passage is presented verbatim, then discussed by the Gemara, though other topics might also be treated in passing. The term 'Gemara' refers to the anonymous editor(s) who compiled discussions, associated somehow with the stated Mishna, by various named rabbis in various periods, putting those discussions in some purposeful order, usually with a commentary binding them together. Other commentators, such as Rashi or Tosafot, may come into play long after the Gemara, asking questions or clarifying points not found explicitly treated by the Gemara. The Mishna is dated at c. 200 CE and the Gemara at about 500 CE.

deductive; it is not logically inconceivable that the ruling might turn out to be different in the two cases on other, more plausible, grounds.

We can rephrase this argument in the standard format for (positive subjectal) analogical argument as follows:

> Given that our case (= major term, P) is similar to the proof-text case (= minor term, Q) in involving the scenario *zeh neheneh ve-zeh lo-haser* (= middle term, R), and that in the proof-text case (Q) the law was so-and-so (obligation to pay, or not, as the case may be) (= subsidiary term, S), it follows that in our case (P) the law should likewise be so-and-so (obligation to pay, or not, as the case may be) (S).

A putative example in our *sugya* of such analogical argument-form is the following. Another rabbi, Rami bar Hama, claims that the solution to the problem posed by R. Hisda is to be found in Mishna *Baba Kama* 2:2, which reads[29]:

> "In what case is this statement applied, that one pays the full value of the food eaten by the animal? It is a case where the animal ate the food on the property of the injured party; but if the animal ate food in the public domain, the owner of the animal is exempt from liability. And even if the animal ate food in the public domain, if the animal derives benefit from eating another's produce in the public domain, the owner pays for the benefit that it derives, just not for the full cost of the food."

This passage of the Mishna comprises three sentences. The first is a reference to a law given in Exodus 22:4. This Torah passage states that "If a man cause a field or vineyard to be eaten, and shall let his beast loose, and it feed in another man's field; of the best of his own field, and of the best of his own vineyard, shall he make restitution."[30] The second sentence in our Mishna is derived from the first by a *davka* (just so) reading, taking it to mean that the liability exists *only if* the loose beast feeds illicitly in a private domain; whence it is inferred that if the problem arose in the public domain, there is no liability (although, logically, partial liability is also a possibility). Note that this is a Mishna ruling based on inference; it is not an explicit Torah given.

The pattern of *davka* inference is always like this: if the proof-text specifically mentions case X ("in another man's field," in the present context), and does not explicitly mention cases other than X (i.e. non-X), then it is assumed that the intent of the omission must have been *to exclude* non-X (namely, here, the public domain). This is a common form of reasoning in Talmudic and rabbinic logic. It should be clear that *davka* inference is inductive, not deductive, since it is logically conceivable (though in fact not the case here) that another text might have been found that included non-X without this implying contradiction (i.e. there could well have been another Torah passage specifying that in the public domain, too, there is liability).

Indeed, even if no Torah passage is found that explicitly provides the missing information, it does not follow that *davka* inference is inevitable and sure. An opposite form of reasoning is possible, and indeed is sometimes practiced; it is called *lav davka* (not just so). One could have in the present context, for example[31], argued that the reason the Torah did not mention an animal eating food in the public domain was because it considered it obvious enough that in such case the animal's owner is liable to pay the food owner full compensation. That is, the argument goes, the Torah

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[29] I here quote the three sentences in the Mishna of interest to us using the translation in Sefaria.org because it is clearer than the one given in the Soncino ed.

[30] Translation taken from Mechon-mamre.org.

[31] Needless to say, I am not here advocating the use of *lav davka* reasoning in the present context. I am merely illustrating the form that a *lav davka* reading would have taken in the present context. I have no interest in contesting the *davka* reading implied in the Mishna.

only mentioned the case of an animal eating food in the private domain requiring full compensation because it considered that it was not so obvious. In this perspective, anything left unattended in the public domain is 'obviously' protected by law, whereas in the private domain the property owner might well be expected to protect all objects therein, say by fencing or a guard dog; and the Torah comes forth to say: "No, even in the private domain the law must protect unattended objects." Such thinking is quite conceivable; so, *davka* reasoning is not deductive, but merely inductive. Likewise, of course, for *lav davka* reasoning.

The second sentence in our Mishna, then, informs us that if a domestic animal illicitly eats food left unattended in the public domain, the animal's owner is not liable to pay the food's owner for his loss. The third sentence informs us that the protagonist (the animal owner) is nonetheless obligated disburse to the antagonist (the food owner) what feeding his animal *would have* cost him, i.e. the amount of money he saved due to his animal feeding illicitly (presumably, a much lesser amount).[32]

We thus have two Mishna rulings that seem contradictory at first blush: the first states that there is no liability (but it means: not the full liability occurring in the private domain); the second states that there is some liability (but it means: a minimal liability equal to the usual cost of ordinary feed). Rami focuses on the last sentence to build his argument. The analogy, as he sees it, is as follows:

> Just as, in Mishna *Baba Kama* 2:2, where the animal owner benefits and the food owner does not suffer loss (in a *de jure* viewpoint, because what he did in fact lose was lost in the public domain), the law was that the former is obligated to pay the latter the minimal cost of feeding (even though he is not liable to pay full compensation);
>
> likewise, in the R. Hisda case, where the squatter benefits (since he disposed of no other place) and the landlord does not suffer loss (since he was content to leave the place empty), the law should be that the former is obligated to pay the latter a minimal rent (even though he is not liable to pay full compensation).

Or putting it in standard form (positive subjectal analogy):

> Given that our case (P) is similar to the Mishna *Baba Kama* 2:2 case (Q) in involving the scenario *zeh neheneh ve-zeh lo-haser* (R), and that in the Mishna case (Q) the law was that the protagonist (animal owner) is obligated to pay the antagonist (food owner) the amount of his benefit (S), it follows that in our case (P) the law should likewise be that the protagonist (squatter) is obligated to pay the antagonist (landlord) the amount of his benefit (S).

As we shall see, such argument can be opposed in various ways. The most obvious counterargument to it would be as follows (positive subjectal analogy with a negative major premise):

> Given that our case (P) is *not* similar to the proof-text case (Q) in involving the scenario *zeh neheneh ve-zeh lo-haser* (R), and that in the proof-text case (Q) the law was so-and-so (obligation to pay, or not, as the case may be) (S), it follows that in our case (P) the law should on the contrary *not*-be so-and-so (obligation to pay, or not, as the case may be) (i.e. *not*-S).

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[32] The exact basis of this additional ruling by the Mishna is not, as far as I can see, explicitly stated or immediately apparent. It could simply be rabbinical fiat. Maybe its basis is obvious to cognoscenti, but I don't know what it is.

Indeed, in the Talmudic narrative under consideration, a third rabbi, Rava, rejects the analogy proposed by Rami, arguing that "in the case of the Mishnah the defendant derived a benefit and the plaintiff sustained a loss, whereas in the problem before us the defendant derived a benefit but the plaintiff sustained no loss." Thus Rava argues (in a more *de facto* spirit than Rami) that in the Mishna the food owner has, objectively, suffered a financial loss (the real value of the food eaten minus the smaller compensation due from the animal owner), whereas in the case at hand the landlord has not done so (since he would not, in fact, have received rent at that time if his place had not been squatted).

This means that Rava does not agree with Rami that the landlord is due compensation from the squatter. Rava thus proposed the following counterargument, put in standard form:

> Given that the present case (P) is *not* similar to the Mishna *Baba Kama* 2:2 case (Q) in involving the scenario *zeh neheneh ve-zeh lo-haser* (R), and that in the Mishna case (Q) the law was that the protagonist (animal owner) is obligated to pay something to the antagonist (food owner) (S), it follows that in our case (P) the law should on the contrary be that the protagonist (squatter) is *not* obligated to pay anything to the antagonist (landlord) (*not*-S).

According to Rava, then, the scenario of the Mishna referred to is that of *zeh neheneh ve-zeh ken-haser* (= this one benefits and that one *does* suffer loss); and this does not correspond to the putative scenario of the case at hand, which is *zeh neheneh ve-zeh lo-haser*. As we have seen earlier, the Gemara explicitly states that in such case, i.e. where the protagonist benefits and the antagonist suffers loss, the former must indeed pay compensation to the latter. For it is obvious, in its view, that if the squatter had no other premises to occupy and the landlord wished to rent the place at that time, there is indeed need to pay rent[33].

Notice that we have come across, here, examples of both a positive argument (similarity between cases) and a negative counterargument (dissimilarity between cases). We thus apparently have, in this *sugya*, examples of two related moods of the argument (analogy and disanalogy of positive subjectal form). Since they involve the same middle term, their major premises are contradictory and they cannot both be valid. Note that Rami and Rava were contemporaries; they were third generation Amoraim (fl. c. 300 CE).

At his point, it should be noted that the Talmud comes to the defense of Rami by means of the following remark: "Rami b. Hama was, however, of the opinion that generally speaking fruits left on public ground have been [more or less] abandoned by their owner [who could thus not regard the animal that consumed them there as having exclusively caused him the loss he sustained, and the analogy therefore was good]." (Note that the explanations given in square brackets in the Soncino edition water down somewhat the position of the Gemara.)

The Gemara is here trying to 'rescue' Rami's argument from Rava's objection by claiming that the food left in the public domain was effectively *hefker*, i.e. mentally given up on by its owner, so that the latter could not blame the animal for its loss; whence, when the Mishna ruled that the animal owner had to pay a small amount, it was not as compensation for a loss sustained by the food owner (as Rava claimed) so much as payment for the benefit received by the animal owner.

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[33] The Gemara also considers that in the event of 'no benefit for the one and no loss for the other', the former is not liable to pay the latter. The scenario of 'no benefit for the one and loss for the other' is not addressed in the Gemara, but (I gather from Jacobs' account, n.3) there is a Tosafot commentary about it. Such a scenario is conceivable; one could for instance refer it to a vandal damaging vacant premises.

In this perspective, then, the Mishna precedent was indeed a case of *zeh neheneh ve-zeh lo-haser* (as Rami claimed) and not a case of *zeh neheneh ve-zeh ken-haser* (as Rava claimed).

The Gemara is here projecting (maybe a couple of centuries later) a thought into Rami's mind that he did not openly express, so as to make him seem to have anticipated Rava's objection and taken it into account. However, the Gemara's intervention turns out to be weak. Jacobs, in an endnote (n.6), informs us of an interesting objection to it by a Tosafist that, in Jacobs' words, "the Talmud cannot mean that the owner has automatically and totally abandoned the food since, if that were the case, there would be no payment at all, the food no longer being his." This observation effectively neutralizes the Gemara's attempted refutation of Rava's counterargument.[34]

So, this additional discussion turns out to be something of a useless digression. We are left with an argument by Rami and a counterargument by Rava, and we need to know which of the two to prefer. Both seem convincing, at least superficially, and it is hard to choose between them. The Talmud is evidently not wholly satisfied with the arguments of Rami and Rava, or even with its own defense of Rami against Rava, since it goes off looking for other arguments that might more convincingly answer the question put by R. Hisda; but it does not make clear why it does that.

For our part, the following **critical remarks** seem relevant. Please note well that I have no halakhic axe to grind. I am not trying to prove the Talmud, or any rabbi mentioned in it, right or wrong. I do not care what the legal outcome of the discussion might be, though I am of course concerned with the logical propriety or inadequacy of the arguments encountered. My ultimate interest in examining this Talmudic passage is to see what lessons can be learned from it for formal logic (and, as will be seen, I did indeed learn some lessons).

As already shown in our theoretical treatment of conflicting analogy, there is no formal way to resolve the conflict between a comparison and a contrast; formally, either thesis might be right. One has to dig deeper into the problem at hand and try to find reasons to prefer one thesis or the other. In the discourse under scrutiny, we can certainly point out that one possible flaw is the variable (or ambiguous or equivocal) use of terms. Each of the predicates 'benefits', 'suffers loss', 'is liable', and their negations, although on the surface seemingly uniform in meaning, is in the course of this discussion (and again as it is extended later on in post-Talmudic commentaries) used in selected restrictive ways, which can be characterized as conventional (or even as subjective or as arbitrary).

Thus, the squatter in R. Hisda's narrative is regarded by the Gemara as having 'benefited' only if, when he occupied the premises, he had no alternative place to stay at his disposal; i.e. only if he needed the place he squatted. (Needless to say an invited guest is not a squatter.) But objectively, one could argue that *the mere fact* that the squatter voluntarily occupied that place implies that he considered doing so as of some value to himself (else he would not have done it). In which case, *all* squatting is benefiting somewhat, and *no* scenario involving squatting could be truly said to involve no benefit to the protagonist. The same can be said for the animal owner (in the Mishna

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[34] There would be no reason for the animal owner to pay anything to the food owner if the latter did not own the food any longer at the time the animal ate it. Jacobs suggests that perhaps the meaning is "not that he [the food owner] has abandoned the food, but that the Torah has abandoned it in declaring that there is no *shen* [i.e. no liability] in the public domain." However, I do not see any significant difference between the Torah abandoning and the food owner abandoning, since the latter would naturally follow from the former. If the food owner abandoned, it was surely because he knew that the Torah abandoned; if he did not know the Torah (or more precisely, the *davka* inference from it), he would have no reason to regard his property as being as good as lost the moment he left it unattended – he would naturally assume or at least hope he would readily recover it upon his return (or else would not leave it unattended). The resulting neutralization of the Gemara's argument is therefore unaffected.

referred to): as of the moment his animal has fed, whether in the private or public domain, he has objectively (albeit fortuitously) benefited somewhat.

Again, the landlord is regarded by the Gemara as having 'suffered loss' only if he was actively seeking or at least mentally desired to rent the place out; otherwise, if he was apparently content to leave the place vacant, he is viewed as not having suffered loss. But one could reasonably argue that he has suffered loss by *the mere fact* that his property was used without his knowledge or permission, even if he was not actively seeking or even desiring to find a tenant (he might perhaps have been keeping the place vacant in case his mother-in-law came to visit). In which case, *all* squatting causes loss, and *no* scenario involving squatting could be truly said to involve no loss for the antagonist. The same thinking applies to the food owner: as of the moment his food has been eaten, whether in the private or public domain, he has objectively suffered loss (even if the law, whether Torah or Mishna, conventionally denies it).

On this basis, i.e. when we insist on *uniform terminology*, both the Mishna case and the R. Hisda case necessarily involve the scenario 'this one benefits, and that one suffers loss' – and Rami is wrong to view them as both *zeh neheneh ve-zeh lo-haser*; while Rava, though partly right in viewing the Mishna case as *zeh neheneh ve-zeh ken-haser*, is partly wrong in viewing the R. Hisda case as *zeh neheneh ve-zeh lo-haser*. Note that it is the Gemara which interprets the squatter as benefiting *restrictively*, only if he had no other premises to occupy, and the landlord as losing *restrictively*, only if he was hoping or trying to rent the place at the time. But since the Gemara's interpretations are restrictive, and it allows for other possible scenarios (notably, 'no benefit for the one and no loss for the other', and eventually 'no benefit for the one and loss for the other'), it is not arguing (as I am here doing) in favor of uniform terminology.

So much for the antecedent scenario (serving as the basis of analogy). As regards the consequent legal obligation (or not), here too we can observe variety in meaning. In the Mishna, following a *davka* (just so) reading of Ex. 22:4, the animal owner is declared exempt from compensating the food owner for the food lost, although the latter is nonetheless, by additional Mishnaic ruling, required to pay the former the (presumably relatively small) amount he would have had to disburse to feed his animal (had not that animal illicitly satisfied its hunger with the more expensive food it found unattended). Here, then, the protagonist is considered as being strictly-speaking 'not liable', even while he is legally obliged to pay the antagonist something; the smaller amount he is required to pay is not considered as falling under the term 'liable'. This is a conventionally *restricted* use of the term 'liable'[35]. Objectively, of course, any obligation to pay any amount is a liability. In that event, the Mishna's verdict is effectively that there is liability, even if one smaller than it might have been. Whence, in the case brought forward by R. Hisda, the verdict ought to be that the squatter must pay the landlord a minimal amount of rent (the minimum market rate for such a property at that time and place).

Granting all these considerations, it appears that the correct application of the Mishna precedent (taken as a whole) to the case at hand would be that the scenario involved is 'benefit for the one and loss for the other', and the resulting legal ruling should be partial compensation[36]. The food

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[35] The fiction being that the antagonist cannot, for his loss, make a financial *claim* (on the protagonist); but the protagonist nevertheless has a *duty* to pay the money he saved (to the antagonist). This is a fanciful distinction because, surely, given the latter legal duty, a legal claim could be made in court.

[36] *Some* compensation is at least implied. The compensation is not *full* because the Mishna has ruled that it cannot be, on the basis of a *davka* reading of Ex. 22:4. But had this Torah passage been read *lav davka*, compensation could well have been full, note. So, the compensation is necessarily *partial*. An additional rabbinical judgment makes it equal to the *minimal* amount the protagonist would have had to disburse had not the events described occurred.

owner does objectively suffer loss, and the animal owner is objectively liable to pay something; and the landlord can also be viewed as suffering loss, and on that basis the squatter can be regarded as liable to pay something. In that event, *neither* Rami's argument by analogy *nor* Rava's counterargument by disanalogy can be claimed to be as accurate as they initially seem. Putting our novel thesis in standard form, we obtain:

> Given that our case (P) is similar to the Mishna *Baba Kama* 2:2 case (Q) in involving the scenario *zeh neheneh ve-zeh ken-haser* (R), and that in the Mishna case (Q) the law was that the protagonist (animal owner) is obligated to pay the antagonist (food owner) the amount of his benefit (S), it follows that in our case (P) the law should likewise be that the protagonist (squatter) is obligated to pay the antagonist (landlord) the amount of his benefit (S).

The Talmud does not take into consideration this simple alternative interpretation, based on uniform terminology. From the start of its reflection, it binds itself to a more complicated approach, from which various logical possibilities arise. Perhaps it opts for this tortuous path because it is not really looking for a solution to the problem (determining a particular legal principle or law) but using the narrative as a convenient occasion to explore different situations and opinions. In that event, it has to keep the issue open and unresolved, even if somewhat artificially, so as to keep the conversation going. (We have seen a clear example of this in the Gemara's gauche attempt to rescue Rami from Rava.) The Talmud's motive is evidently primarily academic and didactic rather than exclusively focused on law-making.

But, so doing, the Talmud misses out on the said additional logical possibility! It never conceives it, let alone propose some credible reason to eliminate it. As we have seen above, the Gemara defines the problem needing solution from the get-go as a search for a precedent in which the protagonist benefits and the antagonist does not suffer loss. It arrives at that putative definition by claiming outright that the two scenarios, in which the former does not benefit and the latter does not suffer loss (for which there would be no liability) or the former does benefit and the former suffers loss (for which there would be liability), are not applicable to the case at hand. And it does not mention or eliminate the third possible scenario (which a Tosafist noticed), viz. that wherein the protagonist does not benefit and the antagonist suffers loss.

The Gemara does not tell us on what basis it has eliminated the said two alternative scenarios it mentions, nor explain why it does not mention the third possible scenario. Yet it adheres with impressive certainty to the fourth scenario (viz. 'this one benefits and that one does not suffer loss'). Most readers allow such offhand (or sleight of hand) claims to pass uncritically because they believe the Gemara has total knowledge and therefore absolute authority. But surely, if the Gemara resorts to reason at all, it must do so consistently and explain all its positions. It must convincingly justify the certainties it displays.

One can readily agree with the Gemara that a squatter who usually pays rent elsewhere would be liable to pay rent to this landlord too, assuming the latter was looking for or wishing for a paying tenant; but *why* would this liability of the squatter disappear if the landlord was not looking to rent his place out and had not given permission for free occupation of his premises? And *why* would a squatter who could have stayed in a friend's place free of charge not be nonetheless liable to pay rent for staying in this landlord's place uninvited, even if the latter was not looking for or wishing for a paying tenant? The Gemara does not justify its fancy fine distinctions, even though they are far from axiomatic.

Step back a moment and consider the absurdity of the Gemara's claim here in the light of common moral standards. Can it be supposed that a homeless vagrant can freely enter and live in (or

otherwise use) premises belonging to a homeowner without the latter's knowledge and permission? Surely that would constitute *theft* of private property, even if temporary and subject to certain conditions (namely, that the squatter could have stayed at other places free of charge and the landlord's place was currently not up for rent). It would be as surely theft as if a stranger cheerfully 'borrowed' someone's automobile for a while without the owner's okay, arguing that his pals usually let him do that and the car was standing idle! Clearly, the Gemara's claim here is effectively a denial of property rights, and a sanction of gross dishonesty. Maybe in those days social norms were that different, but I doubt it.

The only credible statement, I'd say, is that someone squatting a place without permission is always liable to pay some compensation to the landlord, *irrespective of* any conditions relating to either the one or the other. Indeed, he should additionally be prosecuted for trespass! The Gemara nowhere considers or refutes (as it should have) this obvious proposition. Of course, one can imagine a *force majeure* situation – say someone lost in a snowstorm who comes across an empty, potentially lifesaving, cabin – certainly in such an exceptional situation squatting would be morally acceptable. But the Gemara does not refer its permissiveness to mortal danger.

We could go still further in our critique. So far, we have moved away from the parameters imposed on us by the Talmud by proposing a switch from the given middle term 'this one benefits and that one *does not* suffer loss' to the more accurate middle term 'this one benefits and that one *does* suffer loss', and we consequently accepted – through analogy – 'partial compensation' as the subsidiary term.

But we can go deeper and ask how a squatter, who *intentionally* occupies a vacant home, can be compared to an animal-owner, who allowed his animal to roam freely in the public domain *probably inadvertently*[37]. If we say, by analogy to the given case of the stray animal, that the squatter need only pay a minimum rent for his illicit occupation of the vacant home, are we not enticing him to squat? Are we not telling him: 'you can stay in a five-star apartment and need only pay one-star rent for it'? Clearly, this is quite unfair to the landlord. It means that anyone who does not want to pay the full rent can resort to squatting and get away with a much lesser rent-payment.

The way we got to this absurd conclusion, remember, was by reference to a middle term relating to benefit by the protagonist and loss by the antagonist. But we are not logically forced to restrict our search to this sort of middle term! A completely different parameter or set of parameters might be found for use as middle term, with the aim of obtaining a more equitable conclusion. This is assuming we insist on law-making on this subject (i.e. that of squatting) by means of an argument by analogy from precedent. But this is, of course, not the only way to enact a new law. New laws can be established by majority voting in a legislature, for instance.

In short, there are many possible ways for us to 'think out of the box' handed down to us by the Talmud in this matter. We do not need to get mentally bogged-down in the received framework. We can project the goal, then look for the means to it with an open mind. The means, in this context, being one in accord with core Torah principles. And surely, fairness is one of these principles.

It is admittedly very unorthodox to criticize a Talmudic argument without leaving it an escape hatch. Normally, students of the Talmud take for granted whatever it says; and if some 'difficulty' in what it says is found, some convoluted 'resolution' is quickly suggested so as to maintain its

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[37] He could, of course, have done so intentionally in the hope that it would find something to feed on out there. In that case, comparison to the squatter is indeed possible. But in the more common case of accidental straying, such comparison is not appropriate.

overall credibility. But my interest here is not to defend, or even to attack, this document. I am not engaged in 'virtue signaling'. I am just concerned with the logic of the discourse, whatever its purpose or result. My sole intent here is to show that arguing by analogy from a judicial precedent to establish some new legal principle or law is a complex process involving much thought and discussion.

As regards my proposed alternative thesis, viz. that the case under scrutiny (landlord *vs.* squatter) can be derived by analogy from the Mishna case (food owner *vs.* animal owner) through the middle term *zeh neheneh ve-zeh ken-haser* (the subsidiary term then being partial liability), it should be emphasized that I consider this still an *inductive* conclusion. I am not suggesting that it is not open to eventual challenge. There might be some other proof-text or some other inference that belies it or at least surpasses it in credibility. There might, for instance, be analogical argument(s) from some other Mishna(s), arguing though some other middle term(s) and yielding some contrary conclusion(s). We must then somehow weigh the alternatives and decide which is the most convincing. For example, we might find numerically more reasons that support this conclusion rather than that one. Analogy is inductive, not deductive, argument. It involves trial and error.

The above observations have significance for the formal logic of analogy. An important question they raise is: is an analogy valid if the terms used are analogous only conditionally or in specific instances? There is surely a formal difference between the general term 'benefits' and the narrower term 'benefits under such and such conditions' (for example, 'the squatter benefits provided that he has no alternative lodgings at his disposal'). Likewise, the terms 'suffers loss' and 'suffers loss under conditions so and so' (e.g. 'the landlord suffers loss provided he looked for or at least wished for a tenant') are not equivalent but differ in breadth. Again, the terms 'liable' and 'liable conditionally' (e.g. 'the squatter is liable to the landlord only if the law is that he has to pay as rent the full value of the place, not if he only has to pay a lesser rent) – these are not identical terms.

As we have seen in our earlier theoretical treatment of analogical logic, the mere claim that there is an analogy is not necessarily true, even if made sincerely. There may be ambiguity or equivocation in the terminology (whether done innocently or with intent to deceive) which invalidates the attempted inference. The apparent middle term may not be identical for the major and minor terms, and likewise the subsidiary term may lack uniformity. Such problems of scope can be overcome under certain precise conditions, but not always.

Let us try and draw a lesson in analogical logic from the Talmudic example. That is, let us determine under precisely what terminological conditions analogy can be claimed and an argument involving it be declared formally valid. We must first determine whether we truly have a major premise with a middle term (R) true of the whole extensions of the major and minor terms (P and Q); and we must also make sure that the subsidiary term (S) is the same in the conclusion (concerning the major term) as it is in the minor premise (concerning the minor term), or if not, determine what the justification for a difference might be. I deal with the purely theoretical aspects of this issue in detail earlier on in the present essay (in section 3) under the heading of 'Terms of unequal breadth'[38].

In the above Talmudic arguments, the putative middle term is the conjunction '*zeh neheneh ve-zeh lo-haser*'. However, as we have just seen the terms 'benefits' (say, K) and 'does not suffer loss' (say, L) may not be used uniformly. The question is: what happens when the putative middle term

[38] Note for the record that I got involved in the theoretical study of this issue in response to the quandaries posed by the present Talmudic *sugya*. I placed my abstract analysis earlier on in the text to stress its formal significance for all analogical logic, not just for analogy in the Talmud.

is a compound (K + L) composed of more specific or conditional elements? And more to the point, what happens if instead of the generic and unconditional pair of elements K and L, we are faced with more specific or conditional pairs of elements, say K1 and L1 (for the source), or K2 and L2 (for the target). Likewise, what if the subsidiary term, call it M in generic/unconditional form, has different specific/conditional values, say M1 and M2, in the source and target propositions? In such events, our analogical argument would look as follows:

> Source: just as, in the proof-text, where the protagonist has K1 and the antagonist has L1, the law was so and so (say, M1).
>
> Target: likewise, in our case, where the protagonist has K2 and the antagonist has L2, the law must be that so and so (say, M2).

We can reformulate these sentences as if-then propositions, i.e. the source as 'if (K1 + L1), then M1'; and the target as 'if (K2 + L2), then M2'. As we saw earlier on, in our theoretical investigation of positive subjectal analogical argument, these two if-then propositions cannot give rise to a valid analogy if the terms they involve are truly unequal. Precise logical rules are applicable in such event, and they cannot be ignored. Putting the argument in standard (positive subjectal) form, we obtain with the generic terms (K, L, M) the following analogy:

> Given that our case (P) is similar to the proof-text case (Q) in involving the scenario (K + L) (R), and that in the proof-text case (Q) the law was M (S), it follows that in our case (P) the law should likewise be M (S).

But as we shall see, the only valid specific form for this argument is the following 'from minor to major' mood:

> Given that our case (P) is similar to the proof-text case (Q) in involving the scenario (K1 + L1) (R), and that in the proof-text case (Q) the law was M1 (S), it follows that in our case (P) the law should likewise be M1 (S).

The proof of this statement is as follows. Here, the operative middle term must be (K1 + L1), because the minor premise has (and must have) the proof-text case (Q) as its subject. As we have learned earlier, in our theoretical investigation of terms of unequal breadth, the effective middle term (R) must be the broader (more generic, less conditional) one. Therefore, the above argument is valid only in cases where (K1 + L1) is broader than or equal to, and includes, (K2 + L2). In which event, of course, (K2 + L2) must imply (K1 + L1), and the specific compound (K1 + L1) is effectively the generic compound (K + L). If these conditions are met, the argument indeed has a functioning middle term and a working major premise. But if on the contrary (K2 + L2) is broader than and includes (K1 + L1), or if those two terms intersect but do not overlap, or do not even intersect, then the argument is invalid, because it lacks a functioning middle term and a working major premise.

As regards the subsidiary term, since the predicate of the precedent Q in the minor premise has to be M1, the predicate of our case P must also be at least M1. It can however also be M2, provided M2 is broader than or equal to, and includes, M1. In which case, M1 implies M2, and the specific term M2 is effectively the generic term M. In such case, note, we are merely following up the above analogical argument with a syllogistic argument; the analogical argument *per se* is not changed. However, if on the contrary M1 is broader than and includes M2, or if those two terms intersect but do not overlap, or do not even intersect, then the argument cannot conclude with M2 for case P.

It is possible and even likely, given the stringency of these rules of formal logic, that some of the arguments found in the Talmudic *sugya* under consideration, and other narratives, do not constitute logically valid analogies, because they are contrived by means of ambiguities or equivocations,

and wrongly treat some specific/conditional (middle and/or subsidiary) terms as generic/unconditional ones. Analogical argument is not arbitrary rhetoric, but reasoning subject to strict law. Wherever these logical laws are disobeyed, the argument is fallacious.

Let us now apply the above formal tests on Rami's analogical argument as explicated by the Gemara. Rami apparently reasoned as follows.

> Just as, in Mishna *Baba Kama* 2:2, where the animal owner benefits (in that his animal has been fed, and he saved the price of feed) (K1) and the food owner does not suffer loss (he doesn't *de jure* by *davka* inference from the Torah, although he does *de facto* as the Mishna admits) (L1), the law was that the former must pay the minimal cost of feeding (by Mishnaic ruling, albeit not obligated to pay the full price of food consumed to the latter *de jure* by *davka* inference from the Torah) (M1);
>
> likewise, in our case, where the squatter benefits (but only, according to the Gemara, if he was a habitual tenant and had no other place to go for free) (K2) and the landlord does not suffer loss (provided, according to the Gemara, he was content to leave the place vacant) (L2), the law should be that the former must pay the minimal market value of rent (but is not obligated to pay full rent to the latter) (M2).

Notice that Rami ignores (or puts in brackets) a number of things (specified on his behalf by the Gemara), so as to increase impressions of resemblance. Examining this, it appears as if K1 is broadly intended, while L1 is narrower in scope than it is made to seem (since the word 'loss' is not applied to all loss); as for K2 and L2, they are both clearly conditional (since the words 'benefit' and 'loss' are not applied to all events of squatting). K1 could perhaps be viewed as englobing K2, but L1 certainly cannot do the same for L2 (since the limiting conditions are not similar). Thus, the conjunction (K1 + L1) cannot, as formally required, be implied by (K2 + L2). So, I would say that there is an illicit process in this inference; that is, Rami's argument by analogy (as the Gemara presents it) is formally invalid since it lacks an inclusive middle term. As regards the subsidiary term, M1 is more restrictive than it looks, but its restriction could be passed on to M2 *mutatis mutandis*, so there is no problem there.

We have thus shown, by means of one example, that the Talmud can include invalid reasoning by analogy. This is not surprising for, as already said, analogical argument does have complex theoretical rules not always easy to apply in practice. Anyone might well make errors with it, unless very prepared and very careful. As we have seen, Rava rejects Rami's argument; but he does not do so for the reasons of scope here pointed out. Nor does the Gemara show awareness of these problems, although it tries to shore up Rami's argument in reply to Rava's criticism.

Even so, the Talmud evidently senses, if only vaguely, that there is some inadequacy in the arguments by analogy and disanalogy formulated by Rami, Rava, and even the Gemara itself, with reference to Mishna *Baba Kama* 2:2. This is evident, as already pointed out, from the fact that it goes searching for other possible precedents.

The Talmud next attempts to solve the problem posed by R. Hisda with reference to another case, discussed in Mishna *Baba Batra* 1:3, in which the protagonist is the owner of a field, surrounded on all four sides by fields owned by the antagonist; here again, after a long back and forth discussion, the conclusion is moot. The Talmud then refers to yet another discussion, found in Mishna *Baba Metzia* 10:3, in which the protagonist owns the upper storey of a house, while the antagonist owns the ground floor; and again, the analogies proposed are open to debate and

inconclusive. Many more stories, authoritative opinions, and arguments are brought to bear with apparently no indisputable final conclusion.[39]

This results (as often in the Talmud) in unfortunate prolixity. The central issue posed (viz. whether the apparent scenario of *zeh neheneh ve-zeh lo-haser* implies liability or nonliability) is almost lost in a sea of superfluous detail and the reader's mind easily may lose the thread. We have already suggested that the reason for the Talmud's digressions from the primary issue at hand may be that it sees the discussion as an opportunity to communicate in passing other (loosely associated) information it considers worthy of interest in a wider perspective. It is not trying to get to the point, so much as trying to intellectually scan the area around it.

Another important observation is that the discussion (again, as often in the Talmud) does not always result in clear intermediate conclusions, let alone in a practical terminal result that can be posited as halakha. Some statements end effectively with an ellipsis… their finality is left open. (The effective inconclusiveness of the Rami-Rava debate is a case in point.) The writer(s) of the Talmud may have thought the unstated conclusions obvious; but it obviously was not so since subsequent commentators (i.e. Rashi, Tosafot, and many others) are forced to try and elucidate the missing information, and often disagree as to what it might be.

In truth, looking at the above example, albeit armed with a formal analysis of analogical argument in general (which the Talmud authors lacked), I do not offhand see any way to definitively solve the particular problem at hand. The various arguments given in this long Talmudic debate all seem, more or less, reasonably credible to me at first reading. But as shown above with reference to the first set of analogical arguments (Rami's and Rava's), and the Gemara's take on them, closer scrutiny may reveal certain flaws in the reasoning. I must therefore regard the different points of view as all having an element of arbitrariness or subjectivity. The contestants put forward interesting arguments in support of their respective viewpoints, but none apparently settles the matter decisively. My guess is that finally, in this kind of situation, a halakhic ruling is imposed by majority or by authority or by traditional practice, rather than by pure logic.

There is in this Talmudic discourse, then, a lot of obscurity, ambiguity, equivocation, and uncertainty, which makes difficult a finite and definite reading, even if it does have considerable value as thought-provoking and educational material. But such deficiencies need not concern us here, since we were not really interested in solving the specific legal problem at hand, but rather sought to observe the use of analogical logic in the Talmud, and evaluate it by formal means, and perhaps learn lessons from it.

I have here written many pages discussing only the first debate in the present *sugya*. There are many more debates in it, and it would take very many more pages (possibly a whole book) to fully analyze them in equal detail. However, to repeat, my goal here is not to thoroughly analyze the whole *sugya*, but merely to demonstrate through at least one example in it that the Talmud, like many other legal traditions, ancient and modern, near and far, resorts to analogical argument from precedents to derive new legal principles or laws. Having already achieved this goal, I can in good

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[39] R. Louis Jacobs comments (p. 64) that "After the whole *sugya* has eventually arrived at the conclusion that A [the squatter] is not liable, R. Nahman's case is presented for discussion in that, on the surface, it seems to contradict the conclusion towards which the rest of the *sugya* has been leading." But my own impression is that, in view of the mixed *chronology* of the discussion, no definitive final conclusion can really be claimed; if such had been achieved at some point in time, all discussion would have ceased thereafter. Assuming the historicity of the account, it must be ordered chronologically (rather than in a logical or literary progression) to see more clearly and objectively its direction and result.

conscience stop the analysis here; indeed, must do so since I would otherwise be ranging too far off topic (namely, analogical logic in general).

10. More about analogy in the Talmud

Based on the above example, and other readings of Talmudic discourse over the years, I think it is safe to say offhand that the Talmud (including both Mishna and Gemara, in both the BT and JT) makes widespread use of such reasoning. But of course, this proposition still needs to be demonstrated by an exhaustive listing and competent detailed analysis of each and every instance of logical discourse in this massive work[40].

I have already pointed out, in my book *Judaic Logic* (2004), the undercurrents of analogical reasoning in some of the 13 *Midot* (hermeneutic principles) of Rabbi Ishmael, notably in the rules called *kal vachomer* (a fortiori argument), *gezerah shavah* (analogy based on homonymy or synonymy), *binyan av* (causal reasoning), and *heqesh, semuchim, meinyano, misofo* (analogies based textual proximity).

Additionally, in my later book *A Fortiori Logic* (2013), I have listed some Torah passages which can be interpreted as analogical arguments, notably Ex. 2:11-14 (which suggests *gezerah shavah*) and Lev. 10:9-11 (which resembles *binyan av*). The Nakh (the rest of the Jewish Bible) can be expected to contain many examples, too.

I have written extensively about *kal vachomer*, the first rule of R. Ishmael, in my past works and will not repeat myself here.

The second rule of R. Ishmael, the principle of *gezerah shavah*, which is based on the terms having some Biblical wording or intent in common, may be said to constitute simple analogy. This is because (evident) same wording, or (assumed) same 'intent' of different wordings, do not provide a sufficiently substantive explicit predicate (R) in common to the subjects compared (P and Q). Words are explicit, but they are incidental to what they verbalize; therefore, the assumption that the Torah intends them as significant enough to justify an inference is open to debate.

In other words, the traditional Judaic belief (for some people, a dogma) that names are part of the nature of the things they name, if not their very essence, is – as far as formal logic is concerned – only a theory. There is nothing obvious or axiomatic about it. It is a hypothesis that must remain open to scrutiny and testing like any other. Modern linguistics would deny this hypothesis in view of the demonstrable fact that all languages, including Hebrew, have evolved over time. Things do not change in nature just because we change their names.

In any case, *gezerah shavah* inference suggests an argument by analogy of roughly the following (positive subjectal) form: since text P and text Q, found in the Torah, are similar in literal wording or in verbal intent (R), then given that Q implies some information S, it follows that P implies the same information S.

This brings to mind *gematria* and other systems of 'numerology' found in Judaism, which compare the 'numerical value' (variously calculated) of two words, phrases or sentences, and regard their equality (or sometimes, near-equality) as a basis of analogical inference. These exegetic techniques seem to date from Talmudic times (some claim earlier), though they were greatly developed later.

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[40] I hope, and expect, some scholars will eventually dare attempt such an ambitious project; for my part I am already too old to take up the challenge, although I have tried to do a small share of the work.

They are used in haggadic (non-legal) contexts, rather than halakhic ones[41]. I have personally no faith in them[42], and have argued in the past[43] that their probable absurdity could be demonstrated systematically by drawing up a list of the numerical values (under each of the different systems) of every word in the Hebrew dictionary and then grouping all words with the same numerical value together (which should reveal enough contradictory equations, I wager, to dissuade believers).

The third rule of R. Ishmael, the principle of *binyan av*, falls squarely under the heading of complex (positive subjectal) analogy. In fact, our description of complex analogy is an exact description of *binyan av* reasoning. When the rabbis want to extend the scope of a Torah law (S), they show that some new subject (P) has some feature (R) in common with the Torah-given subject (Q), and assuming that this feature is the reason for the law (this assumption constitutes a generalization, even if it superficially may seem to be a direct insight), they carry the law over from the given case to the unspecified case. However, note, sometimes the common ground is not identified explicitly; in which case, of course, the analogy is simple.

As regards the twelfth rule of R. Ishmael, which refers to contextual inferences (*meinyano* and *misofo*, *heqesh* and *semuchim*, and the like), here is how I describe such reasoning in my book *Judaic Logic* (Chapter 10.2): "All these take into account *the textual closeness* of an expression or sentence to certain other(s), and on this basis assume that there exists a conceptual relation between the passages under scrutiny, which makes possible an inference of certain attributes from the context to the expression or sentence." Inference based on context is simple analogy, since it is without explicit explication.

Contextual inference can be cast as positive subjectal analogy, roughly as follows: since text P and text Q are similar in their being placed close together in the Torah narrative (R), then given that Q implies some information S, it follows that P implies the same information S.

Here, the analogy is based on the incidental fact of location of text relative to other text within a Biblical document, not on any substantive motive. Granting that the Torah is Divinely given or inspired, adjacence of texts is not in itself an incredible basis of analogy; it is a formally acceptable basis. However, there is a problem with it, insofar as contextual analogy is not considered throughout the document, but only evoked selectively, in cases where it is convenient for the justification of some legal principle or ruling. This objection would no doubt be rejected by the rabbis, through an argument that there are surely reasons for the close location of all verses in the document even if we humans are not aware of them all. But this is, of course, an appeal to faith, not a proof.

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41 They are often used as homiletic tools in the synagogue. As they make possible surprising connections between narratives or ideas, they grab the attention of auditors in the way a magical trick would. This seems quite acceptable to my mind, provided it is intended playfully, not seriously.

42 I have no faith in them as realistic systems of inference. However, I do personally, like many fellow Jews, use a couple of traditional numbers as *merely conventional symbols*. When I see the number 26 (the primary numerical value of the Tetragrammaton), say on a clock, I am by choice habitually reminded of God and of His mercy. Or again, when I give charity I tend to do so in multiples of 26, or alternatively of 18 (the numerical value of *chai*, the Hebrew word for life), with the intent to benevolently wish mercy or life to the recipients. In my mind, these numbers are mere words, constructed arbitrarily out of numerals instead of letters; I do not imagine a real connection between them and their putative objects, nor refer them to a general system of numerology. Therefore, I do not use them (or any others) superstitiously as lucky numbers, nor use them for any sort of serious inference.

43 Over 25 years ago, in *Judaic Logic*, Appendix 3. No one, to my knowledge, has in the meantime followed up on the idea of a systematic study for the purpose of scientific verification or falsification. Not knowing Hebrew well enough, I cannot do the job.

In some cases, of course, analogy is explicitly proposed in the Torah. For instance, in Deut. 22:26, which compares rape and murder, saying "for as when... even so..." (*ki kaasher... ken hadavar hazeh*). Such analogy is evidently more substantive, and the common ground it suggests might readily be made explicit. It would be interesting to make a listing of all such cases in the whole Jewish Bible (the Torah and the Nakh), as I and others have done for a fortiori argument.

Clearly, analogical argument plays a considerable role in Judaic logic (see my past works for more details and examples). And no doubt similarly in other religious logics, Christian, Islamic, Hindu, Buddhist, and so forth. A lot of work is needed to find all its instances and examine the skill and credibility of each instance. It is also important to know not just the practice of analogy in different traditions, but also just how consciously it is done, i.e. how far each tradition has gone in theoretical reflection on and understanding of what it was doing. In Judaism, we have (as above shown) some theoretical exposition of analogical reasoning in the hermeneutic principles expounded in different lists, although these lists are not as thorough and formal as they could and should have been.

11. Subsumption in analogical terms

Subsumption is the inclusion of a particular instance in a class, or of a narrower class in a wider one. All concept-formation is based on subsumption, and proceeds by identifying the similarities and differences between things, and then grouping together those with certain characteristics in common, and distinguishing them from things without such characteristics. We normally think of subsumption in a positive syllogistic thought process, following Aristotle, and quite rightly, as follows:

> Anything that has certain characteristics B is C;
>
> this item A1 has characteristics B;
>
> therefore, A1 is C.

This informs us that all Bs are C, so that having property B is a *sufficient* condition for the subsumption of an item like A1 under C. The relation of subsumption is stronger in cases where the characteristic B is exclusive to C; that is, where it is additionally given that no non-B are C, so that *only* Bs are C. This implies a negative syllogism, for items like A2 that do not have property B, as follows:

> Anything that lacks certain characteristics B is not C;
>
> this item A2 lacks characteristics B;
>
> therefore, A2 is not C.

In such cases, having the property B is a *necessary* condition, a *sine qua non*, for belonging (as an instance or subclass) to concept C. Where B is both a sufficient and necessary condition for classification as a C, B can be used (if need be) as a defining characteristic of C.

Clearly, all conceptual knowledge is based on this thought process that we call subsumption. Clearly, too, subsumption involves analogical thinking. We can restate the above syllogisms as analogical arguments, forcing things a bit, as follows:

> Given (as above) that A1 has certain characteristics B, it is similar to certain other things which also have B (so far unnamed, call them D); and given (as above) that all B are C, it follows (by syllogism, through all D are B) that all D are C, and thence by analogy that A1 is also C (although we could have obtained the same conclusion syllogistically directly through all B are C, bypassing D). Here, the commonality (viz. that A1 and D have B in common) is the driving force of the (positive) inference.

> Given (as above) that A2 *lacks* certain characteristics B, it is *dis*similar to certain other things which *do* have B (so far unnamed, call them D); and given (as above) that all B are C and no non-B are C, it follows (by syllogism, through all D are B) that all D are C, and thence by *dis*analogy that A2 is distinctively not C (although we could have obtained the same conclusion syllogistically directly through no non-B are C, bypassing D). Here, the distinctiveness (viz. that D has but A2 distinctly lacks B) is the driving force of the (negative) inference.

Note that our casting the two arguments, here, in the form of analogy or disanalogy is somewhat artificial, since the syllogistic path is shorter and clearer (and we are still resorting to syllogism to arrive at the desired results). Not only that, but the two analogical arguments are logically weaker than the two preceding syllogisms, since the syllogisms yield deductive conclusions whereas the analogies yield merely inductive conclusions. However, my purpose here is merely to show the analogical undercurrent of the syllogistic thought; it is not to suggest preference of the analogical statements over the syllogistic ones.

Perhaps if we distinguish between complex and simple analogy the role of such argument in subsumption may be enhanced. There are two kinds of definition in forming concepts. In 'deductive' definition, we form the concept C by defining it from the start through some specified property B; and in such case, the syllogisms shown above are obviously the most natural instruments of subsumption of instances like A1. However, in the case of 'inductive' definition, we do not clearly know at the outset how precisely to define the putative concept C; we sense that there is some property in common and exclusive to certain instances, but we cannot yet say just what it is; so, we coin a term 'C' for a start and then proceed to gradually look for a definition 'B' of it, one capable of subsuming instances like A1 and excluding instances like A2. In the latter case, it is obvious that analogical argument plays a more prominent role, because simple analogy can proceed without explicit specification of the middle term.

My point is just that putting individuals in a class, or subclasses is a wider class, depends on 'seeing' the similarities and differences between the items under consideration, and that underlying thought process is manifestly analogical, whether explicitly or tacitly. How this 'seeing', or direct insight, occurs is something of a mystery. Indeed, it is a big epistemological mystery; and perhaps, precisely because it is such a fundamental power of human consciousness, it is an unsolvable mystery. But what is sure is that if we could not tell the similarities and differences between things, we could not form any concepts.

Man's power of abstraction depends on this faculty of insight into similarities and differences; and subsequent conceptualization depends on consciously differentiating, grouping and naming things on its basis. Animals (at least the higher ones) no doubt can likewise tell similarities and differences between things, since they can recognize edible foods or dangerous predators. But in their case, this faculty seemingly does not proceed on to concept-formation, but remains at a relatively concrete level of sensory memories (sights, sounds, smells, tastes, touch-sensations).

12. Analogy and causality

It would be fallacious reasoning to infer from the fact of *similarity* between two phenomena that the former necessarily *caused* the latter. It can and does happen that analogy implies causality, but it is not always the case. A hypothesis to that effect can of course be proposed in any given case; but such hypothesis must then be tested empirically with reference to the applicable rules of causal logic. In other words, the logical status of such inference is at best inductive, not deductive.

To give an example, Wilhelm Windelband, in his *History of Ancient Philosophy*[44], addressing the claim by some historians that Greek thought owed a great deal to Oriental thought in view of observed similarities between some of their doctrines, points out that such commentators "fell into the error of transmuting analogies into causal relations," and justifies his criticism by pointing out that "equally notable disparities[45] might also have been found" (p. 21). In his view, while admitting that some influence no doubt occurred, the Greeks significantly improved upon any such external inputs.

We might add that individuals in different cultures, or even the same one, can have similar ideas without having influenced each other or been subjects to a common influence. What occurs in such cases is human intelligence responding in like ways to similar data inputs. There is a common cause, but it is not necessarily the simple cause of mutual or common influence; it is a deeper cause. The same reasoning can be applied in other domains, of course. For instance, in biology if two individuals have a similar physical trait (e.g. eye color), it does not necessarily mean that they are siblings or that they have a common parent, or even a remote distinctive ancestor; there may be some other genetic explanation.

Annex: Conflicting a fortiori arguments

It is interesting to compare and contrast the behavior of conflicting analogies to that of conflicting a fortiori arguments. This concerns a fortiori arguments with different middle terms (R1, R2), and a single subsidiary term (S), with the same extreme terms (P, Q), except that their roles as major and minor terms are reversed.

But first let us briefly compare and contrast the structures of analogical and a fortiori arguments.

- In qualitative argument by analogy, the major premise informs us only of the similarity or dissimilarity of the major and minor terms (P, Q) in relation to the middle term (R); and the minor premise and conclusion tell us of the respective relations of P and Q to the subsidiary term (S), with no mention of R. In quantitative analogical argument, the major premise informs us of the quantitative relations of P and Q to R; but the minor premise and conclusion still make no mention of the middle term R. This is inductive inference: given the said premises, the conclusion is only probable (dependent on certain reasonable assumptions).
- Whereas in a fortiori argument, the major premise informs us of the quantitative relations of P and Q to R; and the minor premise and conclusion tell us of the respective relations of the terms P and Q to the term S *through the sufficiency or insufficiency of the middle term R*. This is deductive inference: given the said premises, the conclusion follows necessarily (unconditionally).

Thus, the former kind of argument is simpler than the latter. In the former, the major premise may carry less or as much information as in the latter; but in the former, the minor premise and conclusion make no mention of the middle term (R), whereas in the latter they distinctively do. Keep these differences in mind.

Let us now examine conflicting a fortiori arguments.

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[44] New York, N.Y.: Dover, 1956. Trans. By H. E. Cushman, from the 2nd German ed. (1893).

[45] Disparities means disanalogies. What he is saying is that when considering analogies, we should also consider disanalogies, and weigh the resulting impact of both these factors.

In the case of subjectal a fortiori argument, we need to focus on the following two arguments (one positive and one negative), whose major premises go in opposite directions, and whose minor premises have opposite polarities, and whose conclusions have contradictory implications:

P is more R1 than (or as much R1 as) Q,
and Q is R1 enough to be S;
therefore, P is R1 enough to be S,
whence P is S.

P is less R2 than (or as much R2 as) Q,
(= Q is more R2 than (or as much R2 as) P,)
and Q is R2 not enough to be S;
therefore, P is R2 not enough to be S,
whence P is not S.

Obviously these two arguments are in conflict, because they yield contradictory final conclusions, viz. P is S and P is not S. Their major premises are not formally contradictory – i.e. two terms P and Q may well be comparatively both greater and smaller in relation to different common features (R1, R2). Therefore, the problem lies in the minor premises of these two arguments – and indeed, one implies that Q is S and the other that Q is not S. So, one or the other (or both) of the minor premises of these two arguments must be false.

Keep in mind that a fortiori argument is essentially unidirectional (unlike analogical argument); this is why the above two subjectal inferences go from Q to P. In the special case where the two above a fortiori major premises are egalitarian (i.e. involve 'as much R'), the minor premise and conclusion are interchangeable (as they are in argument by analogy) in each of the two arguments (i.e. the arguments could equally well go from P to Q as from Q to P). But of course, the minor premises are not arbitrary – they must be proposed as true.

The two arguments would yield the same positive or negative conclusion if the two minor premises have the same polarity. If the two minor premises are positive ('Q is R1 enough to be S' and 'Q is R2 enough to be S'), then the conclusions would both be positive ('P is R1 enough to be S' and 'P is R2 enough to be S'), so that 'P is S' would be implied true *by both*. Alternatively, if the two minor premises are negative ('Q is R1 not enough to be S' and 'Q is R2 not enough to be S'), then the conclusions would both be negative ('P is R1 not enough to be S' and 'P is R2 not enough to be S'), so that 'P is not S' would be implied true *by both*. In such cases, there is no conflict between the arguments.

However, if one of the egalitarian a fortiori arguments has a positive minor premise and the other a negative one, as may well occur, their implied conclusions ('P is S' and 'P is not S') would be contradictory, and the arguments would be in conflict.

In the case of predicatal a fortiori argument, we need to focus on the following two arguments (one positive and one negative), whose major premises go in opposite directions, and whose minor premises have opposite polarities, and whose conclusions have contradictory implications:

More R1 (or as much R1) is required to be P than to be Q,

and S is R1 enough to be P;
therefore, S is R1 enough to be Q,
whence S is Q.

Less R2 (or as much R2) is required to be P than to be Q,
(= More R2 (or as much R2) is required to be Q than to be P,)
and S is R2 not enough to be P;
therefore, S is R2 not enough to be Q,
whence S is not Q.

Obviously these two arguments are in conflict, because they yield contradictory final conclusions, viz. S is Q and S is not Q. Their major premises are not formally contradictory – i.e. two terms P and Q may well be comparatively both greater and smaller in relation to different common features (R1, R2). Therefore, the problem lies in the minor premises of these two arguments – and indeed, one implies that S is P and the other that S is not P. So, one or the other (or both) of the minor premises of these two arguments must be false.

Here again, keep in mind that a fortiori argument is essentially unidirectional; this is why the above two predicatal inferences go from P to Q. In the special case where the two above a fortiori major premises are egalitarian, the minor premise and conclusion are interchangeable in each of the two arguments (i.e. they could as well go from Q to P as from P to Q). But of course, the minor premises are not arbitrary – they must be proposed as true.

The two arguments would yield the same positive or negative conclusion if the two minor premises have the same polarity. If the two minor premises are positive (‘S is R1 enough to be P’ and ‘S is R2 enough to be P’), then the conclusions would both be positive (‘S is R1 enough to be Q’ and ‘S is R2 enough to be Q’), so that ‘S is Q’ would be implied true *by both*. Alternatively, if the two minor premises are negative (‘S is R1 not enough to be P’ and ‘S is R2 not enough to be P’), then the conclusions would both be negative (‘S is R1 not enough to be Q’ and ‘S is R2 not enough to be Q’), so that ‘S is not Q’ would be implied true *by both*. In such cases, there is no conflict between the arguments.

However, if one of the egalitarian arguments has a positive minor premise and the other a negative one, as may well occur, their implied conclusions (‘S is Q’ and ‘S is not Q’) would be contradictory, and the arguments would be in conflict.

Clearly, whereas in the case of conflicting analogical arguments the conclusions disagree even though the major premises are compatible and the minor premises are identical – in the case of conflicting a fortiori arguments the conclusions disagree albeit compatibility of the major premises, because of the contrariety of the minor premises – which, though they mention different middle terms (R1, R2), have opposite implications irrespective of middle term. In subjectal argument these implications are ‘Q is S’ and ‘Q is not S’: while in predicatal argument they are ‘S is P’ and ‘S is not P’. Needless to say, such inconsistency means that one (or both) of the given minor premises must be false.

4. RESOLVING PARADOXES

1. Protagoras vs. Euathlus

1. An ancient paradox

One of the many alleged paradoxes that have come down to us from the Greeks is the dispute between Protagoras (of Abdera, ca. 480-410 BCE) and his student Euathlus (about whom nothing more is known). The story is told by Aulus Gellius (Roman, ca. 125-180 CE)[1], that Protagoras, a famous Sophist, and an expensive teacher, agreed with Euathlus to train him in rhetoric, a discipline essential at the time to argumentation in courts of law. The agreement was that Euathlus would not have to pay Protagoras the specified fee (or the unpaid portion of the fee, by some accounts) until he had been fully trained and went on to plead his first case and win it[2].

It is said that after Euathlus completed his course, he did not (for whatever reason) choose to use his newly acquired skills before any court of law, and so he never won or lost any case, and so was contractually not required to pay Protagoras anything. Nevertheless, Protagoras, with motives that we shall presently consider, sued him (in the court of Areopagus in Athens). Euathlus chose to personally defend himself. The following arguments were reportedly put before the court:

a. **Protagoras** argued that he surely ought to be paid the fee, because (i) if the court ruled in his favor, he could on that basis demand payment; and (ii) if the court ruled in favor of the defendant, then the latter would have won his first case and therefore be contractually obliged to pay the fee anyway.
b. **Euathlus** replied that he surely ought *not* to pay the fee, because (i) if the court ruled in favor of the plaintiff, then Euathlus would have lost his first case and therefore not be contractually obliged to pay the fee; and (ii) if the court ruled in favor of the defendant, then he would on that basis be exempt from payment anyway.

Thus, while the plaintiff argued, apparently convincingly, that he was certain to deserve payment however the court ruled, the defendant was in turn able to argue, apparently just as convincingly, that he was certain to be exempt from payment in either event. For this reason, this case is regarded as paradoxical. It is said that the court was so confused by these arguments and counterarguments that it chose to adjourn *sine die* (or, some say, for 100 years) to avoid judgment.

The significance of this legal dispute for logic and philosophy is that it gives the impression that two people can argue dilemmatically and paradoxically in opposite directions and both be right. The enemies of human reason relish this kind of conclusion, since it would put in doubt the reliability and efficacy of human reason. But as we shall now show, the said impression is very superficial. There are, to my mind, at least two possible resolutions of this so-called Paradox of the Court. In fact, although I thought of the second before the first, the latter logically precedes the

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[1] Some 550-600 years after the fact, in his *Attic Nights* (ca. 150 C.E.).

[2] The contract in question was presumably verbal, rather than written, in those days; but we may take it that both parties agreed on its stated clause.

former. I later learned from Peter Suber's survey of the literature on the subject[3] that the first resolution was long before me proposed by Aulus Gellius; the second resolution seems to be novel.

2. First resolution

The simplest solution to this problem is to suppose that the wily Protagoras, seeing that Euathlus was taking his time getting to work, decided to speed things up. Protagoras trapped his pupil by using the above argument, knowing full well that he would lose a first trial, but win an eventual second trial. He knew he would lose a first trial, because the agreement between the parties only obligated Euathlus to pay the fee once he won his first case; it did not obligate him to practice law anytime soon, or even ever. Euathlus foolishly fell into the trap and personally argued the case in court. Had the court not adjourned *sine die*, it would have logically given him victory, thus making Euathlus win his first case. Thereafter, assuming that a second trial was legally permitted – and both the parties' arguments above do make this assumption – Protagoras would have been the ultimate winner. Of course, no second trial would be necessary if Euathlus conceded that having won the first trial he was sure to lose the second, and settled the account forthwith.

In other words, Protagoras's first argument (i) was mere camouflage; he was really relying on his second argument (ii). Euathlus let his vanity get the better of him and formulated two fancy counterarguments, thinking to outdo his teacher. But Protagoras was more cunning than him. The only way Euathlus could have avoided being beaten was by hiring a lawyer[4]. If the lawyer won the first trial, as he could be logically expected to since the only condition for defeat here was not satisfied, Euathlus would not be considered as having personally won his first case in court, and Protagoras would not be able to win a second trial. Thus, the master was indeed superior to the pupil. There was no real paradox, since there was an actual way out of the apparent paradox.

Of course, one might add that Euathlus was in practice the winner, since through his counterargument he managed to so bewilder the court that it gave up trying to judge the matter at all, and he was not forced to pay up. Maybe he hoped for that and he lucked out. But on a theoretical, logical level, in the present perspective, he proved to be not too intelligent. Not only did he foolishly not hire a lawyer to plead on his behalf, but he also wrongly assumed that his argumentation was effective in countering Protagoras'. He kidded himself into thinking that if he won the first trial, he would not have to face a second one. He should have examined his teacher's argumentation more carefully.

Let us now look at the arguments in more formal terms, to clarify them. I shall introduce the following symbols: let P = Protagoras, Q = Euathlus; A = the agreed condition that Q wins his first case in order to be liable, and C = the agreed result that fee must be paid by Q to P. We know at the outset that if A then C, and if not-A then not-C: these are the terms of the agreement. The arguments are as follows (with my critical commentary in *italics*):

a. According to P: (i) if court rules that P wins, then C is true (*but objectively court cannot rule for P, since A not yet true, so this is a non-starter*); whereas if (ii) court rules that Q wins (*as it logically must*), then not-C is true (*at the conclusion of a first trial*); but if Q

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[3] In *The Paradox of Self-Amendment* (Bern: Peter Lang, 1990). The section on the Protagoras v. Euathlus paradox can be read online; it is worth reading, including the notes, and not very long. See at: https://dash.harvard.edu/bitstream/handle/1/10288413/Peter%20Suber%2c%20Paradox%20of%20Self-Amendment%2c%20Section%2020.html?sequence=1#A.

[4] Assuming this was possible in Athens in those days. This is a fair assumption (even if some commentators deny it) since, after all, Euathlus was apparently trained by Protagoras to be a lawyer himself.

thus wins, then A becomes true and C must (*in a later trial*) follow, in which event P finally wins.

b. According to Q: (i) if court rules that P wins, then not-A is true (*at the conclusion of a first trial*); but if not-A is true then not-C must (*in a later trial*) follow, so Q finally wins (*but objectively court cannot rule for P, since A not yet true, so this is a non-starter*); whereas if (ii) court rules that Q wins (*as it logically must*), then not-C is true (*but here Q fails to mention later consequences that P rightly pointed out*).

In conclusion, P is logically the resultant winner; Q's arguments are in fact insufficient to prevent this outcome. P pretends to seek to win immediately; but in truth his aim is longer term victory (in the second round). Q imagines he might lose the first round but win the second or that he might win the first round without having to face a second; but these are all fantasies. It is difficult to understand why the court found this case too confusing – the judge (or judges) can't have been very bright fellows.

3. Second resolution

A more complex solution to the problem is as follows. It is possible that Protagoras sued Euathlus by appealing to *an unspoken clause* of the agreement. The agreement contained *only one explicit clause*, viz. that Euathlus would have to pay Protagoras the fee if and only if he won his first case. If that was so, Protagoras would have no basis for requesting a trial, since that condition had obviously not been satisfied. But since he sued, he must have thought and argued that the agreement included a *tacit (or perhaps implicit) understanding* that Euathlus would practice law within a reasonable time lapse[5], at which time his new skills would be tested in a court and he would be expected to pay if he won. Protagoras couldn't have imagined the judge would allow a trial to proceed, let alone would rule in his favor, without some good reason[6].

Clearly, what the judge was called upon to decide in this trial was (could only have been, in the present perspective) whether this claim by Protagoras, that there was a tacit clause to the agreement, was justified. He could well have justified it by considering that had Euathlus been allowed not to practice law at all or to practice it as late as he chose, the agreement would have surely specified the caveat. He could equally well have rejected it by considering that Protagoras took for granted something he should have explicitly obtained agreement on. So, the case hinges on a tacit issue, rather than exclusively on the explicit clause of the agreement; i.e. there was more to the story than is told.

Furthermore, it is evident from the arguments presented by both parties in this trial that each of them foresaw the possibility of a second trial in which the ruling of the first trial could be reversed. This is logically implied in the second argument of Protagoras and in the first argument of Euathlus. In these two eventualities, a second trial would be needed to finalize the judgment; unless, of course, the losing party freely concedes its inevitable result and settles the account in advance. The judge in the first trial could not decide in favor of either party and then against him in the same breath. A first judgment of win or loss would have to be established before a second judgment could be made, in view of the terms of the contract. So, there is a time factor to take into consideration in analyzing these arguments.

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[5] Protagoras would also, of course, claim that the reasonable delay had expired. If the court agreed with the existence of a tacit clause but disagreed with the claim it was fulfilled, that would merely adjourn the case for a certain amount of time (of their estimate).

[6] Needless to say, the present analysis is made entirely from a logical viewpoint, although a court of law might reason differently or even not reason at all (e.g. bribery, favoritism, pressure).

It should be noted that, whereas the first trial has an uncertain outcome, since it depends on the decision (the judgment call) of the judge regarding an alleged tacit clause to the agreement, the second trial has a foreseeable outcome, since it depends solely on the explicit clause of the agreement.

Clearly, as we shall now show, if we factor the above considerations into the arguments, the paradoxical appearance is easily dissolved. I shall here use the following symbols: let P = Protagoras; Q = Euathlus; A = the explicit condition that Q wins his first case; B = the alleged implicit condition that Q was required to practice within a reasonable amount of time; and C = the fee must be paid by Q to P. The arguments are as follows:

1. According to P: (i) if B then C; and (ii) if not-B then not-C, *but if not-C then (later) if A then C*; therefore, C anyway.
2. According to Q: (i) if B, then C, *but if C then (later) if not-A then not-C*; and (ii) if not-B, then not-C; therefore, not-C anyway.

From this we see that the two parties' arguments are much alike, but each side has cunningly left out part of the consequences (shown in *italics*). P has truncated the consequences of (i) that Q points out, and Q has truncated the consequences of (ii) that P points out. For this reason, they seem to balance each other out. But if we take *all* the subsequent events (in a possible second trial) into account, we get the following more objective joint argument (c), which is clearly non-paradoxical:

(i) if B then C, but if C then (later) if not-A then not-C; and
(ii) if not-B then not-C, but if not-C then (later) if A then C.

So, in (i) the final conclusion is not-C, while in (ii) it is C – which means that there is no paradox. This also shows that, while it cannot objectively be predicted whether P or Q will win the first trial, it can be said that (i) if P wins the first trial, he will lose the second, and if Q wins the first trial, he will lose the second. Obviously, P cannot argue that he has a right to a second trial (if he loses the first) but Q has not; likewise, Q cannot argue that he has a right to a second trial (if he loses the first) but P has not. So, we must take all later events into consideration to logically reconcile all the arguments. Note that if for some reason there is no second trial, there is also no paradox, since the conclusion will be either (i) C or (ii) not-C.

Whatever happens, there is no paradox because neither party can in fact, contrary to initial appearances, claim inevitable victory; victory does not come both ways for both parties, but only one way for each party.

Clearly, here, both parties were employing the common sophistical trick of hiding an inconvenient part of the unbiased argument from the court. Euathlus was a good apprentice of Protagoras', since his counterargument exactly mirrors the latter's argument. That is, there was an element of dishonesty in both their arguments; both were intellectual frauds at heart, knowingly expounding half-truths. Therefore, this fake paradox presents no deep challenge to Logic, contrary to the claims of Relativists. In particular, Protagoras' general claim that "there are two sides to every issue" (*duo logoi*) is shown to be spurious in the present context.

It is worth always keeping in mind that some people involved in philosophy and logic, as in life in general, are sometimes moved by the evil impulse; indeed, some much more than others. They may consciously lie, or subconsciously twist facts and arguments, for a large variety of motives. Usually, lusts for power, fame and fortune play some role. An academic may want the admiration of his peers or of his students; a husband may want to impress his wife; an unemployed may hope to get a job; and so on. It is wrong to look upon all philosophical statements as disinterested. Philosophers and logicians are not all pure scientists or saints.

I might add that the secret of success with finding solutions to philosophical and logical problems, and particularly to paradoxes, is the sincere desire to do so. Many philosophers and logicians approach problems with a negative attitude, not really wanting to solve them, but rather wishing to rationalize their antipathy to human reason through them. The honest researcher is moved by his better impulses; he is sincerely desirous to confirm the effectiveness of the human faculties of cognition – that's precisely why he succeeds.

4. Inadequate resolutions

As earlier mentioned, based on Suber's account of the literature, the first resolution should be attributed to Aulus Gellius, but the second resolution seems to be original. Suber's account shows that the court paradox has been discussed in a number of works over time, but more often apparently from a legal point of view than from a logical one. The legal issues involved are manifold, but most need not and should not be taken into consideration in a purely logical perspective. Why? Because the logician's purpose here is not to decide the case, i.e. who should win or lose, but merely to explain and remove the appearance of paradox.

This remark can be illustrated with reference to the resolution (not mentioned by Suber) proposed centuries ago by Lorenzo Valla (Italy, ca. 1406-57). This attempt at resolution is not adequate because it relies on a thoughtless distinction between payment on account of the court's verdict and that on account of the agreement. I quote an SEP article which describes it[7]:

> "If Euathlus loses the case, he will have to pay the rest of the fee, on account of the verdict of the judges; but if Euathlus wins, he will also have to pay, this time on account of his agreement with Protagoras. Euathlus, however, cleverly converts the argument: in neither case will he have to pay, on account of the court's decision (if he wins), or on account of the agreement with Protagoras (if he loses).… Briefly put, Valla says that Euathlus cannot have it both ways and must choose one or the other alternative: he must comply either with his agreement with Protagoras or with the verdict passed by the judges. If they decide against Protagoras, he may try to reclaim his money in a second lawsuit."

Let us analyze Valla's proposed resolution using the following symbols: let P = Protagoras; Q = Euathlus; and C = the fee must be paid by Q to P.

a. According to P: If Q loses (for whatever reason), then C (by verdict); but if Q wins, then C (by agreement).
b. According to Q: If Q wins (for whatever reason), then not-C (by verdict); but if Q loses, then not-C (by agreement).

Valla's conclusion, as here presented[8], is unclear. Apparently, he puts the onus on Euathlus in particular to "choose one or the other alternative" and comply with either the agreement or the verdict. This tells us nothing, since Euathlus' argument shows he is willing to comply with either, except that he projects both as in his favor. Valla adds that if Protagoras loses a first lawsuit, he

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[7] See *Stanford Enc. of Phil.*, online at https://plato.stanford.edu/entries/lorenzo-valla/. This issue is apparently treated in *Repastinatio dialectice et philosophie.* The author of the article is apparently Lodi Nauta (2013).

[8] I have not read Valla's work. I have to assume that the author of the *SEP* article on Valla correctly presented Valla's reasoning. I suspect he or she may not have, as the details given in the article are rather vague and inconclusive; they do not clarify exactly what resolution of the paradox Valla had in mind. Not everyone is good at logic. (Note that I did write to the author, asking him or her to please verify the summary of Valla's view given in the SEP article, but I got no reply. That the author did not deign to respond confirmed my estimate that this is not a very reliable source: intelligent people confidently welcome reasonable queries.)

may win a second. But here again, this does not resolve the paradox, but only repeats one part of it.

From this we see that Valla has not thought the issues through: he does not consider on what grounds Q might lose or win 'by the court's verdict'; and he does not realize that the second argument by each party, where Q might alternatively win or lose 'by the agreement', in fact refers to a second lawsuit, since the agreement is evidently not about to be implemented voluntarily (Valla's mention of a second lawsuit is placed beyond the four if-then statements, which themselves do not emphasize the temporal sequences involved). In the last analysis, then, Valla does not arrive at a ready resolution of the paradox.

The trouble with Valla's approach is that it effectively takes the initial decision of the court to be arbitrary, i.e. unrelated to the agreement between the parties. It does not consider on what basis the court might judge that Euathlus can lose the first round. This may be acceptable legally, but it is not acceptable logically – and our concern here is with logic. Logically, the idea that Euathlus can lose the first round is a non-starter, if we go by the explicit clause of the agreement. He might lose the first round only if the court grants the supposition that there was a tacit clause to the agreement, such as that he had to practice law sometime soon.

In any case, the verdict for the first round cannot be arbitrary – i.e. irrational, unjustified – but must directly relate to the agreement. This is equally true for the second round. So, no disconnect between verdict and agreement is logically permissible.

Interestingly, my first reaction to the paradox a few years ago[9] was very similar to Valla's. But as soon as I set about seriously considering the issues for the present essay, I realized its weakness and uselessness. The paradox appears neutralized if we insert a distinction between 'payment following court verdict' and 'payment following contract terms' and assign different symbols to these two consequences, say C1 and C2. These two terms may or may not be quantitatively identical; but they anyway refer to distinct events. This measure seems to nullify the paradox, because the consequences of the first and second if-then statements would be different for each party. Thus:

a. According to P: If P wins, then C1; but if P loses, then C2.
b. According to Q: If Q wins, then not-C1; but if Q loses, then not-C2.

It follows that if P wins and Q loses, then C1 and not-C2 are true; and if P loses and Q wins, then not-C1 and C2 are true. Since C1 and C2 are different terms, even if they happen to refer to the same monetary amount, the consequents in each party's argument are not formally contradictory (since the defining motive is different), so there is no paradox. But, to repeat, this approach does not explain why the court would give a verdict inconsistent with the agreement, so it is artificial. For a genuinely logical resolution, we must focus attention on the agreement, and not admit arbitrary verdicts. Arbitrary verdicts just muddy the waters.

Other attempted resolutions I have seen are also flawed, either because they similarly conflate legal and logical issues, or because they do not follow through on all the consequences of all the suppositions. In any case, either of my proposed two resolutions suffices to unravel the paradox; but the two together take care of all eventualities. Note that, in the first resolution, we not only resolve the paradox, but also incidentally decide the case (in favor of Protagoras); whereas in the second resolution, we are content as logicians to resolve the paradox, leaving the task of legal decision to the judge.

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[9] In 2009, in an e-mail to someone.

To conclude: the paradox of the court is due to a number of factors, which must be untangled and taken into consideration if it is to be resolved. (a) The terms 'win/lose' cannot refer to arbitrary judgments by the court; if they do, the paradox may be perpetuated. (b) It must be realized that the arguments put forward by the two parties imply that the process of resolution has two phases: a first trial followed by a second trial (or by a ready concession and settlement without need of a second trial); the appearance of paradox is partly due to overlooking this time factor. (c) There is some vagueness in what is meant by 'the agreement', and all possible interpretations must be taken into account: if the agreement is taken to refer exclusively to the explicit clause, then the first trial concerns that only and is easily decided; but if the agreement is claimed to involve a tacit understanding, then the first trial aims at a decision regarding that tacit clause and the second round deals with the explicit clause.

2. The Sorites Paradox

1. What's a heap?

The Sorites paradox is not a paradox, in the strict sense of the term, but a question. The question is sometimes put in a sophistical manner, so as to make it seem paradoxical. But it can be put in a more straightforward manner, in which case it is seen to be simple though not without importance. The term *sorites* is Latin, derived from the Greek *sōros*, meaning heap.

One way to express the Sorites paradox is: What is a 'heap' (of pebbles, say)? Or, how many pebbles (say) constitute a 'heap' of them? The obvious answer is there must surely be at least one pebble. If you have no pebbles, you do not have a heap, but a non-heap. But is one pebble enough? The obvious answer is: no, you need *at least two* pebbles to make a heap, since heap is a collective term, and one that additionally suggests that the pebbles are stacked one on top of the other (and you cannot stack a non-pebble on a pebble or a pebble on a non-pebble). A single pebble logically counts as a non-heap; heap is intrinsically plural.

Formulated like that, the question is not very problematic. But if it is formulated as follows, it becomes more complicated. If we have many pebbles (say, 100) piled up, we obviously have a heap. What happens if we remove one pebble, do we still have a heap? Yes, 99 make up a heap. What happens if we remove one more pebble, do we still have a heap? Yes, 98 make a heap. And so on, till we come to low numbers, at which stage the sophist wonders whether two pebbles constitute a heap, then one pebble, then no pebble. At this stage, the question appears paradoxical, rather artificially: should we conclude that no pebble makes a heap, or one pebble makes a heap? The sophist thinks we might; the real logician knows we cannot.

2. The use of vague terms

Clearly, the problem here, insofar as there is one, has to do with the exact formulation of initially vague terms. If we do not *at the outset* step back and think about the exact intent of a vague term, of this sort (a term suggesting quantity, as it happens) or any other, we may find ourselves in difficulty *further on* in our discourse. So, we need to stop and think before use of such terms, and preempt any difficulties they might eventually create. In the present case, as we have seen, the term (heap) is inherently plural (and so inapplicable to less than two pebbles). A sophist prefers to complicate the matter, so as to put human reason and knowledge in doubt, as is his wont; but the matter is really simple enough.

In some cases, to be sure, there is a *conventional* element to the definition of a vague term. This can be illustrated with reference to another version of the so-called Sorites paradox: As of how many hairs is a man not bald? Obviously, a man with no hair at all is definitely bald. But would a man with only one hair, or a very small number of hairs, not be considered bald in ordinary discourse? Perhaps so – but only conventionally. If we define bald *strictly*, it implies zero hair; but if we define it *loosely*, it may include cases involving an *arbitrary*, though preferably small, number of hairs, determined by convention – for examples, five or twenty hairs[10]. Clearly, if we want to avoid confusion, nothing stops us from referring to this broadened sense of 'bald' as, more accurately, 'bald or almost bald'.

This issue of vagueness is nothing special – it is not limited to terms giving rise to the Sorites paradox. For instance, a relative term like 'small' (or its relative, 'large') is *inherently vague* and can only be used with precision in specific situations by means of a conventional quantity. Again, when dealing with continua, we may need to set arbitrary dividing lines. For instance, there is no objective dividing line between one color and an adjacent color in the spectrum, and it may be necessary in some circumstances for us to imagine one (e.g. for legal or other practical purposes[11]). Conventional distinctions are part of human thought; but, it is important to stress, they are not all of human thought. There are always objective elements behind conventional ones. For example, the dividing line between blue and green would be somewhere in between what we see as clearly blue and what we see as clearly green – it would never be far on one side or the other, and much less between green and yellow or between blue and indigo or still further afield.

Returning to our alleged paradox, a few more comments are in order. As already stated, there is nothing paradoxical in the concept of a heap, if it is properly defined. Most simply, a heap can be defined as material items placed on top of each other in whatever way, implying that there must be two or more items. A more complex definition would assume that a heap must be pyramidal, i.e. requires at least four such items (three for the base and one on top); but this puts us in no difficulty, as it can be referred to more specifically as a pyramidal heap. Similarly, baldness *stricto sensu* refers to no hair at all; it is nevertheless applied to small quantities of hair, though only roughly-speaking.

There are many vague terms of this sort in our common discourse[12], some of which may require a conventional definition for pragmatic reasons. For example, the term 'crowd' might be taken to refer to a gathering of three or more people, on the basis of the popular saying that "two's company, but three's a crowd;" or we might, say in software used by the police to monitor large groups of people, opt for a larger minimum (say, 50 or 500), set arbitrarily as cause for alarm. The word 'mob' might be preferred when the latter crowd goes on a rampage.

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[10] Or whatever minimal hairiness seems to us subjectively as so close to bald as to be effectively bald.

[11] For example, in Jewish law (*halakhah*) much attention is given to quantitative definitions, notably to the maxima or minima of durations, times o'clock, distances, lengths, volumes, weights, temperatures, monetary values, etc. Initially, such measures were often expressed by the rabbis in vague terms (e.g. 'the volume of an egg'), but later more precise formulations were called for (which different authorities might differently estimate). However, some measures remain subjective (e.g. the estimate of when one is full after eating). See for more details: http://halachipedia.com/index.php?title=Reference_of_Measurements_in_Halacha.

[12] Indeed, this is inevitable on two counts. First, many concrete objects are impossible to precisely define. Where, for example, does an orange end precisely? Is the perfume or heat emanating from the fruit part of it or not? At what points in time and space may such emanations logically be regarded as separate from it? Second, human knowledge being inductive, we cannot always start a concept with a precise definition, but tend to leave it open, to be defined more and more precisely as experience unfolds. In this perspective, the majority of abstract terms we use are open, including terms that may be used to more precisely define other terms – so, here again, vagueness is inevitable. But such ontic and epistemic difficulties do not imply paradox; they simply call for philosophical reflection.

But in any case, there is no real logical problem in such unspecific quantitative expressions. They do not constitute a defect in ordinary language, requiring us to construct an "ideal language" where all terms have single precise meanings[13]. Much less do they call for treatment by means of abstruse symbolic logics (dearly loved by many modern logicians). On the contrary, they demonstrate the versatility and flexibility of ordinary thought and speech; and they witness the fact that much of our linguistic communication has non-verbal undercurrents, which we mostly comprehend very ably. Most of our daily use of vague terms involves *no need* for more clear-cut definition. They are used to suggest things approximately, and are not intended as precise and true affirmations. If the need for precision and truth does arise, it is then addressed in a way that preserves consistency (by explicit convention if necessary).

3. Reasoning with vague terms

To be sure, vague terms can be perilous if we try to reason with them. But vague terms are often used without involving them in argumentative processes Moreover, reasoning with vague terms is not always invalid – there are contexts where the vagueness does not inhibit a reliable conclusion.

In categorical (or hypothetical) syllogism, the rule regarding vague terms (or theses) is the following: The middle item (term or thesis, as the case may be) cannot be vague, because it would provide no guarantee that its intent is the same in the two premises. If the middle item is vague, we cannot be sure of overlap and the conclusion is invalid. On the other hand, the major and minor items can be vague without affecting the argument, and that in all four figures. There is, however, an exception to this rule – when the middle term is vague, but not so vague that overlap is not guaranteed, a valid argument can still be made.

The latter is evident when we consider the following two arguments in the third figure, in which the middle term ensures overlap even though neither premise is universal. The expressions 'most' (more than half) and 'few' (less than half) are vague, insofar as they do not specify exact numbers. But notice the particularity (as against majority or minority) in the conclusion – i.e. the increased vagueness of the conclusion.

Most M are P
and Most M are S;
therefore, Some S are P

Few M are P (which implies that Most M are not P)
and Most M are S;
therefore, Some S are not P

In apodosis, the rules regarding vague terms or theses are the following:

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[13] The resort to an "ideal language" by certain modern logicians to solve a problem of logic is futile. Unable to understand the actual way we real human beings logically deal with certain cognitive difficulties, they try to impose a superficial, artificial and impractical way of thinking on the rest of us. The role of the genuine logician is not to impose imaginary logics, but to understand our natural logical means and thence to perfect and reinforce them. Reasoning by humans should be the central concern of logicians. The natural language way to deal with Sorites paradoxes is to use words more precisely – e.g. instead of calling persons with very few hairs 'bald', to call them 'almost bald'; or more accurately still, if necessary for some practical purpose, 'having (say) one to ten hairs'.

Modus ponens:

If A is B, then C is D,

and A is B (affirming the antecedent);

then, C is D (consequent is affirmed).

The antecedent and the minor premise cannot be vague; else, the conclusion is invalid. However, the consequent could be vague, and the conclusion would still be valid (though also vague).

Modus tollens:

If A is B, then C is D,

and C is not D (denying the consequent);

then, A is not B (antecedent is denied).

The consequent and the minor premise cannot be vague; else, the conclusion is invalid. However, the antecedent could be vague, and the conclusion would still be valid (though also vague).

Here again, in both moods, exception is conceivable, if we know that the major and minor premises overlap, even if we don't know precisely how much they overlap.

Similar rules may be formulated for other varieties of argument, such as dilemma or a fortiori.

The people who claim that vague terms are inherently paradoxical are dishonestly nitpicking, motivated by the desire to impress themselves or others by their ability to find and resolve (contrived) paradoxes, or (worse still) to demonstrate that human knowledge is inevitably paradoxical and therefore futile. Clearly, just as it is dishonest to call a single pebble or no pebble a heap, it is dishonest to call a person with one or more hairs bald[14]. If you indulge in such contradictions-in-terms to start with, you are bound to end up with paradoxes. People who behave thus are not real logicians but sophists. They spin and fabricate – they are not interested in finding ways to true knowledge.

4. Making up fake paradoxes

The original formulations of both the conundrums described above, relating to a heap and to baldness, are attributed to Eubulides of Miletus (*fl*. 4th Cent. BCE), the Megarian logician who also gave us the Liar paradox[15]. He was a student of Euclid, a teacher of Diodorus Cronus, and a contemporary and rival of Aristotle. These puzzles were perhaps not initially presented as paradoxes, but rather as illustrations of a question (viz. where should we draw the line?). This

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[14] As I explain in *A Fortiori Logic*, Chapters 1.4 and 2.5, it is sometimes useful to formulate terms in a way so inclusive that positive, zero and negative values are all embraced by them. This is often done in scientific discourse because it facilitates some calculations and graphs. But it must be well understood that such inclusive terms are inherently undeniable – i.e. they already englobe both an affirmation and its denial. In the present context, we might choose to enlarge terms like heap or bald to include their opposites, for whatever reason; but when we do so we must remain keenly aware of what we are doing. If we do not treat such terms with appropriate care, we should not be surprised if we are forced into contradictions.

[15] A couple of centuries earlier, Epimenides of Knossos declared: "Cretans, always liars," though himself a Cretan, apparently unaware of the contradiction inherent such a statement. Eubulides may have noticed the paradox involved and sought to refine it and strengthen it, since it was not a double one but one easily resolved by saying that possibly not all Cretans are liars or that Cretans do not all always lie (Epimenides being a notable exception).

possibly reflected a dawning consciousness that there are vague terms that may require arbitrary definition in some circumstances. As above shown, this problem is solved easily enough. However, later thinkers tried to make a mountain out of a molehill, and presented the issue in the form of an argument-chain (or sorites, where the conclusion of the preceding argument serves as a premise for the next).

Thus, the bald man puzzle became, in its positive formulation: Surely, a man with one hair is about as bald as one with no hair; and if a man with only one hair can still be called bald, then a man with two hairs qualifies as bald; and if a man with two hairs is bald, then a man with three hairs is bald; etc.; therefore, a man with a thousand hairs can still be considered bald (paradox). Alternatively, the argument could be stated in negative form: if a man with a thousand hairs is not to be regarded as bald, then one with 999 hairs is not to be so regarded either; and if 999 does not qualify, then 998 does not either; and so on… whence, a man with one hair only is not bald; therefore, a man with no hair is not bald (paradox).

Clearly, these arguments are *forced* – they involve some very doubtful and misleading premises. In the positive version, the false premise is that 'a man with no hair can be called bald' *implies* 'a man with one hair can be called bald'. In the negative version, the false premise is that 'a man with one hair cannot be called bald' *implies* 'a man with no hair cannot be called bald'. It is the same false claim of implication (in contraposite form). The paradox is created by the refusal to admit that 'bald', strictly-speaking, means 'hairless'; which refusal is not based on honest logical insight, but on a willful act of illogic. Similarly, in the case of a heap, the trick consists in implying that a single pebble or even no pebble may be considered as a heap.

The inventors of these arguments do not pause and rationally reflect on the underlying issue (i.e. where is the dividing line?) before engaging in an apparent inference process, but instead attempt to bamboozle us into a paradoxical corner. The argument-chain proposed just serves as a smokescreen to conceal the crucial false claim being put over. They are the logical equivalents of pyramid sales, each sale supporting the next without solid foundation. People who are taken in by the tricky move are simply bad logicians, if not shamelessly dishonest. They then pretentiously weave massive and intricate theories around this phenomenon, untroubled by the initial error or lie in their discourse.

To those who argue that a single hair or pebble hardly makes any difference, I would suggest that they make the following simple physical experiment: take an accurate balance with the same weight on both sides, then add a single hair or pebble to one side and watch the scales tip! To those still unconvinced by this, because they dogmatically believe that logic is a matter of fancy and convention, I would suggest (tongue-in-cheek, more or less) that they place, under the heavier scale, a plunger connected to an explosive device strapped to their nose, and then watch Reality blow up in their face! That argument, I think, might finally convince them, if they survived.

As regards Eubulides, we might note in passing his other paradoxes. The most significant is of course the Liar paradox, which as I have shown in detail elsewhere[16] is exceptionally powerful due to the variety of difficulties it involves (but still quite resolvable). Another three paradoxes[17] deal with equivocations in the term 'know', specifically with failure to immediately recognize someone one normally recognizes immediately (such as a close relative or old friend), when the latter is masked or has been away too long or is not looked at attentively enough. Another, the Horns paradox claims that what you have not lost must be in your possession; whence, if you have

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16 See my *A Fortiori Logic*, Appendix 7.4.

17 Namely, the Masked Man, Elektra and Overlooked Man paradoxes.

not lost horns, you must have horns. This apodosis involves a false major premise, since something one has 'never had' may equally (as well as something one 'still has') be characterized as 'not lost' – so the consequent does not follow upon the antecedent; therefore, if one has not lost horns, one cannot be assumed to have horns (since one may well never have had any).

From this short list, it can be seen that Eubulides' queries all give rise to some sort of reflection on logic – reflections on vague terms, on conventional definitions, on equivocations, on term-negations, on self-reference, and various other difficulties that may arise in human discourse. It would be wrong, I think, to assume the motive of such queries to have been teasing or obfuscation (although, to be sure, later skeptics did use such conundrums malignantly, as already mentioned). Rather, I'd say, they served as springboards for earnest reflections and discussions on logic – because it seems unlikely that they were formulated without any attempt to solve the problems they engendered. In some cases, valid explanations or resolutions were no doubt proposed (even though they may not have come down to us), while in other cases the difficulties may have seemed insurmountable. In other words, I doubt that Eubulides was merely sophistical – I'd class him rather as a serious logician.

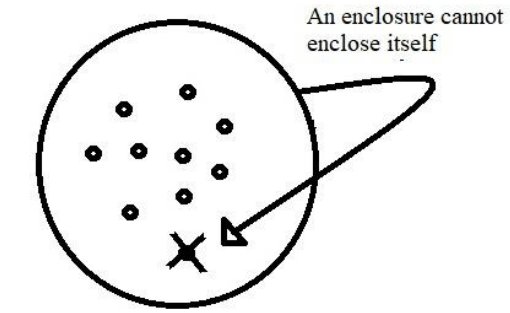
3. The root cause of the Russell paradox

1. My resolution of the Russel paradox

My resolution of the Russel paradox is, simply put, that *no* class is, or can ever be, a member of itself. Self-membership is *unconscionable*. It is therefore no surprise that the unthinking adoption of the idea of self-membership by Russell led him to paradox. The concept of class-membership differs significantly from that of predication. Predication is a natural way of thinking; class-membership is an artificial one, invented by logicians. The concept of class-membership is useful, but it has limitations. The idea of class-membership is very early, being implied already in Aristotle's discourses; but became more consciously used by logicians in modern times.

The original idea is that if any individual thing (call it X) 'has' some property (call it Y-ness), then it is 'subsumed' under the species or genus of Y-having things; i.e. ***if X is Y, then X is a Y*** (notice the use of 'a' to signal that a species or genus, as distinct from a property, is intended by Y). This thought later led to the idea of a 'class with members', defined by: ***if X is a Y, then X is a member of the class "Y"*** (notice the use of inverted commas to refer to a class as distinct from a predicate). Tacitly intended here are the complements: "if X is not Y, then X is not a Y" and "if X is not a Y, then X is not a member of the class 'Y'," note.

But it should be made clear that these two ideas, though related, are quite distinct. While it can be said of anything which is Y, that it is *a* Y; it cannot be said of any class "Y" which happens to be a Y, that it is a member of the class "Y" (i.e. a member of itself). For the idea of class-membership is figuratively the idea of an enclosure (the class) in which a group of things (the members) that are alike in some way (i.e. have some property in common, say Y) are included; and it is *impossible to visually imagine* that the enclosure will be included within itself (i.e. that the class can be a member of itself). One can say it in words, of course; but one cannot draw it on paper.



A class cannot be a member of itself

Thus, the idea of subsumption under a species or genus (of "Y" under Y) and that of class-membership (of "Y" in "Y") are only partly equivalent, and should not be confused. For example, whereas the class "classes" is indeed a class, it cannot logically be a member of the class "classes." The characterizations "is a class" and "is a member of the class 'classes'" are not interchangeable. The class "classes" can, however, still be a member of other classes, notably of the class "not self-member classes." Indeed, no class can be a member of "self-member classes," because there is no such class (as just explained); and consequently, *all* classes are members of "not self-member classes."

Note this well: even though some classes (such as the class "classes") do *subsume* themselves, they cannot reasonably be said to be *members of* themselves. These are significantly different relationships. This signifies that, whatever the value of Y: ***whether the class "Y" is or is not a Y, "Y" is* not *a member of the class "Y,"*** i.e. it is not a member of itself. So, the proposition through which we initially defined class-membership, viz. if X is a Y, then X is a member of the class "Y," has *a formal exception*, viz. the case where the class "Y" is intended by the term X (which is otherwise general).

(Although we have not here dealt with subclasses and overclasses, note that the said exception does not prevent the class "Y" from being a member of the *overclass* "Y-classes;" which, by definition, includes "Y" together with all the subclasses of "Y." This is true because "Y" and "Y-classes" are at different levels, so this does not constitute self-membership. More on this below.)

Another oddity of class logic to note is that, just as "classes" is a class (viz. the class of all classes), "non-classes" can also be said to be a class (viz. the class of all non-classes), without self-contradiction. In the case of predicate logic, such a twist is not possible; i.e. a non-class (i.e. any thing that does not qualify as a class) cannot be said to be a class: only a class (i.e. any thing that does qualify as a class) can be said to be a class. This again proves the divergence between these two ways of thinking. On the other hand, note, "classes" (being a class) cannot consistently be said to be a non-class; and likewise, "non-classes" (also being a class) cannot be said to be a non-class. All this is obvious but worth repeating.

As regards the Russell paradox, it is alleged that the class of "all classes not members of themselves" constitutes a double paradox, because: if we say it is not a member of itself, then it *is a member of itself*, and if we say that it *is a member of itself* then it is not a member of itself. But it is clear, in the light of what we have realized above, that the problem lies in the consequent of the first if-then statement and in the antecedent of the second if-then statement (shown in italics), i.e. in the *false* claim which they contain, viz. that "all classes not member of themselves" can be a member of itself. In truth, *no* class whatever is a member of itself; so, both the if-then statements

through which the Russell paradox is formulated are wrong: they are non-sequiturs – their 'then' clauses do not truly follow from their respective 'if' clauses.

In other words, the resolution of Russell's paradox is that the class labeled "all classes not member of themselves," *just like any and every other class*, is in fact *not* a member of itself, because there is *no such thing* as "a class member of itself;" and this claim does *not* give rise to any self-contradiction. Putting this another way: the term "a class member of itself" is a possible *verbal* construct, but it is *geometrically* unthinkable, and therefore rationally inconceivable; it is a fanciful string of separately meaningful words whose conjunction is in fact meaningless.

2. Why Russell's resolutions are inadequate

Now, Bertrand Russell did formulate a somewhat similar explanation of his paradox, saying:

> "No collection (whole or totality) can contain members that are defined in terms of itself; specifically, no existing collection can ever be a constitutive part of itself"[18].

How does this formulation differ from my denial of the logical possibility of self-membership? Russell's formulation obviously refers indirectly to the collection itself; i.e. he means, more directly put: a collection cannot contain itself as a member because such membership could only be triggered by *defining the collection in terms of itself*, and that is not feasible. Where is the "definition," here? It is the term's formulation itself, viz. "all classes that are members of themselves" in our case.

The proposed definition involves the term that is being defined; i.e. there is an element of self-reference, of circularity. This can be called self-definition; and it is obviously useless, if we do not already know what is intended by other means. But is definition of the class in terms of itself the issue at hand, here? I would say not: this term is merely the end-product of a reasoning process; it does not suddenly come out of the blue. It is this reasoning process that needs to be examined and evaluated.

Is the class of "all classes that are members of themselves" *produced by* self-definition? No – the underlying predicative term "all classes that *are* classes" is not problematic: indeed, all classes are classes. This predication engenders the classificatory term "all classes that *are members of* (the class called) 'classes'," which is not per se problematic, either. What is problematic is that the latter term, using for example the fact that "classes" is a class, and so apparently a member of itself, inductively gives rise to the term "classes that are members of *themselves*." So, the genesis of the latter, problematic term is not through mere self-definition, as Russell seems to think, but through the illegitimate traduction (putative translation of one form into another) of a perfectly legitimate predication into a class-membership proposition.

The said traduction is discovered and proved to be illegitimate by the fact that it gives rise to the Russell paradox. It is this barely noticeable change of form which causes the problem. My proposed resolution is easily proved with reference to the corresponding negative term "all classes that are *not* members of themselves." If the issue were self-definition, as Russell suggests, then this term *too* would be problematic, since it also refers to itself (albeit negatively); yet this term is

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[18] Quotation from *Principia Mathematica* by Nicholas Rescher in *Paradoxes: Their Roots, Range, and Resolution* (Chicago, Ill.: Open Court, 2001), p. 172. This "Vicious Circle Principle" was earlier formulated by Henri Poincaré.

not problematic![19] Clearly, then, the issue at hand is not self-definition, but something else entirely: it is the naïve 'inference' (the traduction) just described.

More accurate is Russell's concluding statement, viz. that "no existing collection can ever be a constitutive part of itself" – but that is not an explanation in itself: it is an evident consequent of the Russell paradox, i.e. a statement that there is obviously some sort of problem with any "existing collection" being "a constitutive part of itself," which implies that such a collection cannot exist. That is not an explanation, but an observation needing an explanation, and the explanation Russell offers revolves around self-definition, as we have seen. But self-definition is not, in fact, genetically involved.

I would say that, although Russell glimpsed that there is a problem inherent in the idea of self-membership (or, alternatively, perhaps, that of non-self-membership) by a class, he did not correctly understand precisely why the problem arose. He wrongly identified the problem as self-definition, i.e. as self-reference in definition. But though this feature is present and significant, it is not the crux of the problem.

In truth, the problem can only be solved through the finer conceptual analysis I have formulated. The problem arises through the assumption that *subsumption under a concept* and *class-membership* are fully parallel (effectively interchangeable) ideas – they simply are not. It is this prior faulty assumption which eventually gives rise to the Russell paradox. Once this is understood, it is easy to see why "self-membership" is unconscionable and we must say categorically that *no* classes are members of themselves, and in that event no Russell paradox arises anymore.

Later on, Russell, seemingly dissatisfied with his earlier proposed resolution of his double paradox, proposed his Theory of Types as a means to this end. In its simplest form, this theory focuses on the differences between the individual items subsumed by classes, the classes themselves, the classes of classes that the latter are subsumed by, and so on. These are successive "types," forming a hierarchy of items from the concrete objects to more and more abstract derivatives of them.[20]

Very briefly put, the theory postulates that a proposition may be universally true of a lower type, but not include a higher type. For example, all classes can be said to be members of the class "classes," without this implying that "classes" is also a member, simply because (the theory claims) it is of a higher type than all the classes it includes. So, "all classes" does not literally mean *all* classes; it can (due to "type" difference) exclude the class "classes" itself. Of course, this claim is quite arbitrary, and merely conceals the Russell paradox behind a smoke screen.[21]

There is some truth to this idea, but it also involves some confusion. There indeed exists a hierarchy between concrete individual items, abstract groupings of items (classes), and more abstract groups of groups of items (classes of classes). However, the Russell paradox does not arise when we try to include a class in a class of classes, i.e. a subclass in an overclass. And of course, the Russell paradox does not arise when we try to include an individual item in a class. The Russell paradox

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[19] Alternatively, if the latter is thought to be problematic, then the former cannot be thought so. In any case, one of the two has to be unproblematic, *for discourse to be at all possible*. I mention this alternative, only because *at first sight* it may seem that the negative term "all classes not members of themselves" is the problematic one, since it apparently results in paradox (whereas the positive one apparently does not). In truth, this negative term is unproblematic, and it is the positive term "all classes members of themselves" which causes the paradox. Note this well.

[20] I will not here get into a detailed analysis of this theory. I refer readers to my previous, more detailed analyses in my books *Future Logic* (Chapters 43-45) and *A Fortiori Logic* (Appendix 7.5).

[21] I refer readers to the excellent exposition and critique of Russell's paradox and his attempted resolutions of it, in William and Martha Kneales' *The Development of Logic*, Chapter 11.

only arises when we try to include a class in itself; or (by extension, since it is also a class) a class of classes in itself.

The Russell paradox *only* arises in attempts to self-include a class, or self-include a class of classes, *in itself*. So, hierarchy is not the issue, here, and the theory of types cannot provide a solution! It is only incidentally true that a class can only group concrete objects, or a class of classes can only group classes less abstract than itself. The reason is not differences in type, but the fact that a class or class of classes cannot, even in imagination, include itself in itself.

Note in passing that, in our above example, the class "classes" is indeed a class of classes since the units it groups are all classes; nevertheless, it remains a class in its own right, and as such apparently belongs under itself, i.e. subsumes itself. The solution is not that it cannot subsume itself (i.e. *be* a class), but only that it cannot thereby be considered as a member of itself (i.e. *be classified as* a class). This is not a matter of hierarchy (type), but a matter of changing predication into classification.

3. Why Rescher's resolution is inadequate

Nicholas Rescher's proposed resolution of the Russell paradox is somewhat but not much different from Russell's first attempt. He applies a general principle that he has formulated, called the Successful Introduction Principle, to the case at hand. He points out that "only when something is properly identified, can it serve as a subject of meaningful discussion;" adding: "With an inappropriately identified pseudo-object… the door to contradiction and paradox is thrown wide open"[22]. In his view, then, the Russell paradox is caused by a failure of proper identification inherent in a class defined in terms of itself.

Thus, Rescher focuses on the problem of self-definition, just as Russell does. His approach differs, as he points out, in that whereas Russell sees self-definition as an ontological issue, Rescher sees it as an issue of communication. For Russell a class cannot exist which is defined with reference to itself; whereas for Rescher, we cannot even begin to use or discuss such a class, because it is as if nothing has been said yet. Both these insights are, in my view, true. But they do not resolve the Russell paradox, as already explained. To repeat, the issue is not one of self-definition, but of passing over from a predicative discourse to a classificatory one. A class like "classes" *is* indeed a class, but "classes" cannot *be a member of* the class "classes." These are two different systems of thought, which do not behave identically in all circumstances.

We cannot form concepts at will, by just stringing some words together arbitrarily in a proposed definition. A verbal definition is a hypothesis subject to verification by logic before it becomes a cognitively-functioning concept. In the case of the definition of class-membership, viz. the proposition that "if X is a Y, then X is a member of the class 'Y'," double paradox is found to ensue in the special case of "if 'Y' is a Y, then 'Y' is a member of 'Y'." Therefore, the rule we

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[22] Op. cit. pp. 172-73. While this SIP principle is intuitively sound, and relevant to the case at hand, it must be said that it is not a general truth. In truth, much of human discourse is made through more or less vague terms. Relatively few terms, if any, are fully and finally defined. This is because our knowledge is essentially inductive, rather than (as many logicians imagine) deductive. We use a word (when we need one) and a working definition (when we have one) as pragmatic tools. Our understanding of their reference is at all times tentative and temporary. As our knowledge grows and our analyses are refined, our terms (hopefully) gain increasing accuracy, become more pointed. So, the principle here proposed by Rescher should not be taken too generally. Meaningful discussion occurs all the time with imperfectly defined words; that is the norm rather than the exception, and we usually manage very well to avoid contradictions nonetheless. Meaning is not essentially a verbal issue, but one of insight and understanding.

tried to formulate (i.e. the definition of class-membership just stated) must have an exception; namely, when X stands for the class "Y."

The Russell paradox is valuable in that it teaches us that an exception must be allowed for in the defining formula. We initially assume the general definition to be valid; but after we find out that it leads to self-contradiction and paradox, we are obliged to retract it in part. We are logically forced to altogether reject the notion of self-membership. This is a very limited and precise retraction, but it restores consistency to class logic. Our hypothetical definition has been adjusted to avoid inconsistency. This is the correct resolution of the Russell paradox.

4. Other incarnations of the Russell paradox

It should be mentioned that Russell realized that the concept of class was at issue somehow, but then he tried to show that the same paradox could be formulated within predication. As Kneale and Kneale rendered it:

> "Instead of the class which is supposed to contain all classes that are not members of themselves let us consider the property of being a property which does not exemplify itself. If this property exemplifies itself, then it cannot exemplify itself; and if it does not exemplify itself, then it must exemplify itself. Clearly, the nature of the trouble is the same here as in the original paradox, and yet there is no talk of classes."[23]

The implications here are that some properties "exemplify" themselves, while some do not. That is, a property Y: in some cases, Y is Y (i.e. has property Y) and in others Y is not Y (i.e. lacks property Y). If Y is Y, then Y exemplifies itself; and if Y is not Y, then Y does not exemplify itself. Is it, however, ever true that Y is Y? Can anything be pointed to that has precisely itself as a predicate? I would say not. Clearly, 'Y is Y' here does not refer to a predicative tautology, like 'white is white'. Rather what we mean by 'Y is Y' here is that 'whiteness is white', i.e. Y-*ness* (*the property* Y) is Y (has the property Y). This is a very different statement, which does not follow from the preceding one.

And the same critique of it can be brought to bear for it as for the classificatory version. We may well say that if X is Y, then X "exemplifies" Y-ness (i.e. is one of the things that has Y-ness), but in the special case where X is replaced by Y-ness (i.e. the property Y), we must make an exception, and deny that 'Y-ness is Y' implies 'Y-ness exemplifies Y-ness (i.e. itself)'. To avoid an eventual Russell paradox, we must at least short-circuit such implication, if not totally reject the consequent (saying that it is never true that Y-ness exemplifies Y-ness), if not even reject the antecedent (saying that it is never true that Y-ness is Y).

Russell claimed that if "the property of non-self-exemplification" exemplifies itself, then it lacks "the property of non-self-exemplification" and so cannot exemplify itself; and if does not exemplify itself, then it has "the property of non-self-exemplification" and so must exemplify itself. To my mind, this means we must reject the very idea of self-exemplification and say that *nothing* exemplifies itself (even if we happen to find some Y-ness which is Y). In that event, we can well say that "the property of non-self-exemplification" does not exemplify itself, without any contradiction arising.

The same thing can be said using other terms: some concepts are "instances" of themselves, while some are not. That is, we can define instantiation by saying: if X is a Y, then X is an instance of Y (i.e. X is an individual within the group called Y); but we must make an exception in the special

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[23] *The Development of Logic* (Oxford UP, 1962), p. 655.

case where X is replaced by Y, and deny that if Y is a Y then Y is an instance of Y. It is clear that Y here refers specifically to a concept, and not just any kind of thing. An example would be "the concept 'concept' is a concept; therefore, the concept 'concept' is an instance of itself."

This is clearly very similar to saying "the class of all classes is itself a class; therefore, the class of all classes is a member of itself;" and a similar paradox is bound to emerge from it, which can be neutralized in a similar manner. That is, we must upon reflection say that *nothing* is an instance of itself, so that in fact "the concept of non-self-instantiation" is not an instance of itself. A similar line of reasoning can be followed with regard to "inclusion" and any other similar relations.

Clearly, none of these issues relate to predication as such, but to more geometrical ideas (classifying, having/belonging, instantiating, including) through which we try to represent predication. These relations are intended as metaphors for predication; but it turns out that these analogies are not perfect. The lesson the Russell paradox teaches us is that these derivative relations all have exceptions (as already detailed above). The Russell paradox does not affect predication as such; predication as such is immune to it. Although Russell was right to investigate whether his paradox meme affects predication as such, he was wrong to conclude that it does; it does not.

5. About the Barber paradox

There is a town with only one barber. Some men in it shave themselves; the rest do not, but are all shaved by the barber. What about the barber? Does he shave himself or not? If he does, he is one of the men who shave themselves (no problem); if he does not, he is one of those shaved by the barber, i.e. by himself (apparent contradiction). Obviously, in his case, the two classes of men overlap: he both "shaves himself" and he is "shaved by the barber." This is hardly problematic: it just tells us that the supposed either-or disjunction between the two groups of men is not really one; his case is a logically implied exception to it. That is, in his case, unlike in other cases, 'being shaved by the barber' does not imply 'not shaving himself'.

How does this compare to the Russell paradox, of which the barber paradox is touted as an illustration? I resolve the Russell paradox by saying that no class is a member of itself; it follows that we can say that the class of "classes not members of themselves" is necessarily not a member of itself, without this giving rise to a contradiction. I resolve the barber paradox differently, by saying that, unlike all other men in town, the barber can well both shave himself and be shaved by the barber (himself). In the Russell paradox, one of the conflicting classes is wholly eliminated. In the barber paradox, the conflict between the classes is softened in a single case (that of the barber himself). Since the two paradoxes are not resolved in the same way, they must be regarded as logically distinct.

4. The self-contradiction of the superposition hypothesis

Those who are watchful never die; those who do not watch are already as dead.[24]

1. Aristotelian logic

A term (a thing of any sort, whether physical, mental or spiritual, whether concrete or abstract, whether named or nameless), say A, and its *negation*, notA (or nonA), are not only different, but contradictory, i.e. incompatible and exhaustive. Logically and really, they cannot both be true at once and cannot both be false at once, in any context or circumstance whatsoever.

The law of **noncontradiction**[25] is that, though A and notA may exist in the same world, perhaps in the same entity in different ways, even spatially side by side and even temporally adjacent, nevertheless they cannot exist in exactly the same way, in the very same place and at the very same time. The law of **the excluded middle** is that A and notA cannot be both absent from the world, or from any part of the world; if they are indeed each other's negations, there is no middle way about them, no third alternative in-between or beyond them, they exhaust the possibilities of existence in that respect, place and time. These two laws are just expressions of the single law of **identity** – which is that each and every existing thing has and can only have a precise identity, whether we know it precisely or not.

The said three laws are known as the laws of thought because they are the basis of all rational discourse about the world. They are called 'axioms' because no denial of them is logically possible, none is rationally conceivable. Antinomy (breach of the laws of thought) is, in fact, not even thinkable; it is, literally, *unconscionable*.

One cannot even visualize contradictions or neither-nor situations, i.e. mentally produce the overlap of an image and its absence, or something in-between an image and its absence. One can verbally formulate a contradictory or neither-nor statement by stringing together words or symbols with an antinomic intention in mind; but such a statement merely articulates the terms A and notA, or neither A nor notA, successively, it does not enact the alleged 'interbeing' of what they refer to.

Ignoring or dismissing these laws can only result in erroneous thought. Thus, if one comes across a coexistence of opposites in a discourse, one can be sure it contains some error; likewise, if a discourse claims a middle ground between something and its negation, this is sure proof of error. In such cases, we must review and revise our thinking if we want to realign it with reality.

The three laws of thought do not imply that things are invariable and static, but on the contrary help us to adapt our knowledge to the continual change and movement evident in the world before us and in us. A thing can differ in space at a given time (in its parts or constituents or components), in time (as it changes aspects or position in space), in quality, in quantity, in motion, and indeed in every possible way – so long, of course, as its particular nature allows for such variations. Nothing, however, can vary in any way that goes against the laws of thought, as already stated.

Note that the laws of noncontradiction and of the excluded middle concern anything (A) and its negation (notA): they apply primarily to 'opposites' (in the narrow sense of this term). 'Negation' is a relation more radical than mere 'difference'. Different things might both be present in a given

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[24] *The Dhammapada*, v. 21. (Juan Mascaro, trans. Harmondsworth, Mx.: Penguin, 1973.)

[25] Also often called the law of contradiction, i.e. *about* contradiction.

place and time: if so, they are compatible. But a thing and its negation cannot do that: they are necessarily incompatible. Different things might both be absent in a given place and time; if so, they are inexhaustive. But a thing and its negation cannot do that: they are exhaustive.

On another tack, let us now consider more broadly any pair of prima facie *different* terms, say A1 and A2. If these two terms happen to fit into the set of conditions given above in the laws of thought, i.e. if they cannot both be present or applicable in every respect at once and if they cannot be both absent or inapplicable in any respect at once, then effectively each is the negation of the other. In such case, A1 = notA2 and A2 = notA1; their relation is an either-or disjunction.

But if besides A1 and A2 we find cases or situations involving a *conjunction* of A1 and A2, and/or we find another possibility, A3, or numerous other *alternatives*, A3, A4, etc., beside them – then, although A1 and A2 are different, they are clearly not each other's negations. In that case, they are still somewhat disjoined, but the disjunction between them is of a weaker sort. The laws of identity, of noncontradiction, and of the excluded middle, remain applicable, but not just to the two initial terms A1 and A2, but to all the terms in the factually given multiple disjunction. In such case, the negation of A1 is the disjunction of A2, A3, etc.; the negation of A2 is the disjunction of A1, A3, etc.; the negation of A3 is the disjunction of A1, A2, etc.; and so forth.

If A3 happens to be a conjunction of A1 and A2, then the disjunction between the latter two is said to be 'inclusive'; that is, A1 or A2 or A3 can in such case be stated as A1 and/or A2 (the A3 alternative being left tacit, implied by the 'and' in and/or). If the three or more disjuncts do not refer to conjuncts (such as A1 and A2), then the disjunction is called 'exclusive' (meaning that each alternative excludes the other). None of this, clearly, goes against the said three laws. Nevertheless, introducing more than two alternatives gives the impression of mitigating them somewhat (without breaching them, to repeat).

Another way the laws of thought may seemingly be softened is by introducing modality into the terms. The terms A and notA cannot be both affirmed or both denied. But the terms *possibly*-A and *possibly*-notA can certainly be both affirmed (this is what we call contingency), without breach of the law of noncontradiction. They cannot, however, be both denied, because that would imply that both A is impossible and notA is impossible, implying that both A and notA are denied, contrary to the law of the excluded middle. It is also of course consistent to deny the one and affirm the other; indeed, A is impossible implies that notA is not only possible but necessary, and likewise notA is impossible implies that A is not only possible but necessary.

Let us now focus on the conjunction 'possibly-A and possibly-notA', which is the most interesting in the present context. First, let us note that there are different meanings of the modality labeled 'possibly', according to the mode of modality intended. In logical modality, contingency means that there is logical indeterminacy in the framework of the given premises. In a more epistemic perspective, this signifies a subjective *uncertainty* on our part as to whether A or notA is in fact applicable in the given context. In natural modality, possibly means potentially, and contingency refers to a factual indeterminacy, i.e. it means that the issue A or notA is *not yet in fact settled*, physically or mentally or spiritually as the case may be.

Let us now focus on the latter conjunction, that between potentially-A and potentially notA, as distinct from actually-A and actually-notA. This refers to natural contingency; but natural contingency varies in intensity.

In a world, or a domain of the world, *ruled exclusively by strict determinism*, the contingency simply means that although either A or notA can be present, depending on unspecified circumstances, nevertheless, one of the two is *necessarily* present (whether we know which of the

two or not) and the other is *necessarily* absent; that is to say, if we had full knowledge of the circumstances involved at a given time, we could surely predict which of the two is present.

But in a world, or a domain of the world, *where indeterminism is possible, whether in the form of natural spontaneity or of volition*, there is no necessity of realization for either potential, so that *even if* we had *full* knowledge of the surrounding forces involved at a given time, we could *not* surely predict which of the two is present. We would be unable to predict this, not because of some cognitive incapacity of ours, but due to the very nature of the facts before us.

Note this well – although the same statement 'potentially-A and potentially-notA' is superficially applicable in both the said scenarios (determinism and indeterminism), in the former prediction is in principle possible, whereas in the latter prediction is in principle impossible. And moreover, the latter scenario is as consistent with the laws of noncontradiction and the excluded middle as the former.

In a deterministic framework, the matter is effectively settled in advance and can, at least in principle (even if in practice it may be close to impossible to do), be predicted; whereas, in an indeterministic framework, whether that of natural spontaneity or that of volition, the matter is *not* settled in advance and therefore *cannot* by any means be predicted. Nevertheless, in the latter framework, it may be possible to establish different probabilities for the two alternatives, i.e. A may be more probable than notA, or vice versa; only, neither alternative may be claimed to have 100% probability.

These are, briefly put, the formal statements that we refer to as Aristotelian logic, because the Greek philosopher Aristotle (384-322 BCE) was the first to formulate them consciously and clearly. Note well that Aristotle did effectively conceive of and take into consideration indeterminism in his discussion of ***the logic of future contingent propositions***, arguing:

> "It is necessary for there to be or not to be a sea-battle tomorrow; but it is not necessary for a sea-battle to take place tomorrow, nor for one not to take place—though it is necessary for one to take place or not to take place." (*De Interpretatione*, 9, 30-33).

Clearly, Aristotle's purpose in this statement was *to defend* the laws of noncontradiction and of the excluded middle – to show their applicability in all situations. Whether the sea battle takes place tomorrow or not cannot be predicted today because (in the sea battle narrative) human volition is involved; we can only be sure of the direction of human acts of will when they definitely materialize. Today, all we can say is that both alternatives are *potential* (equally or unequally so, as the case may be); only tomorrow will we know the answer to the question, when the battle *actually* occurs or *actually* fails to occur. Logically and materially, potentially-A and potentially-notA are quite compatible (though they cannot be both denied); it is only actually-A and actually-notA that are incompatible (and exhaustive too).

In modern times, some people have claimed that 'non-Aristotelian logics' are conceivable simply by denying the said laws. They imagine that these laws of thought are arbitrary claims and can be replaced by their antitheses at will. This delusion is probably due to the widespread resort to symbolic logic, nowadays, which gives silly people the false impression that logic is mere manipulation of symbols, a linguistic game, something ultimately arbitrary. Some of them apparently seek to bring about a 'Copernican revolution' in logic in pursuit of fame; but most of these people evidently oppose Aristotelian logic out of some deep-seated personal resentment against Reason, a yearning to dethrone it and be freed of its strictures.

They formulate seemingly internally consistent 'systems' which superficially appear to support their claim. They grandiloquently call this 'paraconsistent' logic', as if partial consistency is enough to convince us; no – *only wholly consistent discourse can be regarded as truly consistent*

and granted credibility. Such deviant claims are necessarily absurd since they are only made to seem conceivable by devising some internally cogent discourse, without regard to the wider context of knowledge. There is still in them, though, a reference to consistency – that is, to Aristotle's laws of thought – as the seal of their credibility. That is inevitable, because these laws are the very foundation of rational thought.

One cannot use reason to reject reason because such rejection then automatically rejects itself too. An axiom is self-evident because its negation is self-contradictory. Of course, people can choose to think in a totally irrational manner, in order to not even depend on 'internal consistency' (and thus be fully 'consistent' with their choice!); these are nowadays characterized as 'post-modern' or (more recently) as 'woke'. It is all just make-believe; it is not truly logical. Needless to say, people who thoroughly thus divorce themselves from reality are sure to end up in a nuthouse and probably soon after that in a morgue, since our mental health and our very life depend on reason.

2. Superposition and Schrödinger's cat

Following the discovery of certain weird and wonderful phenomena, such as the particle-wave duality, modern physics adopted the concept of 'superposition', which may be briefly described as follows:

> "Quantum superposition is a fundamental principle of quantum mechanics that states that a quantum system can be in multiple states at the same time until it is measured. This is in contrast to classical mechanics, where a system can only be in one state at a time. For example, an electron can be in a superposition of two different energy states, or in a superposition of two different positions. This means that the electron is not in either state definitively until it is measured. When the electron is measured, it collapses into one of the two states."[26]

This concept is commonly perceived and presented as justifying denial of the laws of noncontradiction and of the excluded middle, and thus of identity, at least regarding certain phenomena in quantum mechanics. It is claimed that superposition is *the only possible adequate interpretation and explanation* of the subatomic phenomena in question; so that, effectively, these phenomena constitute *concrete empirical evidence and proof* that the three laws of thought are not universal laws of nature.

I will now explain why this claim is untrue and unsustainable. Note well that it is not my intent to deny in any way the accuracy of the physical observations, experiments, generalizations, and adductions, on which this claim is based. I am not a physicist and I have no axe to grind in that field. I accept whatever findings physicists collectively report and agree upon. My only interest here is the *philosophical spin* that some people have artificially attached to the findings concerned.

The concept of superposition affirms that (simply put) 'a quantum system can be in multiple states at the same time until it is measured'. The suggestion made here is that something A and its negation notA can coexist simultaneously, occupying the exact same space and time in every respect, and/or that there are some states of being in-between them, replacing both of them, this happening *behind the scenes*, as it were, until and unless active observation makes the one or the other alternative to *come to the fore*, giving us the (allegedly false) common impression that A and notA are mutually exclusive (i.e. both cannot occur at once) and exhaustive (i.e. there is no possibility ever of neither occurring) on which the laws of thought are based.

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[26] Quoting Gemini. This suffices for our purpose here.

What is logically wrong with this scenario? What is wrong with it is that it is *necessarily speculative*, i.e. a belief which can only be based on faith because it is incapable of empirical verification. *By definition, we do not know* whether the system is characterized, at the time and place concerned, by event A or notA until we actively test it, and it is this very test that causes the system to actualize one way or the other, right? If this is true, *then how can we possibly know* that the situation prior to actualization is 'superposition' of A and notA? We *logically cannot*.

Clearly, any claim as to what is happening 'behind the scenes' in the said scenario is inherently and inevitably, by the very parameters postulated to start with, based on wild speculation! **The concept of superposition claims *not to know* anything about A and notA while they are in their potential state, and in the same breath claims *to know* that they are coexisting simultaneously there. That is a self-contradiction!** It is, inevitably, claiming something to be both unknowable and knowable.

Of course, to claim that contradictories A and notA can ever actually coexist simultaneously is itself an absurd claim, given the law of noncontradiction; and to claim that there can be a real state of things in-between A and notA is a breach of the law of the excluded middle. Such claims are, logically, already bad enough.

Additionally, the advocates of such claims are imagining that superposition is going on behind the scenes, although they have zero concrete evidence that it is happening there. In other words, they arbitrarily give more credence to their own *imaginative construct* about an invisible, indeed in principle undetectable, phenomenon, than to the laws of noncontradiction and of the excluded middle, which are among the foundations of human *rational thought*. Such a claim is not empirical science but utter fantasy.

But even worse is the *internal inconsistency* of the discourse of those advocating superposition. Even though what is happening behind the scenes is, by their own definition, *unknowable*, they claim to *know* enough about what is happening behind the scenes to tell us, without any empirical proof, indeed admitting the impossibility of such proof, that coexistence of incompatibles and/or the existence of something neither-nor is happening there.

Thus, in addition to fancifully advocating the *ontological* possibility of co-existence of being and not-being and/or of a middle ground between being and not-being, the advocates of superposition resort to self-contradictory discourse, i.e. their *epistemology* is also irreparably flawed.

If inconsistent discourse can be true, then such discourse could equally well be false, and/or both true and false, and/or neither true nor false, or whatever. In other words, the concepts of true and false lose all meaning and all credibility. This is not some sort of discourse, it is the end of all discourse.

It must be emphasized that the concept of 'superposition' *conflates two issues*, using a probably accurate empirical observation to illicitly support an unjustified, and utterly unjustifiable, interpretative speculation.

There is nothing new or antilogical in the underlying phenomenon of *indeterminism* – it is quite conceivable that nature, at the level of quantum phenomena, is capable of true natural *spontaneity*, and is therefore *unpredictable* in principle and not merely in practice, and the findings of modern physics seem to confirm that this indeed happens at that level. ***This is easily assimilated under Aristotelian logic with reference to the logic of future contingents*** (see previous section).

What is wrong with the concept of superposition as it is usually presented, is *the added claim* that natural spontaneity is paradoxical, i.e. implies a breach of the laws of noncontradiction and of the excluded middle. No, it does not; the former (natural spontaneity) simply does not imply the latter (paradoxical 'superposition'). To claim that it does is mere spin. It is dishonest nonsense.

The Austrian physicist Erwin Schrödinger proposed in 1935 a now famous 'thought experiment', involving a cat, apparently in an attempt by him to demonstrate the absurdity of the concept of superposition:

> "In the thought experiment, a hypothetical cat is placed in a sealed box with a radioactive atom and a vial of poison. If the atom decays, it will trigger a mechanism that will break the vial, releasing the poison and killing the cat. However, since the atom is in a state of quantum superposition, it is both decayed and undecayed at the same time. This means that the cat is also in a state of quantum superposition, both alive and dead at the same time."[27]

If I understand it correctly, Schrödinger's point was that since the cat could obviously *not* be both alive and dead at once, but had to be the one or the other, it follows that the atom could not be both decayed and undecayed at once but had to be the one or the other. Schrödinger's purpose was thus *to defend* the laws of thought. The proposed experiment served merely to magnify the invisible quantum events, by tying them to events occurring on a more human scale, namely the life or death of a cat. The cat was incidental; but the assumption that the cat could not be both alive and dead was essential.

It does not look like this thought experiment convinced Schrödinger's colleagues to drop their idea of superposition. It seems to be regarded nowadays as an educationally useful caricatural illustration of that position rather than as a refutation of it. I do not know quite why. Perhaps because the cat might have been observed through a window in the sealed box, or through sensors attached to its vital organs. Perhaps because some people think an intermediate state between life and death may well exist. In any case, the whole thing seems to me to be an unnecessary complication.

The issue at hand is not experimental, it is logical.

Superposition is, as already explained, a mixed-up concept. Aristotelian logic, which insists on the *universal* applicability, without any exception, of the laws of noncontradiction and of the excluded middle, and thus of identity, freely admits that there are events that are unpredictable until they become actual. Such unpredictability may be due to ignorance of some of the parameters involved, if the system concerned is deterministic; or it may be due to the natural spontaneity of some of the factors involved, if the system is indeterministic, as seems to be the case in quantum mechanics; or it may be due to the involvement of volition, which is also indeterminism but of a very different sort.

There is thus *no need* here to appeal to a putative inapplicability of the laws of noncontradiction and of the excluded middle: the proponents of superposition have no logical justification for their revolutionary claim. Superposition clearly does not fall within the bounds of rational discourse, which is inextricably tied to the laws of thought. The proposed concept is, to repeat, mere philosophical spin; it has no empirical physical justification whatsoever and *it is not needed* to explain the known facts.

It is not necessary for the science of Physics, which can readily proceed normally without it. It has nothing to do with Physics; it is a contrived appendage to it. Its motive is anti-rational, a doctrinal stance made fashionable by philosophical skeptics like David Hume. The sad fact is that some people relish the perverse thought that human reason might be debunked. It won't and can't. The empirical phenomena that science discovers cannot ever put the laws of thought in doubt; such doubt would not have the epistemological status of a purely empirical given but would certainly constitute mere interpretation.

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[27] Quoting Gemini again.

And conversely, the laws of thought never put purely empirical findings as such in doubt, but only ever question their interpretations; and by doing that, they make possible significant advances in scientific understanding. For example, the empirically demonstrated constancy of the velocity of light seemed counterintuitive, baffling scientists for years, until the theory of relativity succeeded in explaining it.

However, an illogical concept like superposition cannot be used to explain surprising quantum phenomena like the particle-wave duality. The laws of thought require us to find a more truly rational explanation, one emanating from within Physics itself, rather than from some pseudo-ontology or pseudo-epistemology.

3. Zen 'alive or dead' koans

While on the subject of life and death conundrums, allow me a small digression.

Schrödinger's thought-experiment regarding a cat in a box brings to mind[28] the question "alive or dead?" posed in a Zen koan (Case #55 in *The Blue Cliff Record*[29]). In that story, a Zen master, Tao Wu (China, 768-835), goes with a fellow monk, a student of his, to pay a condolence call at someone's home. The student knocks on the coffin and asks his teacher: "Alive or dead?" The latter replies evasively: "I won't say alive, and I won't say dead!" – and he refuses categorically to settle the matter or verbally elaborate, so angering the student that he strikes and leaves his teacher.

What was the student asking and why did the teacher refuse to answer more fully? Obviously, the student was not asking whether the person in the coffin was physically alive or dead. It happens on very rare occasions that someone appears to be dead yet is in fact still alive, as established by opening the coffin; but this is not what this koan is about. We may take the coffin here to contain a genuine corpse. No, the student's question was obviously more metaphysical than that. He was asking, I suggest, about the possibility of life *after* death.

Buddhism, as is well known, being originally an Indian belief system, believes in the round of birth and death (Samsara – the realm of suffering) and in the possibility of getting out of this round (Nirvana – enlightenment/liberation) through appropriate means (the Eightfold Path). The question arises: what happens to someone who is enlightened/liberated? Is such a one subject to extinction (as Hinayana Buddhism seems to suggest) or does such a one go on existing in a transcendental plane (as Mahayana Buddhism apparently advocates)? In the latter case, the individual effectively becomes undifferentiated from the totality – becomes one with the great All.

I think this was the question really posed by Tao Wu's student. Tao Wu, being a Zen master, considered the question too theoretical and rationalistic to merit a reply. In his view, this issue, that of *oblivion versus eternal life*, is too categorical – the truth of the matter is not expressible in terms of such clear-cut categories. When we are enlightened, we are both of this world and beyond it. This mystic state is not one that can be described in ordinary terms, i.e. in terms familiar to ordinary (non-enlightened) minds. The answer to the question is not philosophical but practical.

Schrödinger's hypothetical cat sacrifice also brings to mind the Zen koan in which master Nan Ch'üan (China, 747-834) makes a point by actually killing a cat (see Case #63 in *The Blue Cliff Record*[30]). Coming across two groups of monks in his monastery arguing over a cat, he grabbed

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[28] Perhaps not accidentally, knowing that Schrödinger displayed some interest in Eastern philosophies. See: https://en.wikipedia.org/wiki/Erwin_Schr%C3%B6dinger.

[29] Tr. Thomas & J.C. Cleary. Boston: Shambhala, 2005.

[30] This is also Case #9 in the *Book of Serenity* (Boston, Mass., Shambala, 2005).

the cat and threatened to kill it unless someone among them immediately gave him some word or gesture demonstrating his understanding of Zen. Not one of them could do that; so, Nan Ch'üan cut the cat in two (a gesture intended as a sample reply to his own question, of course).

This is of course a paradoxical story insofar as Buddhism forbids killing, and yet this Zen master indulged in it. However, Nan Ch'üan obviously went to such an extreme for a worthy purpose – to teach his students the absolute necessity of cutting one's way through to enlightenment, without which one is as good as dead. He taught them in this way that there is no time for vain arguments, that one must take swift action, relying on intuition, when the opportunity arises. It is either-or. The spiritual path is one of life or death, and it is pitilessly demanding.

Another 'alive or dead' koan is found in the commentary to case #22 in the *Book of Serenity*. A visitor asks the Buddha: "Is the sparrow in my hand alive or dead?" to which the latter, straddling the gate, replies: "Am I about to leave or enter?" There are doubtless many more.

5. THE BUDDHIST FIVE SKANDHAS DOCTRINE

1. Prologue

In this essay, I critically comment on the Buddhist 'five skandhas' doctrine. This doctrine is attributed to the Buddha himself and considered as a core belief of Buddhism[122]. However, in my humble opinion, in view of its evident intellectual limitations, this doctrine should not be given such elevated status. Buddhism and its founder have much more intelligent ideas to offer the world. That being the case, the present critique of the five skandhas doctrine should not be taken as a general critique of Buddhism or its founder.

Although often listed in the literature, the five skandhas are rarely clearly defined and expounded on. The Sanskrit word *skandha* (Pali: *khanda*) means 'aggregate' – and apparently refers to a building-block, of the mind or perhaps of the world. In Sanskrit, the five skandhas listed are: *rupa, vedana, samjna, samskara, vijnana* (in Pali: *rupa, vedana, sanna, sankhara, vinnana*). In the dozens of English texts that I have read over the years, I have seen these terms translated in various ways, and with rare exceptions barely explained. It is not made clear whether these terms are essentially phenomenological, psychological, metaphysical, ontological or epistemological. When interpretations are proposed, they differ considerably from one text to another. Nevertheless, this being an important doctrine in Buddhism, it is worth analyzing and evaluating.

2. My own phenomenological reading

Before studying the normative interpretations of these terms, permit me to present my own initial interpretations, even while admitting that they are largely inaccurate historically. That way, the reader will know where I am coming from, and will be better able to follow my thinking. When I first came across the five skandhas in Buddhist books, I took them to constitute a sort of *phenomenology*, i.e. a list of the different categories of being or appearance, one that suggests an ontological and epistemological theory insofar as the list distinguishes and interrelates the categories in certain ways. Consider the following reading:

- *Rupa*, usually translated as 'form', could be taken to refer to *the apparently external and material world*, which contains the phenomena of all shapes and sizes in motion that we seem to witness through our senses, the senses of sight, hearing, smell, taste and touch. This field of experience is quantitatively overwhelming, and takes up most of our existence, but is of course not the whole story, not the whole of our world.
- *Vedana*, usually translated as 'sensation or feeling', could be taken to refer more specifically to *the phenomena we experience as within our personal body*. In a sense, these are part of the external and material world, since our body is apparently part of it; but in another sense, they are closer to home (i.e. more internal) and less material (i.e. containing some phenomena

[122] According to the Wikipedia article on this topic, the American Buddhist monk Thanissaro, in *Handful of Leaves*, Vol. 2, 2nd ed. 2006, p. 309, alleges that the Buddha "never defined a 'person' in terms of the aggregates" and that this doctrine is not pan-Buddhist. To my mind, if he said that (I have not seen it with my own eyes), he may well be right.

notably different from those we experience further afield). In this context, our touch sensations of bodies beyond our own body are feelings, as are all the myriad physical sensations we experience within our bodies, such as sexual feelings (desire, satisfaction), digestive feelings (hunger, thirst, satiety, stomach aches, sensations when urinating or defecating, etc.), and feelings in other internal organs (headaches, heart beats, heartburn, muscular cramps, nerve pains, etc.). Here would also be included emotional reactions experienced within the body, such as love (a flutter or warmth in the heart region), fear (a flutter or warmth in the stomach region), etc. In short, all the pleasures and pains we may be subject to within our bodies, whether they stem from physical or mental causes. Also to be included under this heading would be our sensations of volition (acts of will), i.e. the sense we have that we move our body parts around and our whole body through space; and therefore also our sensations of velleity (pre-volitions, attitudes, intentions). Note however that, while volitions and intentions may have phenomenal aspects, they are largely non-phenomenal; i.e. they are intuited rather than perceived.

- *Samskara*, usually translated as 'mental formations', and sometimes as 'impulses to volition', could be taken to refer to *the inner phenomena we experience through our faculties of memory and imagination* (the latter being voluntary or involuntary manipulation of memory items to produce somewhat new images, sounds, etc.). This includes the images of visualizations, the sounds of verbal thoughts, dreams (during sleep) and hallucinations (the latter being stronger projections, apparently into the space where matter resides, of imaginations). These phenomena resemble those experienced as external and material, in that they also have shape, color, sound, etc., and yet are experienced as substantially different, of a different 'stuff', so much so that we give them a different name (they are characterized as mental, as opposed to material), even if we do regard the mental phenomena, or phantasms, as derivatives of the material ones (through memory of experiences). Such mental phenomena obviously can and do condition (variously incite or otherwise affect) subsequent more overt actions.
- *Samjna*, usually translated as 'perception', but often as 'apperception', 'conception' or 'cognition', could be taken to refer to our various *objects of cognition*, i.e. whatever we intuit (non-phenomenal concretes), whatever we perceive apparently through the physical senses or mentally through memory and imagination (phenomenal concretes), and all the abstractions and theories (based on the preceding items) that we construct through conceptual insight and reasoning (including negation, similarity, dissimilarity, etc.). Thus, *samjna* would include our non-phenomenal impressions (apperceptions), our phenomenal experiences (perceptions), and the concepts and thoughts (conceptions) emerging from the preceding through which we get, not merely to experience things, but also to order and interrelate them, and thus to understand them (or be confused by them) to various degrees. Thus, note well, *samjna* focuses on objects in the context of their being cognized, i.e. as contents of cognition (and not as objects apart from cognition).
- *Vijnana*, usually translated as 'consciousness', could be taken to refer to *the fact of cognition*, the cognizing, as against its object (content), and its subject (the self apparently doing the cognizing). Consciousness has to be listed separately because it is substantially different from any of the other categories in our enumeration. Note well that, to assure a complete enumeration, this term in my view would have to include both the *relation of cognition* and *the apparent self or soul* which is related by it to the object. This refers to the self which we all routinely intuit – even though Buddhists deny the latter's reality and consider it as illusory. This understanding is not entirely foreign to Buddhist practice, which tends to use the terms 'consciousness' and 'mind' in an ambiguous manner that sometimes really (though typically

without frankly admitting it) intends the self (i.e. the one who is conscious)[123]. Moreover, it should be stressed that the self not only cognizes, but also wills and values – i.e. that *volition and valuation* are among its powers as well as cognition, and that these three faculties are interdependent and do not exist without each other.

That is to say in our present perspective: while *rupa* refers to external and material objects, *vedana* to more specifically bodily objects, and *samskara* to mental objects, and while *samjna* identifies these same categories of objects as contents of cognitive acts, *vijnana* refers to the implied knowing (and willing and valuing) acts and to the spiritual entities (ourselves) apparently engaged in them. From this we see that the various phenomenological categories here enumerated overlap somewhat: *rupa* includes at least part of *vedana*, *samskara* is a side-effect of *rupa* and *vedana*, *samjna* includes the preceding experiences and adds their more complex conceptual and rational products, while *vijnana* focuses on the subject and the relationship of consciousness (and volition and valuation) between it and these various concrete and abstract objects.[124]

The above phenomenological account is merely, to repeat, my personal projection: it is the way I have in the past tended to interpret the five skandhas doctrine in view of the terminology used for it in English. This is the way I, given my own philosophical background, would build a theory of knowledge and being if I was forced to use these five given terms, even while aware that such theory contains some non-orthodox perspectives.

3. A more orthodox psychological reading

However, Buddhists and other commentators present these terms in a rather different light. I will use as my springboard an interesting account I have seen on this topic by Caroline Brazier in *Buddhist Psychology*[125]. Let us first look at this *psychological* approach, which I think is close to the original intent of the five skandhas doctrine, given that Buddhism is concerned with 'enlightening and liberating' people rather than with merely informing them to satisfy their curiosity. She writes:

> "The skandhas are the stages in a process whereby the self-prison is created and maintained. At each stage, perception is infiltrated by personal agendas that create distortion. Delusion predominates…. Each of us continually seeks affirmation that we are that person who we have assumed ourselves to be. Situations that disturb this process are avoided or reinterpreted, and the self appears to become more substantial" (pp. 92-93).

Her exposition of the stages is as follows (summarily put, paraphrasing her). The first stage is *rupa*, which is finding indications of self in everything we come in contact with; i.e. grasping onto all sorts of things because they reinforce our belief in having a self, and indeed one with a specific identity we are attached to. Next in the process comes *vedana*, which refers to our immediate value-judgments in relation to things that we come across (people, events, whatever); we may find them attractive, repulsive or confusing – but in any case, we have a visceral reaction to them that

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123 I have often in my past writings pointed out the vagueness of the terms mind and consciousness in the discourse of Buddhist philosophers, and explained there how it allows them to get away with much fallacious reasoning.

124 Note that in my listing, *samjna* is placed after *samskara*, which is not the usual order of listing. I could also have placed *samjna* after *vijnana*, since the latter category adds objects of cognition to be considered by the former. However, *vijnana* also has *samjna* as one of its objects, since the latter involves consciousness and a conscious subject; so the chosen order of presentation seems most logical.

125 London: Robinson, 2003.

affects our subsequent responses to them. Thirdly comes *samjna*, which consists in spinning further fantasies and thoughts around the things we have already encountered and initially reacted to; due to this, we are unconsciously carried off into certain habitual behavior-patterns. *Samskara* refers to these action and thought responses which we have, through repetitive past choices, conditioned ourselves into doing almost automatically. Finally comes *vijnana*, which refers more broadly to the mentality (perspectives and policies) we adopt to ensure our self is well-endowed and protected in all circumstances.

These five stages constitute a vicious circle, in that the later stages affect and reinforce the earlier ones. They ensure that we enter and remain stuck in the cycle of birth, suffering and death. The important thing to note is that the purpose of this psychological description is to make us aware of the ways we ordinarily operate, so that we may over time learn to control and change those ways, and become enlightened and liberated. As Brazier puts it: "Buddhism is not a matter of just going with the flow. It is about changing course" (p. 95). In this approach, the skandhas doctrine is a practical rather than theoretical one. It is a 'skillful means', rather than an academic exposition. It is concerned with the ways we commonly form and maintain of our 'self'.

Needless to say, this looks like a very penetrating and valuable teaching[126]. The question for us to ask at this point, however, is whether it is entirely correct. That is to say, assuming the above sketch is an accurate rendition of the Buddhist theory of human psychology, is this the way we ordinary (unenlightened, unliberated) human beings actually function? Brazier, being a committed Buddhist, takes this for granted rather uncritically. I would answer that though this theory seems partially correct, it is certainly not fully so. What we have here, at first sight, is a portrait of someone who is (very roughly put): very narrow-minded (*rupa*), instinctive (*vedana*), irrational (*samjna*), habitual (*samskara*), and selfish (*vijnana*). This may fully describe some people, and it may partly describe all of us, but it is certainly not a complete picture of the ordinary human psyche.

What is manifestly missing in this portrait are the higher faculties of human beings – their intelligence, their reason and their freewill. It could be argued that these higher faculties are present in the background, in the implication that people can (and occasionally do) become aware of their said lowly psychological behavior and make an effort to overcome it. But if so, this should be explicitly included in the description. That is to say, intelligence, reason and freewill should be presented as additional skandhas. But they are not so presented – it is not made clear that humans can function more wisely, and look at the facts of a situation objectively and intelligently, and decide through conscious reasoning how to best respond, and proceed with conscious volition to do so. In any case, these higher faculties are routinely used by most people, and not just used for the purpose of attaining enlightenment and liberation.

Why are these higher faculties, which are common enough, even if to varying degrees, not mentioned in the Buddhist account as integral factors of the human psyche? I would suggest that the main reason is that the self (or soul) has to be dogmatically kept out of it[127]. The central pillar of the Buddhist theory of enlightenment and liberation is that our belief that we have a self is the

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[126] One that could be, and no doubt is, used in meditation.

[127] It is interesting to note in passing how modern physicists, biologist, psychologists and philosophers tend to similarly studiously ignore the human soul and its functions of cognition, volition and valuation, in their respective accounts of the world, life and humanity. But whereas Buddhism's motive is to protect its dogma of no-self, the motive of modern 'scientists' is to protect their dogma of universal materialism and determinism. The intellectual sin involved in both cases is to deliberately make things look simpler than they are so as to make them fit more easily into one's pet theory.

deep cause of all our suffering, because a self is necessarily attached to its own existence, and the way out of this suffering is to realize that we do not really have a self and so do not need to attach to anything. In such a context, the human psyche must necessarily be described as essentially reactive and stupid, like a ship without a helmsman, at the mercy of every wind and current. Buddhism does regard humans as able to transcend these limitations, by following the ways and means taught by the Buddha in the Dharma; but it does not (in my opinion) fully clarify the psychological processes involved in self-improvement, no doubt due to the impossibility of verbally describing them with precision and generality.

Brazier does go on to describe how Buddhist psychology conceives transcending of the skandhas. She does so in terms of the 'five omnipresent factors' being transformed into 'five rare factors' "through spiritual practice." But of course, that account does not clearly say *who* is doing the spiritual practice, and *what faculties* are involved. It does not acknowledge that the individual person involved (the self) has to realize (through intelligence and reason) the need for and way to such transformation, and then proceed to bring it about (through complex volitional thoughts and actions). The self and its higher faculties are not given due recognition (because, as already explained, such recognition would go against the Buddhist dogma of no-self). This is not a fault found only in Brazier's account, but in all orthodox Buddhist accounts.

Understandably, Buddhism, particularly its Zen branch, rejects excessive intellectualism. Admittedly, intelligence, reason and freewill are all very well in principle, as tools for human betterment; but used in excess – or simply misused or abused – they can also and often do exacerbate human delusion and suffering. The intellect can be compulsively used to weave complex webs that distance its victim from reality rather than bring him or her closer to it. We can by such excess become more and more artificial and divorced from our true nature. Of that danger there is no doubt; it is observable. But intellectualism is surely not the whole story concerning our said higher faculties. Surely, they play a big role in improving our understanding and behavior, both in everyday life and in longer-term more intentionally spiritual pursuits.

Moreover, we have to ask whether the five skandhas doctrine, even taken at face value, is truly consistent. We are told that *rupa* consists in viewing things in relation to self rather than objectively, that *vedana* consists in immediate likes or dislikes, that *samjna* consists in making up associations, that *samskara* consists in conditioning, and that *vijnana* consists in selfish mentality – and it is all made to seem simple and mechanical. But is it? The Buddhist account itself tells us that these events are interrelated, i.e. stages in a process. Therefore, beneath each of them there must be complex mechanisms at play. *Rupa* must involve a certain sense of self and of its identity, to be able to select information of interest. *Vedana*, however instantaneous it may seem, cannot be immediate since it must be filtered through the subconscious scale of values of the person concerned. *Samjna* presupposes that there are older mental contents to which it associates new mental contents. *Samskara* refers to habits, which imply programming by repetition. And *vijnana* in turn implies storage of information and of valuations.

Furthermore, even if we grant that the five skandhas reflect common *tendencies* within the human psyche, it is introspectively evident that normally the self can in fact, at every one of these stages, intervene through free will based on rational considerations and conscious valuations. That is to say, faced with the ego-centricity of *rupa*, we can still choose to view things more objectively; faced with thoughtless valuations of *vedana*, we can still choose to evaluate things in a more balanced manner; faced with wild associations of *samjna*, we can still choose to put things in context more accurately; faced with our bad *samskara* habits, we can still choose to resist temptations or overpower resistances; faced with native *vijnana* selfishness, we can still choose to act with larger perspectives in mind. The human psyche is not a mechanical doll, driven by forces

beyond control – there is a responsible soul at its center, able (whether immediately or gradually) to impose its will on the rest of the psyche. Buddhists cannot consistently deny all this, since they do believe in and advocate self-improvement, as the Noble Eightfold Path makes clear.

4. A plainly mechanistic thesis

This brings us to the crux of the matter, the *determinism* tacitly involved in the five skandhas doctrine. The skandhas are imagined by Buddhists as *dharmas*, i.e. as "a series of consecutive impersonal momentary events," as Vasubandhu put it[128]. No one is making them happen, they just happen each one caused by the ones preceding it and causing the ones succeeding it. They do not happen to someone, either, even if they seem to. They are "linked to suffering," but no one suffers them. Clearly, there is logically no room, in this conception of psychological processes, for a person actually cognizing, understanding, evaluating, reasoning, deciding, choosing and engaging in action. Not only is the person removed, but the acts of cognition, valuation and volition are also removed. They are reduced to mere momentary electrical disturbances in the mental cloud[129], as it were. They are no longer special *relations* between a subject or author and other things in the mind or body. This doctrine is, really, crass reification of things that are definitely not entities.

The five skandhas is clearly a mechanistic thesis, even if it is mitigated in a subterranean manner by the Buddhist faith in the possibility of enlightenment and liberation. In this view, logically, such spiritual attainment is itself merely the product of a chain of impersonal mental events, with no one initiating them and no one profiting from them[130]. This state of affairs is claimed to be known by means 'deep meditation', although it is not made clear who is doing the meditating, nor by means of what faculties or for what useful purpose. Clearly, objectively, however deep such meditation it could not possibly guarantee the verity of the alleged insights, but must needs submit them to logical evaluation in accord with the laws of thought. Scientific thought cannot accept any deep insights, or any revelations based on them, at face value; it demands rational assessment of all claims.

In truth, granting that there is some truth to the psychological processes described by the skandhas doctrine, they must be viewed more restrictively as processes of *ego*-building, rather than so radically as processes of *self*-invention. They refer, not to ways that 'we' (a never explained yet still repeatedly used grammatical subject) imagine the self or soul to exist, but to ways that we (the truly existing soul, our real self) construct and maintain a particular self-*image* that we think

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[128] Quoted or paraphrased (not clear which) in *Buddhist Scriptures*, edited and trans. by Edward Conze (Harmondsworth, Mx.: Penguin, 1959). Vasubandhu was a Buddhist monk and major philosopher, fl. 4th to 5th cent. CE in Ghandara (a kingdom located astride modern-day Pakistan and Afghanistan). His philosophical posture is today normative, at least among the Mahayana, but it was opposed by a Hinayana school called the Personalists, which lasted for many centuries as of 300 BCE and involved a good many monks (e.g. an estimated 30% of India's 200,000 monks in the 7th cent.). See pp. 190-197.

[129] Modern 'scientists' (I put the word in inverted commas deliberately, to signify criticism) would say much the same, but would place the electrical disturbances on the more physical plane of the brain and nervous system. The idea that the mind is a sort of very sophisticated computer is untenable, for exactly the same reasons that the idea of skandhas is untenable.

[130] One Victoria Lavorerio, in a paper called "The self in Buddhism," has written: "If following Descartes we say that where there is a thought there is a thinker, the Buddhist would respond 'where there is a thought, there is a thought'." While rather witty, this statement is of course inane, since its author does not grasp the logical absurdities of the Buddhist no-soul thesis (and that, even though she quotes a couple of arguments of mine regarding them), but merely seeks to position herself fashionably. See her essay here: http://www.academia.edu/1489808/The_self_in_Buddhism.

flattering or securing. What is evident in honest, non-dogmatic meditation is that, while such processes can surely *influence* our mental and physical behavior, i.e. make things easier or more difficult for us, they do not normally *determine* it. An influence, however strong, can always (with the appropriate attitude and effort) be overcome. At almost every moment of our existence, we remain free to choose to resist these mental forces or to give in to them. If we but make the effort to be aware, to judge and to intervene as well as we can, we remain or become effective masters of our fate.

It is only because we indeed exist as individuals, and have these powers of cognition, valuation and volition, that we can observe, identify, understand and overcome the impersonal forces described by the five skandhas doctrine. Therefore, in fact, the said doctrine, far from constituting an exhaustive listing of the basic building blocks of the human psyche, at best depicts just some surface aspects of much more complicated events and structures. Not only is the list incomplete in that it lacks overt reference to the human self and its higher faculties, but additionally its presentation of the five lower faculties (even assuming that these five faculties indeed exist) is rather superficial. For all the above reasons, and yet others, I view the five skandhas account of human psychology as deficient.

As regards enlightenment, liberation and wisdom, these are impossible without a soul and its faculties of cognition, volition and valuation. Enlightenment means perfect cognition by the soul, i.e. a consciousness as high, wide and deep and accurate as can be for the person concerned. Liberation means perfect volition by the soul, i.e. a will as free of obstructions and as powerful as can be for the person concerned. Wisdom means perfect valuation by the soul, meaning full understanding of good and bad coupled with behavior that is accordingly fully virtuous and non-vicious. Enlightenment, liberation and wisdom are concepts only applicable to sentient beings (notably to humans and other animals, and perhaps in some sense to plants); they are irrelevant to non-spiritual entities (i.e. material and/or mental entities devoid of soul, such as skandhas, computers or fantasy creatures).

5. The metaphysical aspects

The *Encyclopaedia Britannica* (EB) defines the skandhas as "the five elements that sum up the whole of an individual's mental and physical existence." It lists them as "(1) matter, or body, the manifest form of the four elements—earth, air, fire, and water; (2) sensations, or feelings; (3) perceptions of sense objects; (4) mental formations; and (5) awareness, or consciousness, of the other three mental aggregates [i.e. items 2-4]."

In most accounts I have seen, this theory is presented as descriptive of what constitutes a person. Some accounts I have seen, however, apply it more broadly, viewing the five skandhas as the constituents of the phenomenal world. In any case, this theory clearly contains an ontological thesis, insofar as it acknowledges two kinds of phenomena, the material (the first skandha) and the mental (the other four skandhas)[131]. Moreover, note in passing, since the above definition mentions the 'four elements', it includes a physical theory, one admittedly very vague and by today's standards rather useless[132]. Secondly, the skandhas doctrine has some epistemological

[131] I assume that the Yogacara, Mind-Only, school would advocate that matter is a sort of mental phenomenon. In that case, they would presumably advocate that the skandhas theory concerns not only personality but the whole phenomenal world.

[132] It is worth noting, of course, that the fact that this simplistic, though ancient and widespread, theory of physics (with reference to the 'elements' of earth, air, fire and water, or similar concepts) is advocated by Buddhism is proof

implications, in that it identifies sensations or feelings, perception of sense objects, and so on – implying our ability to know *of* such things, presumably by introspection.

Furthermore, the said source (EB) explains that "The self (or soul) cannot be identified with any one of the parts, nor is it the total of the parts. All individuals are subject to constant change, as the elements of consciousness are never the same, and man may be compared to a river, which retains an identity, though the drops of water that make it up are different from one moment to the next." This statement is the metaphysical element in the skandhas doctrine, since it involves important claims regarding the ultimate nature of individuals (i.e. persons, people).

This explanation reminds us that *the philosophical motive* of the skandhas doctrine is to buttress the Buddhist claim that we have no self or soul (*anatta*). According to this doctrine, we are only clusters of the listed five material and mental phenomena, which are in constant flux, unfolding as a succession of events, each new event being caused by those before it and causing those after it. It is stressed that none of the skandhas is the self, and neither is their sum the self. The self is not something apart from them, either. What we call the self is a mere illusion, due to our conflating these ongoing, causally-linked events and giving them a name.

The no-self idea is usually expressed by saying that the human being is 'empty of self'. This is presented as one aspect of a wider metaphysical doctrine of ultimate 'emptiness' (*shunyata*), applicable to all things in the phenomenal world. Initially, I suggest, Buddhist thought sought *to replace* the self we all naturally assume we have with the five skandhas. Since the doctrine of ultimate emptiness needed to be applied to the apparent self, an explanation of apparent selfhood was provided through the doctrine of the five skandhas. The self does not really exist; it is only made to appear to exist due to the play and interplay of the five skandhas. However, consistency required that the five skandhas be empty *too*. This was later acknowledged, for instance, in *The Heart of the Prajnaparamita Sutra*, which stated:

> "Form is emptiness, emptiness is form… The same is true with feelings, perceptions, mental formations, and consciousness."[133]

Here, the five skandhas, thanks to which the self seems to us to exist even though it is in fact empty, are *affirmed to be empty too*, note well. All phenomenal existents are empty, and this includes the skandhas too. The question might then well be asked (by me, at least): *if the skandhas are equally empty, what ideological need have we of them? Why can we not just as well admit the existence of the self or soul, and call* it *'empty' too, directly?* This is of course a significant flaw in the doctrine of the skandhas – it shows the idea of them to be *logically redundant*. If the motive behind it was to explain the emptiness of self, it was not only unnecessary but useless, since the emptiness of skandhas also had to be admitted! Logically, far from simplifying things, the skandhas hypothesis complicated them.

In other words, the Heart Sutra could equally well have stated: "self is emptiness, and emptiness is self;" or even: "soul is emptiness, and emptiness is soul." And indeed, it could be argued that soul, being more insubstantial (less phenomenal) than the skandhas, is closer to emptiness than the skandhas are.

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that this doctrine is not the product of any 'omniscience'. If the Buddha indeed formulated it or accepted it, he cannot be said to have been 'omniscient' since this is not an accurate account of the physical world. This being the case, it is permitted to also doubt he was 'omniscient' in his understanding of the mental or spiritual world. Of course, it could be argued that he appealed to the four elements theory only because it was commonly accepted in his day, in the way of a 'skillful means', without intending to endorse it.

[133] Given in full in Thich Nhat Hanh's *The Heart of Understanding*.

6. Soul and emptiness

There are obviously two concepts here to clarify – (a) soul and (b) emptiness. Additionally, we must (c) examine their interrelation.

(a) The term ***soul*** refers to an entity of spiritual substance, i.e. of a substance other than the substances that material or mental things seem to have. Soul has no phenomenal characteristics – no shape or color, no sound, no flavor, no odor, no hardness or softness, no heat or cold, etc. That is to say, it cannot be cognized by external perception (using the five sense organs) or by internal perception (using the proverbial mind's eye, and its analogues, the mind's ear, etc.). This does not mean it cannot be cognized by some other, appropriate means – which we can refer to as *apperception* or *intuition*.

Just because the soul is not phenomenal, *it does not follow* that it does not exist. Buddhists apparently cannot understand this line of reasoning. In the West, David Hume (Scotland, 1711-76) evidently had the same problem. Looking into himself, he could only perceive images and thoughts, but no soul. Obviously, if you look for something in the wrong place or in the wrong way, you won't find it. If you look for something non-phenomenal in a field of phenomena, you won't find it. If you look for color with your ears or for sound with your nose, you won't find them. To look for the soul, you just need to be intuitively aware. All of us are constantly self-aware, even though we cannot precisely pinpoint where that self is. There is no need for advanced meditation methods to be aware of one's soul – it is a common, routine occurrence.

Note well that I am not affirming like René Descartes (France, 1596-1650) that soul is known through some sort of *inference*, namely the famous *cogito ergo sum*, i.e. "I think therefore I am." We obviously can and do know *about* the soul through such rational means, i.e. through abstract theorizing – but our primary and main source of knowledge *of* the soul is through direct personal *experience*, which may be referred to as apperception or intuition. So, my approach is not exclusively rationalist, but largely empiricist. In this, note well, my doctrine of the soul differs radically from the Cartesian – as well as from the Buddhist.

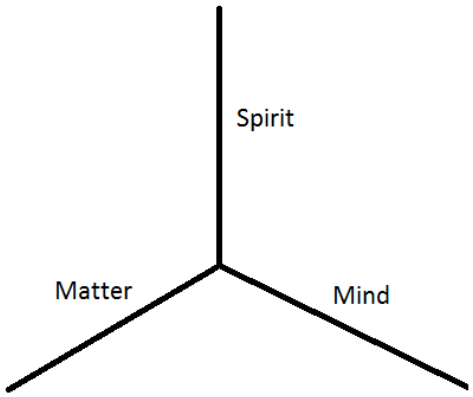
According to Buddhist dogma, one cannot perceive the soul in meditation; if one observes attentively one only finds various mental phenomena (the five skandhas, to be exact). But I reply that the soul is manifestly a *non-phenomenal* object and should not be conflated with such overt phenomena. We all have a more or less distinct '*sense of self*' most if not all of the time, without need of meditation.

This is obvious from the very fact that everyone understands the word 'self'. Buddhism admits this sense of self, but absurdly – quite dogmatically – takes it to be 'illusory'. Having at the outset dismissed this significant 'sense' (intuition) as irrelevant, it is not surprising that it cannot find the soul (i.e. the human self) in the midst of the phenomena of mind (the five skandhas)! Note this well – **Buddhism has no credible argument to back its no-soul thesis**. It *begs the question*, calling the sense of self illusory because it believes there is no self, and claiming that it knows by introspection that there is no self while rejecting offhand the ordinary experience of self we all have. As a result of this manifest error of reasoning, if not outright doctrinaire dishonesty, Buddhism becomes embroiled in many logical absurdities.

Just as one would not look for visual phenomena with one's hearing faculty or for auditory phenomena with one's visual faculty, so it is absurd to look for spiritual things (the soul, and its many acts of consciousness, will and valuation) with one's senses or by observing mental phenomena. Each kind of appearance has its appropriate organ(s) of knowledge. For spiritual things, only intuition (or apperception) is appropriate.

To understand how the soul can exist apparently in midst of the body and mind (i.e. of bodily and mental phenomena) and yet be invisible, inaudible, etc. (i.e. non-phenomenal), just imagine a three-dimensional space (see illustration below). Say that two dimensions represent matter and mind and the third applies to spirit. Obviously, the phenomena of mind will not be found in the matter dimension, or vice versa. Similarly, the soul cannot be found in the dimensions of matter and/or mind, irrespective of how much you look for it there. Why? Simply because its place is elsewhere – in the spiritual dimension, which is perpendicular to the other two. Thus, it is quite legitimate to claim awareness of the soul even while admitting that it has no phenomenal (matter-mind) characteristics.

Figure 4. Matter, mind and spirit presented as three dimensions of existence



Note well that the above illustration of the spiritual as located in another dimension is intended as merely figurative, and not to be taken literally, because the concept of dimensions is itself a material-mental concept based on the perception and projection of space. Even the idea of time as a fourth dimension relative to the three dimensions of space is mere analogy; all the more so, the idea of spirit as a further dimension (or maybe a set of dimensions) is somewhat artificial. The simple truth is that **spirit cannot really or fully be expressed in material or mental terms, being so very different, truly sui generis**. We might likewise object to the image of mind as a distinct dimension (or set of them) in comparison to matter, but mind does have some phenomenal characteristics in common with matter whereas spirit cannot be said to be at all phenomenal. So, to repeat, the above analysis of these three domains with reference to dimensions is merely a convenient metaphor.

Furthermore, it would be epistemologically quite legitimate to claim the existence of soul on purely abstract, conceptual grounds. This is justifiable with reference to the principles of adduction. One can hypothesize an entity, if such assumption serves to explain various observable concrete phenomena. In the case of soul, the 'phenomena' involved are our commonplace experiences of cognition, volition and valuation. These experiences are largely intuitive too, but they make their manifest mark in the fields of mind and body. We experience cognition whenever we perceive or conceive anything. We experience volition whenever we think or do anything. We experience valuation whenever we like or dislike anything. Soul explains all these experiences by means of a central entity. This is akin to, say, in astronomy, discovering a planet invisible to our telescopes by observing the displacement of other celestial bodies around it. This is inductive logic.

But in truth, soul is not a mere abstraction; it is a concrete (though spiritual) thing that can be cognized directly using our inner faculty of intuition, to repeat. One error Buddhists make is to confuse entity and essence. The claim of a soul is not a claim of essence, but of entity. The soul is not the essence of the body, or even of the body-mind complex – it is a distinct entity that resides, somehow, in the midst of these phenomena, and affects them and is influenced (and perhaps also affected) by them, but does not have the same nature as them. It is a substance, but a very different and insubstantial substance, as already pointed out. Indeed, to call soul an entity or substance is really just *metaphor* – analogical thinking. In truth, soul is so different from the other constituents of the world that it can only be described by means of analogy – it cannot really be reduced to anything else we know of.

We can see the said philosophical error made, for instance, in the *Milinda-panha*, a non-canonical but orthodox Theravada (Pali) text[134]. Here, Milinda questions Nagasena, after the latter claims not to really exist. He asks him very pertinent questions such as who, then, is it that eats, engages in spiritual practices, keeps morality, gains merit, etc. The latter replies by giving the example of a chariot, pointing out that no part of the chariot can be considered as the chariot, nor even the combination of all the parts. Milinda, whose questions were excellent, is very easily taken in by Nagasena's answers. But (to my mind) we need not be.

For a start, a chariot cannot be considered as analogous to a person. We do not look upon a chariot as like a person, for the simple reason that it does not have capacities of cognition, volition and valuation. To look for the analogue of a soul in a chariot is to commit the red herring fallacy. Moreover, while it is true that a chariot contains no 'core entity' which can be so called, and it is true that no one part or combination of its parts can be used to define it, it still has an 'essence'. A chariot, as a man-made object, is defined by means of its purpose or utility – as a horse-drawn carriage, used for transport and travel, especially in war or hunting or racing. Its essence is an *abstraction*, not a concrete entity. Certainly, all the required parts must be there to form a functioning chariot, but these parts can be changed at will for other parts *like them* (though not for other parts unlike them: e.g. one cannot replace a wheel with a shoe). The one constant in it is the said abstract purpose or utility.[135]

The same reasoning does not apply to persons, obviously. So, Nagasena's argument was in fact beside the point. As already mentioned, a soul is not an essence, but a core (spiritual) entity. It therefore cannot be viewed as one of the five skandhas, nor as the sum of those skandhas, as the Buddhists rightly insist. It can, however, contrary to Buddhist dogma, be viewed as one of the parts of the complete person, namely the spiritual part; but more precisely, it should be viewed as the core entity, i.e. as the specific part that exclusively gives the whole a personality, or selfhood. This is especially true if we start wondering where our soul came from when we were born, whether it continues to exist after we die, where it goes if it does endure, whether it is perishable, and so forth.

This brings us to the question as to *whether the soul is eternal or temporary*, or (in more Western terminology) whether the soul is immortal or mortal. Eternal would mean that it has existed since the beginning of time and will exist till the end of time. Temporary would mean any shorter period

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[134] See Conze, pp. 147-151. The dialogue is given in full here. Milinda (Gk. Menander) was the "Greek ruler of a large Indo-Greek empire [namely Bactria] in the late 2nd century BC." Nagasena was a senior Buddhist monk. The text was, according to EB, "composed in northern India in perhaps the 1st or 2nd century AD (and possibly originally in Sanskrit) by an unknown author."

[135] Similarly, a river, though not man-made, can be defined by means of abstractions. This is said with reference to the analogy proposed by EB earlier on.

of time, though it may be very long indeed. Temporary could mean as long as the current body lives, or it could mean for many lifetimes – and that with or without physical bodies.

It seems that Indian philosophy had no place for temporary souls, only eternal souls or no-souls – with regard to soul, it was all or nothing[136]. However, this disjunction is philosophically untenable. It is conceivable that the soul is an epiphenomenon of the living human (and more broadly animal, or at least higher animal) body, which comes into existence with it and ceases to exist when it does. Or it may be that this temporary soul lasts longer, transmigrating from body to body or maybe existing without a body. We do not know (at least, I don't); but what is sure is that these are conceptual possibilities that cannot be ignored. Certainly, non-Buddhist humanity has found them conceivable, since many religions are based on such alternative beliefs.

As regards the eternal soul, the question is whether such a soul can or cannot be liberated from the (alleged) cycle of birth and death. Does eternity of the soul logically imply its eternal imprisonment in suffering? I do not see why. It is conceivable that the eternal soul was once happy, then somehow fell into suffering, but can still pull itself out of its predicament through spiritual practices. It may well be, even, that its liberation depends on a spiritual program like the Four Noble Truths of Buddhism; i.e. on realizing that it is in a vicious circle of suffering, that this suffering is caused by attachment and can be cured by non-attachment, and that such non-attachment can be cultivated through the Noble Eightfold Path. So, there is nothing inherently contradictory to Buddhism in the assertion of an eternal soul. I am not advocating this, only pointing out that it can consistently be advocated contrary to established dogma.

What is sure, in any case, is that the no-soul idea is logically untenable. Buddhists have never squarely faced the logical problems it raises and honestly tried to solve them. They are always inhibited by the fear of being regarded by their peers as heretical holders of 'false views'; so, they keep repeating the no-soul catechism and keep trying to justify it (using absurd means such as the tetralemma, which puts forward the nutty idea that something can both be and not-be, or that something can both not-be and not-not-be). The use of the five skandhas doctrine as an explanation of the (alleged) illusion of selfhood simply does not convince any honest observer, as above shown. Buddhist preachers say that individuals should not take Buddhism on faith, but try and think the issues through for themselves, and they will see the logic of it. But when someone does so, and comes to a different conclusion and rejects one of their clichés, they are nonplussed if not hostile.

The truth is that it is impossible to formulate a credible theory of the human psyche without admitting the existence of a soul at its center. *Someone* has to be suffering and wanting to escape from suffering. A machine-like entity cannot suffer and cannot engage in spiritual practices to overcome suffering. Spiritual practice means, and can only mean, practice by a spiritual entity, i.e. a soul with powers of cognition, volition and valuation. These powers cannot be equated to electrical signals in the brain, or to events in the skandhas. They are *sui generis*, very miraculous and mysterious things, not reducible to mechanical processes. Cognition without consciousness *by a subject* (a cognizing entity) is a contradiction in terms; volition without a freely willing *agent* (an actor or doer) is a contradiction in terms; valuation without *someone at risk* (who stands to gain or lose something) is a contradiction in terms. This is not mere grammar; it is logic.

An important question as regards the soul is whether it is the same throughout its existence, or alternatively it spiritually changes (for better or worse) over time. This issue is important as it

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[136] For instance, in the Hindu Bhagavad-Gita, the *atman* (individual soul) is said by Krishna to be: "unborn, undying; never ceasing, never beginning; deathless, birthless; unchanging for ever." (*Bhagavad-Gita, The Song of God*. Markham, Ont., Canada: Mentor, 1972.)

could affect responsibility, and reward or punishment (karma, in Buddhism). Granting that the soul is responsible for its acts of will *at the time* of such actions, is it just for the soul to receive the consequences of such actions *at a later time*? Should I pay in my old age for the vices of my youth that I no longer indulge in, or get the belated rewards for my youthful virtues even if I no longer have them? If the soul is unchanging through time, the answer would obviously be yes. But if the soul does evolve or devolve over time, the answer might at first sight seem negative. Can it still be said that *the same person* involved in such case?

Different solutions to this problem might be proposed. First, we should emphasize that much of the karmic load (for good or bad) of our lives is placed in our mental and bodily dimensions, our mind and body. The question here posed is whether some of the karmic load is placed in our spiritual dimension, our soul. If we say that the soul is constant, we must place all apparent spiritual changes related to it in its mental and physical environment. Thus, the same soul as a baby has more limited powers of walking, talking, etc.; as an adult, his intellectual and bodily powers reach their peak; in old age, they gradually deteriorate.

Moreover, if one thinks and acts in a saintly manner, one is likely to have a pleasant inner life and probably outer life too; whereas, if one thinks and acts in a depraved manner, one is likely to have an unpleasant inner life and probably outer life too. But what of in some supposed afterlife, when the soul is without body or mind? The choices a person makes at any given time reflect its total circumstances at that time. If I am the same across time, then in principle if I were put back in the same circumstances I would react the same way to them. This would seem contrary to the principle of free will, which is that whatever the surrounding influences the soul remains free to choose – and is therefore ultimately unpredictable.

A better position to adopt may be that proposed by Buddhism in the context of the five skandhas doctrine. I am referring to 'the Burden Sutra' expounded by Vasubandhu[137]:

> "The processes which have taken place in the past cause suffering in those which succeed them. The preceding Skandhas are therefore called the 'burden', the subsequent ones its 'bearer' [of the burden]."

We could adapt the same idea to the soul (instead of the skandhas), and say that since its present existence is caused by its past existence, it is in a real sense at all times a continuation of its past, carrying on not only its existence but also its good and bad karma. In this way, even if the soul (the 'bearer') has undergone inner changes, it remains responsible for its past deeds (the 'burden'). The past becomes cumulatively imbedded in the present and future. In that case, we must ask the question: what changes are possible within a soul? Is it not a unitary thing? Can it conceivably have parts? This would seem to take us back full circle to a psychological description, such as the one proposed in the five skandhas theory!

However, I would suggest that such questions are not appropriate in the spiritual realm, which is not quite comparable to the material and mental realms. The soul, being non-phenomenal, cannot be thought of as having size or shape or even exact location, or as increasing or decreasing in content – these concepts and others like them being drawn from the phenomenal realms. We should rather accept that we cannot describe the soul, any more than we can truly fathom its ultimate workings. Just as cognition, volition and valuation are *sui generis* world-events, so is the soul too special to fit into any simplistic analogies.

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137 In Conze, again (p. 195).

It should be added that the view of the soul here proposed is not very far, in many respects, from the Buddhist notion of Buddha-nature. Consider the following brilliant statements by Son Master Chinul[138]:

> "The material body is temporal, having birth and death. The real mind is like space, unending and unchanging….
>
> The material body is a compound of four elements, earth water, fire, and air. Their substance is insentient; how can they perceive or cognize? That which can perceive and cognize has to be your Buddha-nature….
>
> In the eyes, it is called seeing. In the ears, it is called hearing…. In the hands, it grabs and holds. In the feet, it walks and runs…. Perceptives [sic] know this is the Buddha-nature, the essence of enlightenment. Those who do not know call it the soul….
>
> Since it has no form, could it have size? Since it has no size, could it have bounds? Because it has no bounds, it has no inside or outside. Having no inside or outside, it has no far or near. With no far or near, there is no there or here. Since there is no there or here, there is no going or coming. Because there is no going or coming, there is no birth or death. Having no birth or death, it has no past or present…."[139]

Clearly, the "real mind" which is "like space," the "Buddha-nature" which alone can "perceive and cognize," that which sees and hears and grabs and walks, i.e. that which is the Subject of acts of consciousness and the Author of volitional acts, corresponds to what we commonly call the soul, even if the said writer refuses to "call it the soul." It is noteworthy that, despite the Buddhist dogma that cognitive and volitional acts do not imply a self, this writer seems to advocate that they do (even while virtuously denying selfhood). Is then the difference between these concepts merely verbal? I would say not. The idea of the soul suggests individuation (in some realistic sense), whereas that of Buddha-nature has a more universal connotation (with apparent individuality regarded as wholly illusory).

(b) Let us now examine the Buddhist concept of ***Emptiness***. Note at the outset that I make no claim to higher consciousness, and have no interest in engaging in fanciful metaphysical speculations using big words. I write as a logical philosopher, an honest ordinary man intent on finding the truth without frills. By 'emptiness', most Buddhists do not mean literal vacuity, or a void (non-existence). It may be that some Hinayana thinkers understood the term that way, but I gather Mahayana thinkers viewed it more positively (or ambiguously) as referring to 'neither existence nor non-existence'. The latter expression is meant to reject both excessive belief in the existence of the phenomenal world (Eternalism) and excessive belief in the non-existence of the phenomenal world (Nihilism). It is intended as a golden mean – a 'middle way'.

However, as regards this concept of 'middle way', it is inaccurate (quite muddle-headed, in fact) to say, as Buddhists do, that this emptiness is 'non-dualistic', suggesting that it literally *includes* all opposites, i.e. allows of effective contradiction. All that can be said is that emptiness comprises everything that is positively *actual*, whether in the past, present or future. Just as actuals are never contradictory, i.e. just as contradiction never occurs in reality at any time or place (not even, upon reflection, in the mind), so emptiness does not admit of contradictions. Contradiction is certainly illusory, and any claim to it is necessarily false. 'Non-dualistic' must be taken to mean (more accurately) unitary, undifferentiated. It refers to the actual positive, not to any imagined negative.

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138 Korea, 1158-1210.

139 *Classics of Buddhism and Zen*, vol. 1. Pp. 417-419, 424.

Often, it is implied that Emptiness corresponds to the Absolute, the Infinite, Ultimate Reality, the Original or Primordial ground of Being, or of Mind, the One, Nothingness, the Noumenon, and so forth. This concept, and some of the terminology used for it, are of course not entirely foreign to other philosophies and religions.

From its Pre-Socratic beginnings, Greek philosophy has sought for the underlying unity of the many, what lies beneath the variegated phenomenal world, the common ground of all things we commonly experience, from whence things presumably come and to which they presumably go (as it were). These ideas culminated in Neo-Platonism in late Antiquity, and returned to Western philosophy in the late Middle Ages and in the Renaissance in various contexts. Comparable notions are also found in Judaism, Christianity and Islam, and of course in other Indian religions, notably Hinduism – especially in their respective more mystical undercurrents. Greek philosophy has of course influenced these various religions, and they have also demonstrably influenced each other, in this respect. There has also no doubt been influences from and to Buddhism, as the above mentioned *Milinda-panha* attests, being a dialogue between a Greek king and a Buddhist monk.

With regard to our bodies, or to matter in general, it is often argued that though they appear varyingly 'substantial' (including gases and liquids with solids), if we go deeper into their composition, as we nowadays can, we shall find mostly empty space, with only very rare particles of mass, which are just pockets of energy anyway, connected by insubstantial fields of force. But the obvious reply to that is that this would still not be total void; i.e. even if matter is not as full and substantial as it at first appears, that does not mean that there is nothing in it *at all*.

Nevertheless, I do not think that the Buddhist concept of emptiness applied to matter refers to this empty space with very rare substantiality. Rather, I think that it refers to the assumed universal and unitary common ground of all things, which is conceivable as *pure existence, prior to any differentiation* into distinct entities, characteristics or events. This root existent cannot be described or localized, because to do so would be to ascribe to it some specific character or location to the exclusion of another.

With regard to mental phenomena, by which I mean the stuff of memories and derived phantasms, which apparently occur our heads, they seem much less substantial than material ones, but nevertheless they are *phenomenal* insofar as we perceive them as having colors, shapes, sounds, and perhaps also (though I can't say I am sure of it) also odors, flavors and feelings of touch. We must also in this context pay attention to concrete feelings and emotions which appear to occur in our bodies or heads, which we would collectively classify as touch-sensations.

It is worth noting the importance touch-sensations play in our view of matter: the 'solidity' we ascribe to matter is defined in terms of the resistance we *experience* when we push it, pull it or squeeze it, as well as with regard to the evident *relative* duration of the object at hand. No matter how much empty interspace matter may in fact contain, the experience of solidity (to various degrees) remains, and strongly determines our sense of 'materiality'. Mental phenomena, in this context, appear far less 'solid' than material ones, being able to dissolve more quickly and to be relatively more malleable (and in some respects less so). The Buddhist adjective 'empty' should not be taken to mean 'devoid of solidity', for solidity (as just explained) is a *phenomenological* given and therefore cannot be denied.

Additionally, in my view, we must take into consideration, as mental 'phenomena' in an expanded sense (more precisely, 'appearances'), objects of intuition like self, consciousness, volition and valuation, even though they are quite *non-phenomenal*, i.e. devoid of color, shape, sound, etc. All these existents can also, and all the more so, be regarded as empty, if we understand the concept correctly as above suggested.

According to Buddhism, this root and foundation of all existence, which is somewhat immanent as well as transcendental, can be known through meditation, or at any rate when such meditation attains its goal of enlightenment. In this, Buddhism differs from Kantian philosophy, which views the noumenal realm as in principle unattainable by the human cognitive apparatus (even though Kant[140] evidently claimed, merely by formulating his theory, quite paradoxically, to at least know *of* it).

Nevertheless, the two agree on many points, such as the characterization of the phenomenal realm as illusory while the noumenal is real. What is clear is that emptiness refers to a universal and unitary substratum, which is eminently calm and quiet, and yet somehow houses and even produces all the multiplicity and motion we perceive on our superficial plane. The world of phenomena rides on the noumenal like ocean waves ride on the ocean. Water and waves are essentially one and the same, yet they are distinguishable by abstraction; likewise, with regard to the noumenal and phenomenal.

Mention should be made here of the Buddhist theory of *the codependence or interdependence of all dharmas*. According to this theory, everything is caused by everything else; nothing is capable of standing alone. That precisely is why everything (i.e. all things in the world of phenomena) may be said to be empty – because it has no 'own being' (*svabhaha*). This means that not only we humans, and all sentient beings, are empty of self, but even plants and inanimate entities are empty. This may sound conceivable at first blush, but the notion of interdependence does not stand serious logical scrutiny. The claim that everything is a cause of everything is a claim that there is at least a partial, contingent causative relation between literally *any two* things. But such causative relation must needs be somewhat exclusive to exist at all[141]. So, the idea put forward by Buddhist philosophers is in fact fallacious, a 'stolen concept'.

It should also be said that the term 'emptiness', insofar as it is intended negatively, i.e. as indicative of *privation of* existence, is necessarily *conceptual*. We can say that being comes from and returns to non-being, but it must be acknowledged that this is something that cannot be known by direct experience, whether ordinary or meditative, but only by conceptual insight. The simple reason for this is that negation cannot be an object of perception or intuition, but can only be known by *inductive inference* from an unsuccessful search for something positive[142]. Only positives can be experienced. All negative terms are, logically, necessarily conceptual; indeed, negation is one of the foundations of conceptual thought. Thus, any claim that one has *purely experienced*, in the most profound levels of contemplation, the Nothingness at the root of Existence, is not credible: reasoning (even if wordless) was surely involved.

For Buddhism, the original ground is something impersonal, though some might view it as a sort of pantheism. For the above mentioned major religions, the original ground is identified with God. In my opinion, such identification is more credible, because I do not see how the conscious, willful, and valuing individual soul could emerge from something greater that is not itself essentially conscious, willful, and valuing. These faculties being higher than impersonal nature, their ultimate source must potentially have them too. In Jewish kabbalah, for instance, the human soul is viewed as a spark of the Divine Soul (a chip off the old block as it were). We are in God's image and likeness in that, like Him, we have soul, cognition, volition and valuation, although to an infinitesimal degree in comparison to His omniscience, omnipotence and all-goodness. But in any

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140 Immanuel Kant (Germany, 1724-1804).

141 See my *The Logic of Causation*, Chapter 16.3, for a full refutation.

142 See my *Ruminations*, Chapter 9.

case, it is clear that there is some considerable agreement between the various philosophies and religions.

(c) Let us now consider soul *in the context of* emptiness. Is the concept of self or soul logically compatible, or (as the Buddhists claim) incompatible, with that of emptiness? Can a soul find liberation from its limitations and suffering, or is it necessarily stuck in eternal bondage to birth and death, deluded by endless grasping and clinging to things of naught? Is liberation only possible by giving up our belief the soul? If the answer to these questions is in accord with Buddhism, the five skandhas doctrine would seem to be useful; but if a soul can (through whatever heroic efforts of spiritual practice) extricate itself from the phenomenal and reach the noumenal, then that doctrine would seem to be, at best, redundant, if not ridiculous.

Consider, first, a temporary soul (whether its existence is limited to one lifetime or it spans several lifetimes, either in a body or disembodied). Such a soul, necessarily, like all other impermanent existents that have a beginning and an end, has come from emptiness and will return to emptiness; it is created and conditioned, by the uncreated and unconditioned One. Moreover, a temporary soul might even be regarded as eternal in the sense that it has a share in eternity, not only when it temporarily exists manifestly as a distinct entity, but even before its creation and after its apparent destruction, when it is still or again an undifferentiated part of the original ground. So, no problem there, other than finding out precisely how to indeed liberate it (no mean feat, of course).

A problem might rather be found with regard to an eternal soul, and this is no doubt what caused Buddhists to be leery of the very idea of self (which they regarded as necessarily eternal, remember). The problem is: if the individual soul (or anything else, for that matter) stands side by side with the ultimate reality throughout eternity, then how can it ever merge with it? No way to liberation would seem conceivable for a soul by definition eternally separate from emptiness. But even here, we could argue that the separation of the distinct soul from the universal unitary matrix is only illusory; i.e. that all through eternity this indestructible soul is in fact a constant emanation from the abyss and really always imbedded in it. What makes an illusion (e.g. a mirage or a reflection) illusory is not how long it lasts (a split second or a billion years), but its relativity (a mirage is due to refraction of light from an oasis, a mirror image of the moon is due to reflection of light from the moon). So, in truth, even an eternal soul can conceivably be reconciled with emptiness. I am not affirming the soul is necessarily eternal in that sense, but only pointing out that it conceivably could be so.

In conclusion, the skandhas idea serves no purpose with regard to the requirement of emptiness. Indeed, it is highly misleading, since it is based on false assumptions concerning other doctrinal possibilities. Buddhists cling to this idea for dear life, but without true justification. Clearly, the position taken here by me is that logically we can well claim that people have a soul, and reject the orthodox Buddhist belief that what we call our self is nothing but a cluster of passing impersonal events, without giving up on the more metaphysical doctrine that at the root of spiritual (i.e. every soul's) existence there is 'emptiness' as here understood.

Just as we can say that apparently substantial material, or concrete mental, phenomena are ultimately empty, so we can say that the soul each of us consists of is 'substantial' in its own rarified, spiritual way and at the same time ultimately empty, i.e. at root just part of the universal and unitary ground of all being. In other words, contrary to what Buddhist philosophers imagined, it is not necessary to deny the existence of the soul in order to affirm its 'emptiness', any more than it is necessary to deny the existence of the body or mind in order to affirm their 'emptiness'. That is to say, there is no logical necessity to adopt the five skandhas idea, if the purpose of such position is simply to affirm 'emptiness'.

7. In conclusion

The fact of the matter is that the no-soul thesis is riddled with contradictions. We are told by Buddhists that we can find liberation, but at the same time that we don't even *exist*. We are told to be conscious, but at the same time we are denied the power of cognition – i.e. that the soul is the *subject* of cognitive events. We are told to make the effort to find liberation, but at the same time we are denied possession of volition – i.e. that the soul is the *free agent* of acts of will. We are told to make the wise choices in life, but at the same time we are denied the privilege of value-judgment – i.e. that the soul is capable of *independent and objective* valuation.

The no-soul thesis is upheld in spite of these paradoxes, which were well-known to Buddhist philosophers from the start. What is the meaning of spirituality without a spirit (soul, self)? Who can be liberated if there is no one to liberate? Why and how engage in spiritual practice if we not only do not exist, but also have no power of consciousness, volition or valuation? Why bother to find release from suffering if we do not really suffer? Who is writing all this and who is reading it? The no-soul thesis simply cannot be upheld. The soul can well be claimed to be ultimately 'empty' in the aforesaid sense, but the thesis of five skandhas instead of a soul is logically untenable.

We have seen that the five skandhas doctrine cannot be regarded as an accurate description of the human psyche in its entirety. It is not a thorough phenomenological account, since it ignores mankind's major higher faculties – intelligence, rationality and freewill. It focuses exclusively on some petty aspects of human psychology, the five skandhas, without openly acknowledging the more noble side of humanity, which makes liberation from such pettiness possible. It has metaphysical pretensions, with ontological and epistemological implications – notably, the idea that we are empty of soul, devoid of personality – but it turns out that this idea does not stand up to logical scrutiny, being based on circular arguments and foregone conclusions.

Thus, whereas the five skandhas thesis may have at first seemed like an important observation and idea, which applied and buttressed the more general Buddhist thesis of emptiness, and at the same time provided a spiritually useful description of human psychology, it turns out to be a rather limited and not very well thought-out creed. This does not mean that it has no worth at all, but it does mean that it is far less important than it is made out to be.

This being said, I hasten to add that the present criticism of this one doctrine within Buddhist psychology and philosophy is not intended as a blanket belittling or rejection of Buddhist psychology and philosophy. Certainly, Buddhist psychology and philosophy have a great deal more to offer the seeker after wisdom than this one doctrine. It is rich in profound insights into the human psyche and condition, which every human being can benefit from. This is evident already in the opening salvo of Buddhist thought, the Four Noble Truths, which acknowledge the human condition of suffering and identify the psychological source of such suffering in clinging to all sorts of vain things, and which declare the possibility of relief from suffering through a set program of spiritual practices.

In the Buddhist conception of human life, our minds are poisoned by numerous cognitive and volitional and emotional problems. At the root of human suffering lies a mass of ignorance and delusion about oneself and the world one suddenly and inexplicably finds oneself in. These give rise to all sorts of unwise desires, including greed (for food, for material possessions) and lust (for sexual gratification, for power), and aversions (fears, hatreds). The latter impel people to act with selfishness (in the more pathological sense of the term), and in some cases with dishonesty or even

violence (coldness and cruelty), and generally with stupidity. But Buddhism proposes ways to cure these diseases, so its outlook is essentially positive.

Clearly, Buddhism has a particularly 'psychological' approach to life. It is also distinguished by its businesslike, 'no blame' approach to spirituality, which is no doubt why many people in the West nowadays are attracted to it. Unlike most of the other major religions, notably Judaism and its Christian and Islamic offshoots, it does not try to make people feel guilty for their sins, but rather encourages them to deal with their problems out of rational self-interest. It is thus less emotional and more rational in many ways.

Judaism too, for instance, includes psychological teachings, although perhaps to a lesser extent. One of the main features of Judaic psychology is the idea that humans have two innate tendencies – a good inclination (*yetzer tov*) and a bad inclination (*yetzer ra'*)[143]. These two inclinations *influence* a person for good or for bad in the course of life (physical life and spiritual life), but they never *control* one, for human beings are graced with freewill. This means that come what may, a man or woman is always (at least, once adult) responsible for his or her choices. This ethical belief in freedom of choice and personal responsibility is present in Judaism since its inception, as the following Biblical verse makes clear: "*Sin coucheth at the door; and unto thee is its desire, but thou mayest rule over it*" (Gen. 4:7)[144]. Knowing this, that one indeed has freedom of will, one can overcome all bad influences and forge ahead towards the eternal life.

In Buddhism, we may discern a similar possibility of taking full responsibility for one's life in the very first chapter of the *Dhammapada* (1:1-2). "If a man speaks or acts with an impure mind, suffering follows… If a man acts with a pure mind, joy follows." Further on in the same document (12:163) is a statement suggesting that man's inclination is more on the side of bad than good: "It is easy to do what is wrong, to do what is bad for oneself; but very difficult to do what is right, to do what is good for oneself." But personal responsibility for both bad and good is nonetheless clearly (v.165) declared: "By oneself the evil is done... by oneself the evil is not done." Although, as we have seen, the five skandhas doctrine gives people the impression that they are not responsible for their deeds, we see here that this is not really the message of Buddhism, which generally enjoins strong spiritual effort in the direction of self-liberation and thence of liberation for all sentient beings.

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[143] This two-inclinations psychological thesis of course stands in contrast to three other theses: that humans have only a good inclination (optimism), or only a bad inclination (pessimism), or no natural inclination at all (neutrality). This is an interesting issue that deserves a longer discussion. The difference between these four theses is moot, if we consider that all this is about *influences* on the soul, and not about determinism or fatalism; the soul remains free to choose whether influenced one way or the other to greater or lesser degree. I think the point of the Jewish doctrine is simply this: to make the individual aware that he is constantly under pressure from influences of various sorts, some good and some bad, and that he is wise to at all times *identify with* the positive ones and *avoid identifying with* the negative ones; i.e. to regard the true ultimate desire of his soul as the good and to regard the bad as delusive nonsense.

[144] The idea of 'inclination to evil' may also be traced to the Bible, namely to Gen. 8:21, which quotes God as stating that "the inclination of man's heart is evil from his youth." (That is said after the Deluge; earlier, in Gen. 6:5, it is said more pessimistically that "every inclination of the thoughts of his [man's] heart is only evil all through the day." Commentators explain the difference by suggesting that the Deluge made man wiser. Maybe the difference between the terms "the inclination of man's heart" and the "inclination of *the thoughts of* man's heart" has some significance.)

6. ALL THE MORE

1. A synthesis of old and new information

After I distributed my essay "The 46 *Qal vachomer* Arguments in the Tanakh" (drawn from my 2013 book *A Fortiori Logic*[145]) to many people in my mailing lists, one of the recipients, R. Yaacov Gabay[146], responded by pointing out to me the existence of older lists of a fortiori arguments found in the Jewish Bible, lists given in traditional Jewish literature, which might contain cases that I was not aware of and did not mention in my said essay. I asked the rabbi to research this literature for me, because my knowledge of Hebrew is insufficient for such a demanding task, and he kindly accepted to do that. I told him that I would, for my part, process and report the information.

Note that the present study has two goals. Its primary purpose is to draw up a list of all Biblical a fortiori discourse discovered by anyone to date; to achieve this, one must grasp the forms of a fortiori argument and be able to interpret Biblical text accurately. The secondary goal is historical – to find out who first discovered each instance of such discourse; and as far as possible to find out whether eventual subsequent listings of the case were independent or influenced by a predecessor. Both these tasks require extensive reading in the (mostly Hebrew) sources. The a fortiori arguments might be flagged by authors in scattered commentaries; or they might be collected by them in lists. The authors might reveal the predecessors that influenced them explicitly, or only by allusion.

Most of the literature research for the present study was carried out by Gabay; after which I analyzed, evaluated, and compiled, the findings. We have to date identified **a total of 72 instances of *qal vachomer* in the Tanakh**. This is already a larger number than any previously published by anyone (to our knowledge). My previous list included only 47 instances; so, the present research has added 25 cases to it. I count in my lists as instances of a fortiori argument only those that are clearly enough intended as such within the Biblical text and, of course, are also logically valid.

Gabay thought of and investigated five rabbinical works with lists of Biblical a fortiori arguments, namely: *Yefeh Toar* by R. Shmuel Yaffe Ashkenazi (16th Cent.), *Netivot Olam* by R. Zvi Hirsch Katzenellenbogen (19th Cent.), *Midrash Tannaim* by R. Zeev Wolf Einhorn (19th Cent.), *Mattat Yah* by R. Mattityahu Strashun (19th Cent.), and *Berure HaMidot* by R. Chaim Hirschensohn (20th Cent.). For my part, I thought of and investigated Rashi's *Commentary on the Tanakh* (11th Cent.), and R. Louis Jacobs' "The Qal Va-Homer Argument in the Old Testament" in *Rabbinic Thought in the Talmud* (20th Cent.).[147]

All the credit for finding 23 of the additional cases belongs to Gabay. To be precise, he did not discover these cases, since they were already known to past commentators; but he certainly re-

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[145] I often refer to books by the initials of their titles. Thus, for my books: Judaic Logic = JL, A Fortiori Logic = AFL, All The More = ATM.

[146] Currently the rabbi of the Hekhal Haness congregation in Geneva, Switzerland (where I reside). Hereinafter, I refer to the rabbi simply by his last name, as is customary in academic papers.

[147] Einhorn and Hirschensohn were briefly mentioned in my AFL, following mention of them in Jacobs's essay.

discovered them for the purpose of our modern cumulative listing. His research has also made it possible for me to clarify who (at least apparently) discovered each case first. The research work relating to Rashi was, however, done by me; and this added one more case (of Rashi's) to our expanded list. A second added case was found by me by analogy to a case listed by Hirschensohn. Additionally, of course, I brought over findings by Jacobs and myself mentioned in my past books. Otherwise, my job in this project has consisted merely in critically evaluating and organizing the data and in composing the present paper.

The following, briefly put, is **the full list of 72 Biblical occurrences** of *qal vachomer* proposed in the present essay: **Genesis 3:22-23**, **4:14, 4:24, 6:3, 11:6, 14:23, 17:17, 18:12, 39:8-9, 44:8; Exodus 6:12, 6:30; Numbers 12:14; Deuteronomy 31:27, 32:39; Judges 14:16; 1 Samuel 14:29-30, 14:39, 17:37, 21:6, 23:3; 2 Samuel 4:10-11, 11:11, 12:18, 12:21, 16:11; 1 Kings 8:27; 2 Kings 5:12, 5:13, 10:4, 18:23-24, 18:35; Isaiah 1:3, 10:11, 20:6, 36:8-9, 36:20, 66:1 (a & b); Jeremiah 2:11, 8:7, 12:5 (a & b), 25:29, 45:4-5, 49:12; Ezekiel 3:4-7, 14:13-21, 15:5, 33:24; Jonah 4:10-11; Psalms 25:8-9, 78:20, 94:9 (a & b), 94:10; Proverbs 11:31, 15:11, 17:7, 19:7, 19:10, 21:27; Job 4:18-19, 9:13-14, 15:15-16, 25:5-6, 35:13-14; Esther 9:12; Daniel 2:9, Nehemiah 13:26-27; 2 Chronicles 6:18, 32:15**.

See chapter 3 (below) for a more detailed listing and explication of these 72 cases, and chapter 17 for a detailed tabulation of them.

Note in the above list that there are **15 instances in the Torah proper**. The statistics per Biblical book (so far as we know to date) are as follows: Genesis (10), Exodus (2), Numbers (1), Deuteronomy (2), Judges (1), 1 Samuel (5), 2 Samuel (5), 1 Kings (1), 2 Kings (5), Isaiah (7), Jeremiah (7), Ezekiel (4), Jonah (1), Psalms (5), Proverbs (6), Job (5), Esther (1), Daniel (1), Nehemiah (1), 2 Chronicles (2); the other books have none.

Of the above listed 72 cases, 47 cases were already listed in my 2013 study (AFL, appendix 1); these are again presented and explicated below in chapter 3. The **25 cases newly encountered in the present (2023-4) study** are the following[148]: **Genesis 3:22-23**, **4:14, 6:3, 11:6, 14:23, 17:17, 18:12, 39:8-9; Exodus 6:30; Deuteronomy 32:39; 1 Samuel 14:39; 2 Samuel 11:11, 12:21; 2 Kings 5:12, 18:35; Isaiah 1:3, 10:11, 20:6, 36:20; Jeremiah 2:11, 8:7; Ezekiel 3:4-7; Psalms 25:8-9; Proverbs 17:7; Job 35:13-14**; these are also presented and explicated below in chapter 3.

The following table specifies the **logical forms** of these 72 cases, the symbols meaning: **s** for subjectal, **p** for predicatal, **+** for positive, **-** for negative, and **&** for proportional (see chapter 2 for a full explanation of these terms).

[148] Thus, to be sure, the 25 cases comprise: 8 cases from Genesis, 1 case from Exodus, 1 case from Deuteronomy, 1 case from 1 Samuel, 2 cases from 2 Samuel, 2 cases from 2 Kings, 4 cases from Isaiah, 2 cases from Jeremiah, 1 case from Ezekiel, 1 case from Psalms, 1 case from Proverbs, and 1 case from Job.

A fortiori argument form		Quantity (all)	Of which, a crescendo (**&**)
positive subjectal	**+s**	28	5
negative subjectal	**-s**	20	3
positive predicatal	**+p**	18	1
negative predicatal	**-p**	6	0
Total		72	9

I have classified all cases in each author's listing in the following categories of authorship: (A) 'historic first' (the author concerned is apparently historically the first to have mentioned or listed this case); (B) 'independent' (the author concerned is evidently *not* historically the first to have mentioned or listed this case, but apparently found it independently); and (C) 'derived' (the author concerned is known to – or may reasonably be assumed to – have learned of this case from another who preceded him).

In a few cases, classification into one of these three categories is rather difficult, because someone may have noticed that a Biblical passage has an a fortiori intent, but either not formulated what the *qal vachomer* might look like or wrongly formulated it. In such cases, when I formulate the argument for him or correct him, should I count the case as one of my findings, even though I did not notice the case by myself, or still consider it as his? And what if my understanding of the narrative is based on some commentary, say by Rashi? I have tried to be as fair to all (including myself) as I could. In any case, I am always transparent.

The following table summarizes the **achievements of the various authors/works**.

Author/Work		All	A	B	C
Eliezer b. Jose / Baraita 32 Midot	EbJ	3	3	0	0
Oshaya b. Hama / Genesis Rabbah	GR	11	8	0	3
Rashi / Commentary on the Tanakh	R	25	14	0	11
S. Y. Ashkenazi / Yefeh Toar	SYA	24	8	5	11
Z. H. Katzenellenbogen / Netivot Olam	ZHK	31	9	11	11
Z. W. Einhorn / Midrash Tannaim	ZWE	24	0	1	23
M. Strashun / Mattat Yah	MS	52	14	7	31
Ch. Hirshensohn / Berure ha-Midot	HH	41	8	1	32
Avi Sion / Judaic Logic	AS	34	5	18	11
L. Jacobs / Rabbinic Thought in the Talmud	LJ	33	2	7	24
Avi Sion / A Fortiori Logic	AS	47	1 (+5)	21	20
Avi Sion + Yaakov Gabay / All The More	AS+YG	72	0 (+6)	21	25+20

It is worth listing the historic-first discoveries (i.e. column A) of each of the authors mentioned in the above table. The sum of the totals is, of course, 72 cases. **Genesis Rabbah** (all possible authors and commentators lumped together[149]): Genesis 44:8, Exodus 6:12, Numbers 12:14, Deuteronomy 31:27, 1 Samuel 23:3, Jeremiah 12:5 (a & b), Ezekiel 15:5, Proverbs 11:31, and Esther 9:12. To which must be added, from another place in the same document, at least: Genesis 4:24; total: 11 cases. **Rashi**: Genesis 6:3; 2 Kings 5:13; Ezekiel 14:13-21, 33:24; Proverbs 15:11, 17:7, 19:7, 19:10, 21:27; Job 9:13-14, 15:15-16, 35:13-14; Daniel 2:9; Nehemiah 13:26-27; total: 14 cases.

Ashkenazi: 1 Kings 8:27; 2 Kings 10:4; Isaiah 20:6; Jeremiah 25:29, 45:4-5, 49:12; Job 4:18-19, 25:5-6; total: 8 cases. **Katzenellenbogen**: Deuteronomy 32:39; 1 Samuel 14:29-30; 2 Samuel 4:10-11, 12:18, 12:21, 16:11; 2 Chronicles 6:18; Psalms 25:8-9, 78:20; total: 9 cases. **Einhorn**: 0 cases known. **Strashun**: Judges 14:16; 1 Samuel 14:39; 2 Samuel 11:11; 2 Kings 5:12, 18:23-24, 18:35; Isaiah 1:3, 10:11, 36:8-9, 36:20; Jeremiah 2:11, 8:7; Ezekiel 3:4-7; Jonah 4:10-11; total: 14 cases. **Hirschensohn**: Genesis 3:22-23, 4:14, 11:6, 14:23, 17:17, 39:8-9 and Exodus 6:30. Plus Genesis 18:12, added on; total: 8 cases. **Sion in JL**: 1 Samuel 17:37, 21:6; Psalms 94:9 (a & b), 94:10; total: 5 cases. **Jacobs**: Isaiah 66:1 (a & b); total: 2 cases. **Sion in AFL**: 2 Chronicles 32:15; total: 1 case. **Sion and Gabay in ATM**: 0 cases.[150]

R. Eliezer ben Jose ha-Gelili (2nd Cent. CE), in the *baraita* on the 32 hermeneutic rules of Biblical interpretation, in rules 5 and 6, distinguishes between **explicit *(meforash)* and implicit *(satum)*** *qal vachomer*. This is an important distinction, which I support wholeheartedly. My lists of Biblical a fortiori arguments therefore only include explicit cases, to the exclusion of implicit ones. The reason for that policy is that explicit cases are based on literal reading of the text, i.e. reading wholly 'out of' the text; whereas implicit cases are based on overly interpretative reading of the text, i.e. reading partly 'into' the text. The latter readings add something to the text, some narrative or homiletic element; they involve 'reading between the lines', i.e. interpolation. These are only general criteria; more specific criteria can and will be posited as we go along.

That does not mean that 'explicit' cases are always entirely explicit; often some part of the intended a fortiori argument (in most cases it is one of the premises, usually the major, though in some cases it is the conclusion) is unstated – but the a fortiori *intent* of the literal text is quite evident or unavoidable. And that does not mean that 'implicit' cases are not as a fortiori as claimed – the problem with them is only that the a fortiori argument proposed is not strictly part of the received text, because the text *can be read in some other way(s)* than the a fortiori way proposed. Thus, my lists may reject a case for one of two reasons: either because the proposed a fortiori argument is not explicit, but merely implicit, even though it is formally valid; or because it is not a formally valid, even though intended or claimed as a fortiori, or because it is not at all a fortiori in intent.

Note that proposed cases judged by me to be merely implicit or invalid or not a fortiori are relatively rare. I am solely responsible for the ad hoc judgments made here; Gabay was occasionally consulted, but the final judgment was always mine. Sometimes, as will be seen, my judgment differs from that of some other commentator; but I can justly claim my decisions to be the more reliable because of my comprehensive theoretical studies in a fortiori argument as such in my books *Judaic Logic* (1995) and *A Fortiori Logic* (2013). See chapter 2, below, for a brief

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149 Including: Eliezer b. Jose, 3 cases: Jeremiah 12:5 (2 cases), Esther 9:12.

150 Note that the cases listed in brackets in AFL (+5) and ATM (+6) are so listed to indicate that they are still historic firsts (A) in my name. I could not list them as mere independent findings (B), being the same author of these different books.

review of the formalities of a fortiori reasoning which I elaborated in these past works. I always, of course, give precise reasons for such decisions; they are never arbitrary. In some cases, the decision is explicated at length, being based on hours of narrative, linguistic and logical analysis[151].

The only exception I have made in the past to the said principle of only accepting explicit *qal vachomer* cases in my lists is the case of Esther 9:12. Although I personally doubt that its original intent was *qal vachomer* – because its language is exceptional and its a fortiori form is not manifest[152] – I have kept it in my lists anyway because it is included in the classical GR list of ten cases.

Thus, I have categorized the few cases left out from the list of 72 valid and explicit cases as either (D) 'judged as at best implicit only' or (E) 'judged as invalid' (the latter meaning 'apparently a fortiori but not really so' or 'not at all a fortiori'). Of all potential cases encountered, 7 cases were classed as D, namely: Genesis 6:9, 17:20-21; Leviticus 10:19; Habakkuk 2:4-5; Psalms 15:4, 15:5; Job 28:17. And 7 cases were classed as E, namely: Genesis 3:1, 27:37; 2 Samuel 10:16; Ezekiel 23:39-40; Proverbs 6:30, 10:17, 19:7b.

It should be clear that rejection of a proposed case is just as important as acceptance of one. Obviously, one cannot accept offhand all cases proposed, but must subject each one to careful scrutiny. When an author proposing a dud case specified in what way he thought the verse(s) concerned could be read as a fortiori, or I could at least guess what reading he may have had in mind, I could analyze the proposal and state precisely why I rejected it as either implicit (7 cases) or invalid (4 cases). But when the author did not clarify his reading, or maybe Gabay did not report his exact reading to me, then if I could not see a possible a fortiori interpretation, I just declared the case as not a fortiori by default (3 cases). Of course, I am not infallible, and have occasionally made mistakes of appreciation in my past works. But I keep an open mind and am always open to being corrected.[153]

In the following chapter, I offer the reader some background in formal logic, needed for better understanding of the findings in the present study. In chapter 3, I list the 72 (47+25) recognized cases, and briefly paraphrase them to show more clearly their a fortiori intent. In subsequent chapters, I examine, in chronological order, the lists attributed to the various commentators here considered. These are based, to repeat, on the work of the following authors: the author(s) of Genesis Rabbah, Rashi, S. Y. Ashkenazi, Z. H. Katzenellenbogen, Z. W. Einhorn, M. Strashun, Ch. Hirschensohn, A. Sion (1995), L. Jacobs, and A. Sion (2013).

There may, of course, be still other literary sources which mention Biblical a fortiori arguments that we have not yet come across and examined; maybe even many of them[154]. There may even,

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[151] The reader should take the time to read such occasional detailed exegeses, as they are very instructive in various ways.

[152] Here, the Hebrew sentence *meh assu*, literally: [in the provinces] "what have they done?", which in the context most literally means: "how many people have they killed?", is taken by a fortiori advocates to mean: "how many *more* people have they killed?" Clearly, the assumption that the king thought that *more* people were killed is not inevitable; he could have been asking an open question.

[153] For example, in JL I erroneously reject Genesis 3:22-23, 11:6, and 17:7; whereas in the present study I accept them after learning just how Hirschensohn interpretated those cases.

[154] S. Goltzberg, in his 2010 essay "The A Fortiori Argument in The Talmud" mentions "the forgotten a fortiori arguments," without however saying how many he thinks there are or listing them. Apparently, he draws this information from Moshe Koppel's *Meta-Halakha. Logic, Intuition And The Unfolding Of Jewish Law* (Northvale, NJ,

of course, be scattered mentions of such arguments that we have overlooked in the sources already examined. Our list therefore must be said to comprise *at least* 72 cases, those found by us *so far*.

2. The valid forms of a fortiori argument

Before proceeding further, the reader should get a theoretical grounding in a fortiori logic, by studying the valid forms of copulative argument listed below, on which all analyses, classifications, and listings, in the present essay are based. **The symbols P, Q, R, S, refer respectively to the major, minor, middle, and subsidiary terms.**[155]

With regard to *structure*, an a fortiori argument consists of three propositions: **a major premise, a minor premise, and a conclusion**. The major premise compares the major and minor terms (P and Q, respectively) by means of the middle term (R); the minor premise relates the minor or major term to the subsidiary term (S) through the *sufficiency* (or not) of the middle term; and the conclusion accordingly relates the major or minor term to the subsidiary term through the sufficiency (or not) of the middle term.

This applies to *'pure' (non-proportional) a fortiori*; in the case of *'a crescendo' (proportional) a fortiori*, there is an additional premise (called the premise of **proportionality**) describing the concomitant variation of R and S, which (note well) affects the quantity of S in the conclusion.

There are two *figures* of a fortiori argument: the subjectal and the predicatal; the difference between them is one of orientation. In the former, P and Q are subjects and S is a predicate; in the latter, P and Q are predicates and S is a subject; R is a predicate in both figures. Each figure has two *polarities*: in the positive one, the minor premise and conclusion are positive propositions; and in the negative one, they are negative.

The following are the **valid moods** of copulative a fortiori argument:

a. The **positive subjectal** {symbol **+s**} mood (= minor term to major, i.e. Q to P):

P is more R than (or as much R as) Q (is R),

and Q is R enough to be S;

therefore, all the more (or equally), P is R enough to be S.

Given the same major premise, inference from P to Q (is R enough to be S) would be *invalid*. Except in cases where P and Q are equally R, of course.

Above is the *pure* mood; the corresponding *a crescendo* mood (added symbol **&**) would add:

If, *moreover*, (for things that are both R and S,) we find that:

S varies in proportion to R, then:

knowing from the above minor premise that: if R = Rq, then S = Sq,

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Jason Aronson, 1987). I did find this book (in English), and looked into it; but I found no reference to, let alone listing of, Biblical a fortiori arguments in it.

[155] P is called the major and Q the minor, because usually P is greater than Q relative to R, though in some cases P and Q are equal in magnitude.

it follows in the conclusion that:

if R = *more than* Rq = Rp, then S = *more than* Sq = Sp:

i.e. from Q is R enough to be S conclude that P is R enough to be more than S.

b. The **negative subjectal** {symbol **-s**} mood (= major term to minor, i.e. P to Q):

P is more R than (or as much R as) Q (is R),

yet P is R *not* enough to be S;

therefore, all the more (or equally), Q is R *not* enough to be S.

Given the same major premise, inference from Q to P (is R *not* enough to be S) would be *invalid*. Except in cases where P and Q are equally R, of course.

Above is the *pure* mood; the corresponding *a crescendo* mood (added symbol **&**) would state (note that the conclusion is no different than in the pure mood):

P is more R than Q (is R),

and P is R *not* enough to be *more than* S;

and S varies in proportion to R;

therefore, Q is R *not* enough to be S *at all*.

Clearly, the conclusion *cannot* merely be that Q is R not enough to be *more than* S, because if Q were still R enough to be S (though not more than S) that would contradict the claim that S varies in proportion to R, because P would then be R enough to be S and indeed proportionally more than S. Of course, if the premise of proportionality is denied, then the conclusion *would* be that Q is R not enough to be more than S; but in that case, the subsidiary term is not really just 'S' but 'more than S'.

c. The **positive predicatal** {symbol **+p**} mood (= major term to minor, i.e. P to Q):

More (or as much) R is required to be P than (as) to be Q,

and S is R enough to be P;

therefore, all the more (or equally), S is R enough to be Q.

Given the same major premise, inference from (S is R enough to be) Q to P would be *invalid*. Except in cases where P and Q are equally R, of course.

Above is the *pure* mood; the corresponding *a crescendo* mood (added symbol **&**) would add:

If, *moreover*, (for things that are both R and P or Q,) we find that:

R varies in proportion to S, then:

knowing from the above minor premise that: if S = Sp, then R = Rp,

it follows in the conclusion that:

if S = *less than* Sp = Sq, then R = *less than* Rp = Rq;

i.e. from S is R enough to be P conclude that less than S is R enough to be Q.

d. The **negative predicatal** {symbol **-p**} mood (= minor term to major, i.e. Q to P):

More (or as much) R is required to be P than (as) to be Q,

yet S is R *not* enough to be Q;

therefore, all the more (or equally), S is R *not* enough to be P.

Given the same major premise, inference from (S is R not enough to be) P to Q would be *invalid*. Except in cases where P and Q are equally R, of course.

Above is the *pure* mood; the corresponding *a crescendo* mood (added symbol **&**) would state (note that the conclusion is no different than in the pure mood):

More R is required to be P than to be Q,

and *less than* S is R *not* enough to be Q,

and R varies in proportion to S;

therefore, S *at all* is R not enough to be P.

Clearly, the conclusion *cannot* merely be that *less than* S is R not enough to be P, because if S were still R enough to be P (though less than S not so) that would contradict the claim that R varies in proportion to S, because S would then be R enough to be Q and indeed proportionally less than S would be so. Of course, if the premise of proportionality is denied, then the conclusion *would* be that less than S is R not enough to be P; but in that case, the subsidiary term is not really just 'S' but 'less than S'.

Note that, in all the above four (or eight) moods, the comparative expression in the major premise may be "P is *as much* R as Q" in lieu of "P is *more* R than Q;" this is because the former 'egalitarian' possibility is in fact the limiting case of the latter 'superior' formula (i.e. > here means ≥). The comparative premise is sometimes expressed in the form "Q is *less* R than P;" this is of course okay provided that in such case too P refers to the greater quantity of R, and Q refers to the lesser.

It is useful to remember that a valid a fortiori argument goes "from minor to major" (i.e. from the minor term, Q, to the major term, P) if it is positive subjectal or negative predicatal, and it goes "from major to minor" (i.e. from the major term, P, to the minor term, Q) if it is negative subjectal or positive predicatal. Thus, if an argument goes from minor to major and is positive, we know it is subjectal; but if it does that and is negative, it is predicatal. Whereas, if it goes from major to minor and is positive, we know it is predicatal; but if it does that and is negative, it is subjectal.

Also remember that the difference between a crescendo argument and purely a fortiori argument depends on whether the subsidiary term (S) and the middle term (R) vary together; variation in the middle term alone is not indicative of a crescendo. The concomitant variation of these two terms, usually proportional, may on occasion be inversely proportional. In the latter cases, of course, the a crescendo arguments must be modified accordingly, with one of these terms increasing while the other decreases.

The above are, to repeat, the eight valid logical forms of *copulative* a fortiori argument, including four pure moods and four corresponding proportional moods. There are another eight valid moods, resembling these, of *implicational* a fortiori argument. In implicational a fortiori argument, categorical propositions are replaced by hypothetical ones, meaning that the terms are replaced by theses and the copula (is) is replaced by words of implication (if-then). Note that implicational arguments can usually be re-formulated in simpler, copulative form.

Regarding **objections** to a fortiori arguments. These often occur in Talmudic and other debates, and therefore need to be understood. A *qal vachomer* (if correctly formed) is a deductive argument, meaning that the conclusion necessarily follows from the premises. But like all deductive arguments (including syllogisms, for instance), you can put its premises in doubt: i.e. you can show one or more of them to be factually untrue or at least uncertain. This does not affect the formal *validity* of the argument, but it does impact on its factual *credibility*.

The following example illustrates this point. Deuteronomy 31:27 reads (Moses is the speaker): "Behold: while I am yet alive with you this day, ye have been rebellious against the Lord; and how much more after my death [ye will be rebellious]?" This can be paraphrased as follows: People are more unfaithful after their leader's death than while he is alive (major premise); the Israelites during Moses's lifetime are unfaithful enough to rebel (minor premise); therefore, they will after his death be unfaithful enough to rebel (conclusion). The Maharsha[156] credibly objects that the people might well repent and become more faithful after Moses's death, as a result of their realization that he is no longer there to pray on their behalf. This objection effectively puts the major premise in doubt, showing that it could turn out to be factually untrue. A retort to this objection is possible, by saying (as the Maharsha himself effectively does) that even if Moses's worry might in the future (as indeed happened eventually) turn out to be unjustified, the way things looked at the time Moses uttered his reproach justified it.

The logic of a fortiori argument was first correctly formulated by me in my 1995 book *Judaic Logic* (JL), and further developed in my 2013 book *A Fortiori Logic* (AFL). Readers are referred to my book AFL, chapters 1 and 2, for more details, including the precise procedures for *validation or invalidation* of all moods of such argument. These procedures are obviously important, but too long-winded to be repeated here.

It is important to be aware that a fortiori argument differs significantly from **argument by analogy**, in that a fortiori depends for its inference on there being a *sufficient* quantity of the middle term ("R enough"), implying that *a threshold value* of it (i.e. R) must be reached or passed in the minor premise for the conclusion to be validly drawn, whereas this feature is absent in mere analogy. As a result of this difference, a fortiori argument is deductive (meaning that given the premises, the conclusion is 100% sure), whereas analogical thinking is inductive (meaning that given the premises, the conclusion is only probable). One should therefore be careful not to confuse these two forms of reasoning.

The following are, simply put, the valid moods of analogical argument. Keep in mind, however, that, because such argument is inductive, equally valid conflicting analogies are possible. Note that here (unlike in a fortiori argument) P and Q are interchangeable throughout; i.e. the argument holds in either direction. Analogy may be qualitative or quantitative:

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[156] R. Shmuel Eidels (Poland, 1555-1631). In *Chiddushei Aggadot* on Sanhedrin 37a.

Qualitative analogy (this resembles pure a fortiori argument somewhat, note):

Positive subjectal (+**s**): P and Q are alike in some respect, e.g. in that both have R. And Q is S; so, P is probably S. (Or: And P is S; so, Q is probably S.) Negative moods (**-s**): Same major premise. And Q is not S; so, P is probably not S. (Or: And P is not S; so, Q is probably not S.)

Positive predicatal (+**p**): P and Q are alike in some respect, e.g. in that R has both. And S is Q; so, S is probably P. (Or: And S is P; so, S is probably Q.) Negative moods (**-p**): Same major premise. And S is not Q; so, S is probably not P. (Or: And S is not P; so, S is probably not Q.)

Quantitative analogy (this resembles a crescendo argument somewhat, note): given that R and S vary jointly:

Positive subjectal (+**s**): If P is greater than Q with respect to R, then if Q is S (Sq), it follows that P is probably proportionately more S (Sp>Sq) (from inferior to superior). If P is equal to Q with respect to R, then if Q is S (Sq), it follows that P is probably proportionately as much S (Sp=Sq) (from equal to equal). If P is lesser than Q with respect to R, then if Q is S (Sq), it follows that P is probably proportionately less S (Sp<Sq) (from superior to inferior). Negative mood (**-s**): Whether P is greater than, equal to, or lesser than, Q with respect to R, then if Q is not S, it follows that P is probably not S, and conversely, if P is not S, it follows that Q is probably not S.

Positive predicatal (+**p**): If P is greater than Q in relation to R, then if S (Sq) is Q, it follows that probably proportionately more S (Sp>Sq) is P (from inferior to superior). If P is equal to Q in relation to R, then if S (Sq) is Q, it follows that probably proportionately as much S (Sp=Sq) is P (from equal to equal). If P is lesser than Q in relation to R, then if S (Sq) is Q, it follows that probably proportionately less S (Sp<Sq) is P (from superior to inferior). Negative mood (**-s**): Whether P is greater than, equal to, or lesser than, Q with respect to R, then if S is not Q, it follows that S is probably not P, and conversely, if S is not P, it follows that S is probably not Q.

All this is briefly presented here to make sure the reader does not confuse a fortiori argument with analogical argument. See my work *The Logic of Analogy* (2023) for more information on this interesting topic.

3. Merged list of 72 *qal vachomer* arguments in the Tanakh

The following is the listing and brief analysis of the 72 *qal vachomer* arguments so far known by us to occur in the Tanakh, including the 47 cases that I listed in 2013 (in AFL, appendix 1), and the 25 cases newly found in the present study (all but two of them outcomes of Rav Gabay's research in the literary sources)[157]. I here give, for each case, a brief analytic paraphrase, consisting of only the minor premise and conclusion (by means of an if-then statement), since the major premise is easy to construct given these two components. The logical form of the argument is indicated symbolically in brackets {}, as **s** or **p**, + or **-**, and where applicable **&**. The first

[157] The sources of my Biblical quotations (of translations) vary. For the cases I listed in the past, see there. For cases recently added, my main source was (as I recall) https://mechon-mamre.org/p/pt/pt0.htm, However, in some cases, I preferred https://www.chabad.org/library/bible_cdo/aid/63255/jewish/The-Bible-with-Rashi.htm. But note that in some cases, I modified the received text somewhat, usually with a more literal rendering, if I deemed this necessary for highlighting the a fortiori intent of the verse or even just its meaning. Sometimes, I added some text in brackets to the given text.

discoverer, to our knowledge so far, of each case is indicated by means of his name's initials[158]. When I have substantially amplified or modified or replaced an author's reading, I may add my initials AS to his. Cases newly identified in the present study are marked as NEW.

Genesis 3:22-23. God: "Behold (*hen*), the man is become as one of us, to know good and evil; and now (*ve-atah*), lest he put forth his hand, and take also of the tree of life, and eat, and live forever. Therefore (*ve*), the Lord God sent him forth from the garden of Eden, to till the ground from whence he was taken." {**+s**} If man having already eaten of tree of knowledge of good and evil is excessively godlike enough to have to be expelled from Eden, then if man additionally now ate of the tree of life, he would be even more excessively godlike enough to have to be expelled from Eden. (NEW – First found by HH/AS.)

Genesis 4:14. Cain: "Behold (*hen*), Thou hast driven me out this day from the face of the land; and (*ve*) from Thy face shall I be hid; and (*ve*) I shall be a fugitive and (*ve*) a wanderer in the earth; and it will come to pass (*ve-hayah*), that whosoever findeth me will slay me." {**+s&**} The four listed curses alone are severe enough to fully punish Cain's crime of murder of Abel; therefore, the four curses plus the said threat to life are severe enough to punish that crime and more. (NEW – First found by HH/AS.)

Genesis 4:24. Lemekh ben Methushael: "If (*ki*): Cain shall be avenged sevenfold, then (*ve*): Lemekh [shall be avenged] seventy and seven-fold." {**-s&**} If an intentional killer is not abhorred enough to be punished immediately, then an unintentional killer will remain unpunished for a much longer time. (First found by GR.)

Genesis 6:3. God: "My spirit shall not quarrel concerning man forever, in that also (*beshagam*) he is flesh; and (*ve*) his days shall be a hundred and twenty years." {**+s**} Unrepentant sinning. by a high powerful creature made of fire or some hard substance, is insubordinate to God enough to merit annihilation in 120 years; all the more, unrepentant sinning, by a lowly weak creature made like man of mere flesh, is insubordinate to God enough to merit annihilation in 120 years. (NEW – First found by R/AS.)

Genesis 11:6. God: "Behold (*hen*), they are one people, and they have all one language; and this is what they begin to do; and now (*ve-atah*) will it not be withheld from them, all that they have planned to do?" {**+s**} The people beginning to build a city and a tower are rebellious enough to be opposed by God; therefore, the people succeeding to build a city and tower would be rebellious enough to be opposed by God. (NEW – First found by HH/R/AS.)

Genesis 14:23. Avram: "I will not (*im*) take a thread nor a shoe-latchet nor aught that is thine, so that (*ve*) thou shouldest not say: I have made Avram rich." {**-p**} If Avram wishes to avoid being indebted enough to refuse what is due to him, then perforce Avram wishes to avoid indebtedness enough to refuse what is not due to him. (NEW – First found by HH.)

Genesis 17:17. Avraham: "Shall (*ha-le*) a man a hundred years old beget? and shall (*ve-im*) Sarah, a woman of ninety, bear?" {**+s**} If Avraham (a hundred-year-old man) had a younger wife (of naturally childbearing age), it would be difficult enough for him to beget; all the more so, with his actual wife Sarah (a ninety-year-old woman, way past childbearing age) it would be difficult enough, indeed even more difficult, for him to beget. (NEW – First found by HH.)

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[158] GR=Genesis Rabbah, EbJ=Eliezer ben Jose HaGelili, R=Rashi, SYA=S.Y. Ashkenazi, ZHK=Z.H. Katzenellenbogen, ZWE=Z.W. Einhorn, MS=M. Strashun, HH=Ch. Hirschensohn, LJ=L. Jacobs, AS=A. Sion (1995, 2013).

Genesis 18:12. Sarah: "After I am waxed old, will (*hayta*) I be rejuvenated? Also (*ve*) my lord being old?" {+**s**} Sarah (now ninety years old, past menopause) would, with Avraham (now one hundred years old) if he were currently younger than he is, have had difficulty enough having a child; all the more so, she can expect, with him now in fact quite aged, have difficulty enough, indeed even more difficulty, to have a child. (NEW – First found by HH/AS.)

Genesis 39:8-9. Joseph: "But he refused, and said unto his master's wife: 'Behold (*hen*), my master, having me, knoweth not what is in the house, and he hath put all that he hath into my hand; he is not greater in this house than I; neither hath he kept back anything from me but thee, because thou art his wife. How then (*ve-ekh*) can I do this great wickedness, and sin against God?'" {+**s**} If Joseph stole any property that he was permitted to use, he would be dishonest enough to be committing a grave sin; all the more, if Joseph stole any property that he was not permitted to use (such as his master's wife), he would be dishonest enough to be committing a grave sin. (NEW – First found by HH.)

Genesis 44:8. Joseph's brothers: "Behold (*hen*): the money, which we found in our sacks' mouths, we brought back unto thee out of the land of Canaan; how then (*ve-ekh*) should we steal out of thy lord's house silver or gold?" {+**p**} If the accused were honest enough to return found goods, then they must have been honest enough not to steal anything. (First found by GR.)

Exodus 6:12. Moses: "Behold (*hen*): the Children of Israel have not hearkened unto me; how then (*ve-ekh*) shall Pharaoh hear me?" {-**s**} If the Israelites, who have much faith, have not had enough of it to hearken to a prophet like Moses, then the chief of the Egyptians, who has far less faith (if any), will not have enough of it to do so. (First found by GR.)

Exodus 6:30. Moses: "Behold (*hen*), I am of uncircumcised lips; so how (*ve-ekh*) will Pharaoh hearken to me?" {-**p**} If Pharaoh would have patience not enough to hearken to Moses were he not handicapped; all the more, Pharaoh will have patience not enough to hearken to Moses he being in fact handicapped. (NEW – First found by HH/AS.)

Numbers 12:14. God: "If (*ve*) her father had but spit in her face, should she not (*ha-lo*) hide in shame seven days? [Similarly, since God is angry with her,] let her be shut up without the camp seven days." {+**s**} If someone causing paternal anger is culpable enough to deserve seven days isolation, then someone causing Divine anger is culpable enough to deserve seven days isolation. (First found by GR.)

Deuteronomy 31:27. Moses: "Behold (*hen*): while I am yet alive with you this day, ye have been rebellious against the Lord; and how much more (*ve-af ki*) after my death [ye will be rebellious]?" {+**s**} If the people during Moses's lifetime are unfaithful enough to rebel, then they after his death will be unfaithful enough to rebel. (First found by GR.)

Deuteronomy 32:39. "See now (*reu ata*) that (*ki*) I, even I, am He, and there is no god with Me; I kill, and I make alive; I have wounded, and I heal; and (*ve*) there is none that can deliver out of My hand." {+**p**} If no one other than God is powerful enough to kill and revive, to wound and heal (like He does, alone); then surely, no one other than He is powerful enough to deliver out of His hand (i.e. to stop Him killing and reviving, Him wounding and healing, at will). (NEW – First found by ZHK.)

Judges 14:16. Samson to his wife: "Behold (*hine*), I have not told it [the solution to my riddle] to my father nor my mother, and (*ve*) shall I tell [it to] thee?" {-**p**} If Samson was not trusting enough to tell the secret to his parents, then he won't be trusting enough to tell it to his wife. (First found by MS.)

1 Samuel 14:29-30. Jonathan: "See (*reu*): because (*ki*) I tasted a little of this honey, how (*ki*) mine eyes are brightened. How much more (*af*): if (*ki*) haply the people had eaten freely today of the spoil of their enemies which they found, then (*ki*) would there not have been a much greater slaughter among the Philistines?" {+**s**&} If someone eating a little honey is energized enough to have his eyes brighten, then people eating lots of food are energized enough to do that and much more. (First found by ZHK.)

1 Samuel 14:39. Saul: "Though (*ki*) it [the crime of eating despite the king's prohibition] be in Jonathan my son, (*ki*) he shall surely die." {-**s**} If my son is not worth enough to me to escape my killing him if he ate, then no one else is worth enough to me to do so. (NEW – First found by MS.)

1 Samuel 17:37. David: "The Lord who saved me from the paw of the lion and the paw of the bear [= innocent animals], He will [surely] save me from the hand of the Philistine [= willful enemy]." {+**p**} If David had spiritual credit enough to be saved from innocent creatures, then he has credit enough to be saved from evil ones. (First found by AS[159].)

1 Samuel 21:6. David: "'Of a truth, (*ki im*) women have been kept from us about these three days; when (*be*) I came out, at that time (*ve*) the vessels of the young men were holy, though (*ve*) it was but a common journey; how much more then (*ve-af ki*) to-day, when there shall be holy bread in their vessels [have we avoided women]?'"{+**p**} If we were virtuous enough to practice abstinence on a common journey, then we are virtuous enough to do so on a special day like today. (First found by AS.)

1 Samuel 23:3. David's men: "Behold (*hine*): here in Judah, we are afraid; how much more if (*ve-af ki*) we go to Keilah against the armies of the Philistines [will we be afraid]?" {+**p**} If we lack confidence enough that we feel fear while on our own territory (Judah), then we will lack confidence enough that we will feel fear when on enemy territory (Keilah). (First found by GR.)

2 Samuel 4:10-11. David: "When (*ki*): one told me saying, 'behold, Saul is dead' and (*ve*) he was in his own eyes as though he had brought good tidings, then (*va*) I took hold of him and (*ve*) slew him in Ziklag in the way of reward. How much more when (*af ki*) wicked men have slain a righteous man in his own house upon his bed, then now (*ve-atah*) shall I not (*ha-lo*) require his blood of your hand and (*ve*) take you away from the earth?" {+**s**} If someone who merely announced the death of Saul, David's respected adversary, was judged wicked enough to deserve execution, then the people who actually killed a respectable man, Ish-bosheth, the son of Saul, who did David no harm, must be judged wicked enough to deserve execution. (First found by ZHK.)

2 Samuel 11:11. Uriah to David: "The ark, and Israel, and Judah, abide in booths; and my lord Joab, and the servants of my lord, are encamped in the open field; shall (*va*) I then go into my house, to eat and to drink, and to lie with my wife?" {-**s**} If the rest of the army etc. are not worthy enough to merit the listed pleasures, then I am not worthy enough to merit such pleasures. (NEW – First found by MS.)

2 Samuel 12:18. David's servants: "Behold (*hine*): while (*be*) the child was yet alive, [David's sorrow was so great that] we spoke unto him, and (*ve*) he hearkened not unto our voice; how then (*ve-ekh*) shall we tell him that the child is dead, so that (*ve*) he do himself some harm?" {+**s**&} If David while his child still lived was sorrowful enough to be utterly distracted, then David now that the child has died will be sorrowful enough to cause himself some harm. (First found by ZHK.)

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[159] Actually, noticed by one Mark Leroux in 2001, though not formulated by him. Also, long before that, the 19th Cent. commentator Malbim thought that this verse has a fortiori intent, but he did not correctly formulate it.

2 Samuel 12:21. David's servants to him: "What (*mah*) is this thing that thou hast done? Thou didst fast and weep for the child, while it was alive; but when (*ve-ka-asher*) the child was dead, thou didst rise and eat bread?" {**+s&**} If David while the child still lived was sad enough to fast and weep, then David after the child died should have been (but was not) sad enough to do same and even more. (NEW – First found by ZHK.)

2 Samuel 16:11. David: "Behold (*hine*): my son, who came forth from my body, seeketh my life [still, I do not react]; how much more (*ve-af ki*): now [in the case of] this Benjamite, should [I order] you: let him alone and let him curse [me]; for the Lord has bidden him." {**+p**} If David was self-controlled enough to avoid reacting under attack from his own rebellious son, then David will be self-controlled enough to avoid reacting under attack from a more remote enemy. (First found by ZHK.)

1 Kings 8:27. Solomon: "But (*ki*) will God in very truth dwell on the earth? Behold (*hine*), heaven and the heaven of heavens cannot contain thee; how much less (*af ki*) in the case of this house that I have builded." This is repeated in 2 Chronicles 6:18. {**-s**} If the heavens are not big enough to contain God, then an earthly house is not big enough to do so. (First found by SYA.)

2 Kings 5:12. Naaman: "Are not (*ha-lo*) Amanah and Pharpar, the rivers of Damascus, better than all the waters of Israel? may I not (*ha-lo*) wash in them, and be clean?" {**-s**} If the rivers of Damascus, in which I have often bathed, were not good enough to heal me, the waters of Israel cannot be good enough to do it. (NEW – First found by MS.)

2 Kings 5:13. Naaman's servants: "Had the prophet bid thee do some great thing, wouldst thou not (*ha-lo*) have done it? how much more when (*ve-af ki*) he [merely] saith to thee: wash and be clean [you should do it]!" {**+s**} If the prophet making some difficult request would have seemed powerful enough in your eyes to succeed in healing you, causing you to obey him, then his making an easy request suggests he may be more powerful than you expected and indeed powerful enough to heal you, and should cause you to obey him. (First found by R.)

2 Kings 10:4. The rulers of Jezreel in Samaria: "Behold (*hine*): the two kings [Joram and Ahaziah] stood not before him [Jehu]; how then (*ve-ekh*) shall we stand [before him]?" {**-s**} If the two kings (who were powerful men) were not strong enough to resist Jehu, then we (who are comparatively weak) are not strong enough to do so. (First found by SYA.)

2 Kings 18:23-24. Rabshakeh (emissary of the king of Assyria): "Since (*ve*) thou puttest thy trust on Egypt for chariots and for horsemen, I will give thee two thousand horses if (*im*) thou be able on thy part to set riders upon them. [But you are not able to do even that, and so cannot hope to defeat us.] How then (*ve-ekh*) canst thou [without gift of horses] turn away the face of one captain, even of the least of my master's servants?" {**-s&**} This is repeated in Isaiah 36:8-9. If you had 2000 horses, you would not have enough power to defeat the Assyrian army, then without such a gift you surely do not have enough power to do so, not even to defeat a minor captain of it. (First found by MS.)

2 Kings 18:35. Rabshakeh, in the name of the king of Assyria: "Who are they among all the gods of the countries, that (*asher*) have delivered their country out of my hand, that (*ki*) the Lord should deliver Jerusalem out of my hand?" {**-s**} If the gods of the already-conquered other countries were not strong enough to prevent my conquests, then Israel's God is not strong enough for that. (NEW – First found by MS.)

Isaiah 1:3. God: "The ox knoweth his owner, and the ass his master's crib; but Israel doth not know, My people doth not consider." {**+s**} If the said animals are smart enough to obey their

masters, then the people should (in principle, though they are not so in fact) be smart enough to obey their master (God). (NEW – First found by MS.)

Isaiah 10:11. God reports the thoughts of Sennacherib (the Assyrian king): "Shall I not (*ha-lo*), as (*ka-asher*) I have done unto Samaria and her idols, so (*ken*) do to Jerusalem and her idols?" {**+s**} If Samaria's idols were weak enough to allow its and their own destruction, then Jerusalem's idols [being the same graven images] will be weak enough for the same outcome. (NEW – First found by MS.)

Isaiah 20:6. God quoting the inhabitants of around Ashdod: "Behold (*hine*), such is our expectation [viz. to be led away captive like Egypt and Ethiopia], whither we fled for help to be delivered from the king of Assyria; and how (*ve-ekh*) shall we [the Judeans, who are weaker than the other nations,] escape?" {**-s**} If Egypt and Ethiopia, nations to which we are wont to flee for help, were not strong enough to avoid defeat from the king of Assyria, then we Judeans are surely not strong enough to do so. (NEW – First found by SYA.)

Isaiah 36:8-9. Rabshakeh (emissary of the king of Assyria): "[Since] thou puttest thy trust on Egypt for chariots and for horsemen, I will give thee two thousand horses, if (*im*) thou be able on thy part to set riders upon them. [But you are not able to do even that, and so cannot hope to defeat us.] How then (*ve-ekh*) canst thou [without gift of horses] turn away the face of one captain, even of the least of my master's servants?" {**-s&**} (First found by MS.) This is the same narrative as 2 Kings 18:23-24.

Isaiah 36:20. Rabshakeh, in the name of the king of Assyria: "Who are they among all the gods of these countries, that (*asher*) have delivered their country out of my hand, that (*ki*) the Lord should deliver Jerusalem out of my hand?" {**-s**} If the gods of the already-conquered other countries were not strong enough to prevent my conquests, then Jerusalem's God is not strong enough for that. (NEW – First found by MS.) This is the same narrative as 2 Kings 18:35.

Isaiah 66:1. God: "The heaven is My throne, and the earth is My footstool; where (*eizeh*) is the house that ye may build unto Me? And where (*eizeh*) is the place that may be My resting-place?" {2 instances, both **-s**} There are clearly two antecedents and two consequents here; so. there are really *two* arguments, intertwined, though their intent is roughly the same[160]; viz.: (a) If His heavenly throne is not big enough to house God, then an earthly house is not big enough to do so; and (b) if the entire earth is not big enough as a resting-place for God, then a delimited place is not big enough for that. (First found by LJ.) Note that their narrative is comparable to 1 Kings 8:27 and 2 Chronicles 6:18.

Jeremiah 2:11. God: "Hath a nation changed its gods, which yet are no gods? But (*ve*) My people hath changed its glory for that which doth not profit." {**+s**} If the gods of other nations are credible and worthy enough to remain unchanged by their nations, then Israel's God is (i.e. should be) credible and worthy enough to remain unchanged by His nation. (NEW – First found by MS.)

Jeremiah 8:7. God: "Yea (*gam*), the stork in the heaven knoweth her appointed times; and the turtle and the swallow and the crane observe the time of their coming; but (*ve*) My people know not the ordinance of the Lord." {**+s**} If the said animals are cognitively efficacious enough to know certain facts relevant to them, then God's people are (i.e. should be) cognitively efficacious enough to know certain facts relevant to them. (NEW – First found by MS.)

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[160] I wrongly conflated them in AFL, as did Jacobs. They are two instances, not one.

Jeremiah 12:5. God: (a) "If (*ki*) thou hast run with the footmen and (*ve*) they have wearied thee, then how (*ve-ekh*) canst thou contend with horses [and not be wearied]?" (b) "and if (*u*) in the land of peace, thou dost [only just] feel secure; how then (*ve-ekh*) wilt thou do so [feel secure] in the wild country of the Jordan?" {2 instances, both **-p**} If you are not strong enough to cope with the easier challenges, then you are not strong enough to cope with the more difficult ones. (First found by EbJ.)

Jeremiah 25:29. God: "For (*ki*), lo (*hine*), I begin to bring evil on the city whereupon My name is called, and (*ve*) should ye [who are less virtuous] be utterly unpunished?" {**-s**} If those calling on my name are not absolved enough to escape my wrath, then you less virtuous folk are not absolved enough to escape my wrath. (First found by SYA.)

Jeremiah 45:4-5. God: "Behold (*hine*), that which I have built will I break down, and that which I have planted I will pluck up; and this in the whole land. And (*ve*) seekest thou [who is less valued] great things for thyself?" {**-s**} If the things I worked for are valued by Me not enough to escape being undone, then the things you work for are valued by Me not enough to escape being undone. (First found by SYA.)

Jeremiah 49:12. God: "Behold (*hine*), they to whom it pertained not to drink of the cup shall assuredly drink; and (*ve*) art thou [who is more guilty] he that shall altogether go unpunished?" {**+s**} If people who are not reprehensible are implicated enough to be punished, then people who are reprehensible are implicated enough to be punished. (First found by SYA.)

Ezekiel 3:4-7. God to Ezekiel: **"**Son of man, go, get thee unto the house of Israel, and speak with My words unto them. For (*ki*) thou art not sent to a people of an unintelligible speech and of a slow tongue, but to the house of Israel; not to many peoples of an unintelligible speech and of a slow tongue, whose words thou canst not understand. Surely, if (*im lo*) I sent thee to them, they would hearken unto thee. But (*u*) the house of Israel will not consent to hearken unto thee; for (*ki*) they consent not to hearken unto Me; for (*ki*) all the house of Israel are of a hard forehead and of a stiff heart." {**+s**} If dimmer peoples would be intelligent enough to listen to the words I send through you, then the house of Israel is (i.e. should be) intelligent enough to listen to My words. (NEW – First found by MS.)

Ezekiel 14:13-21. God: "Son of man, when (*ki*) a land sinneth against Me by trespassing grievously, and I stretch out My hand upon it… and send famine upon it… [Or] if I cause evil beasts to pass through the land… Or if I bring a sword upon that land… Or if I send a pestilence into that land… though Noah, Daniel, and Job, were in it, as I live, saith the Lord God, they shall deliver neither son nor daughter; they shall only deliver their own souls by their righteousness… How much more (*af ki*) when I send My four sore judgments against Jerusalem, the sword, and the famine, and the evil beasts, and the pestilence, to cut off from it man and beast." {**-p**} If such holy men lack sufficient spiritual credit to prevent the execution of each of the four negative decrees (penalties for sins committed) separately, then they lack enough to stop all four of these decrees together. (First found by R/AS.)

Ezekiel 15:5. God: "Behold (*hine*): when (*be*) it [the vine-tree] was whole, it was not meet for any work; how much less when (*af ki*) the fire hath devoured it and (*ve*) it is burned, shall it then (*ve*) yet be meet for any work?" {**-s**} If when whole the vine-tree was not in good condition enough to be useful; then now when damaged it is not in good condition enough to be useful. (First found by GR.)

Ezekiel 33:24. God: "They that inhabit those waste places in the land of Israel speak, saying: Avraham was one, and he inherited the land; but (*va*) we are many; the land is given us for

inheritance." {+**s**} If one man is important enough to inherit the land, then many men are important enough to inherit the land. (Obviously, though God is reporting this argument, He is not its author. It is not very credible, and rightly rebutted in the verses 25 and 26: it is not numbers but moral worth that makes possible inheritance of the land.) (First found by R.)

Jonah 4:10-11. God: "Thou hast had pity on the gourd, for which thou hast not laboured, neither madest it grow, which came up in a night, and perished in a night; and (*va*) should I not have pity on Nineveh, that great city, wherein are more than sixscore thousand persons that cannot discern between their right hand and their left hand, and also much cattle?" {+**s**} If a mere gourd etc. can be appreciated enough to be cared for (as by Jonah), then a great city etc. can be appreciated enough to be cared for (by God). (First found by MS.)

Psalms 25:8-9. "Good and upright is the Lord; therefore (*al ken*) doth He instruct sinners in the way. He guideth the humble in justice; and He teacheth the humble His way." {+**p**} The proposed a fortiori reading seems to be: If God is good and upright enough to instruct sinners in the way; then He is good and upright enough to guide the humble in justice and teach him His way. (NEW – First found by ZHK.)

Psalms 78:20. Asaph: "Behold (*hen*): He struck a rock, then (*ve*) waters flowed and (*u*) streams burst forth. In that case (*gam*): bread He can give; is there any doubt that (*im*): He will prepare meat for His people?" {+**p**} If God is powerful enough to draw water from a rock[161], then He is powerful enough to feed His people with bread and meat. (First found by ZHK.)

Psalms 94:9-10. Moshe: "He who implanted the ear, does He not (*ha-lo*) hear?" "If (*im*) He formed the eye, does He not (*ha-lo*) see?" "He who chastises nations, does He not (*ha-lo*) reprove [the individual]?" (3 instances, all 3 {+**p**}) If God is powerful enough to implant the ear and form the eye, then He is powerful enough to hear and see. If God is powerful enough to chastise nations, then He is powerful enough to reprove individuals. (First found by AS.)

Proverbs 11:31. Solomon: "Behold (*hen*): the just man shall be recompensed on earth: how much more (*af ki*) the wicked and the sinner [shall be recompensed on earth]." {+**s**} If the just man is imperfect enough to be recompensed on earth, then the wicked and sinner are imperfect enough to be recompensed on earth. (First found by GR.)

Proverbs 15:11. Solomon: "Hell and destruction are before the Lord; how much more (*af ki*): the hearts of the children of men [are before the Lord]?" {+**p**} If God is powerful enough to look into hell and destruction, then He is powerful enough to look into people's hearts. (First found by R.)

Proverbs 17:7. "Overbearing speech becometh not a churl; much less (*af ki*) do lying lips a prince." {+**p**} If wrongful speech is unbecoming enough in a churl, then it is unbecoming enough in a prince. (NEW – First found by R.)

Proverbs 19:7. Solomon: "All the brethren of the poor do hate him, how much more (*af ki*): do his friends go far from him?" {+**p**} If the poor man is disliked enough that his brothers avoid him, then he is disliked enough that his friends avoid him. (First found by R.)

Proverbs 19:10. Solomon: "It is not seemly for a fool to have luxury; how much less (*af ki*): for a servant to have rule over princes [would be seemly]." {+**s**} If for a fool to have luxury is

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[161] The subject of "he struck a rock" could be Moses, but the cause of the water gushing from it must be God. Likewise, it is God that provides bread and meat. This is obvious from the Torah account (Ex. 17:6, 16:12).

inappropriate enough to be unseemly, then for a servant to have rule over princes is inappropriate enough to be unseemly. (First found by R.)

Proverbs 21:27. Solomon: "The sacrifice of the wicked is an abomination; how much more (*af ki*): brought with a bad intention?" {**+s**} If the sacrifice of a wicked person brought with a good intention is abominable enough to be rejected, then the sacrifice of a wicked person brought with a bad intention is abominable enough to be rejected. (First found by R.)

Job 4:18-19. Eliphaz the Temanite: "Behold (*hen*): He puts no trust in His servants, and (*u*) His angels he charges with folly; how much more (*af*): those who dwell in houses of clay, whose foundation is in the dust [does He distrust and charge with folly]?" {**+p**} If God is perspicacious enough to judge His servants/angels as untrustworthy and unwise, then He is perspicacious enough to judge mere human beings as untrustworthy and unwise. (First found by SYA.)

Job 9:13-14. Job: "God will not withdraw His anger; the helpers of Rahab did stoop under Him. How much less (*af ki*) shall I answer Him, and choose out my arguments with Him?" {**-s**} If Rahab's helpers were not worthy enough to argue with God, then Job is not worthy enough to do so. (First found by R.)

Job 15:15-16. Eliphaz the Temanite: "Behold (*hen*): He puts no trust in His holy ones; and (*ve*) the heavens are not clean in His sight. How much less (*af ki*): one who is abominable and filthy, man, who drinks iniquity like water [does He trust or consider clean]!" {**+p**} If God is demanding enough to judge His holy ones as untrustworthy and the heavens as unclean, then He is demanding enough to judge mere human beings as untrustworthy and unclean. (First found by R.)

Job 25:5-6. Bildad the Shuhite: "Behold (*hen*): even the moon has no brightness, and (*ve*) the stars are not pure in His sight; how much less (*af ki*): man, who is a worm, [is bright and pure in His sight]?" {**+p**} If God is perfectionist enough to judge the moon as obscure and the stars as impure, then He is perfectionist enough to judge mere human beings as obscure and impure. (First found by SYA.)

Job 35:13-14. Elihu: "Indeed (*akh*) God will not hear vanity, neither will the Almighty regard it. Yea, when (*af ki*) thou sayest thou canst not see Him – the cause is before Him; therefore, wait thou for Him." {**+s**} If God ignores the vain ones for a while, then He will ignore the doubters for as much (or maybe more) time. (NEW – First found by R.)

Esther 9:12. Ahasuerus says: "In Shushan the capital, the Jews have slain and destroyed five hundred men and the ten sons of Haman; in the rest of the king's provinces, what (*meh*) have they done? [i.e. surely many more!]" {**+s&**} If the Jews in Shushan have found and destroyed as many as 500 anti-Semites, then the Jews in the provinces have found and destroyed many more than 500 of their enemies. (Doubtfully a fortiori intent, in my view; but I kept the case because it is traditionally taken for granted.) (First found by EbJ.)

Daniel 2:9. Nebuchadnezzar: "Thus (*lahen*): tell me the dream, and (*ve*): I shall know that you can declare its interpretation to me [since it is more difficult to tell it than to interpret it]." {**+p**} If Daniel is powerful enough to tell the dream, then he is powerful enough to interpret it. (First found by R.)

Nehemiah 13:26-27. Nehemiah: "Did not Solomon king of Israel sin by these things? yet among many nations was there no king like him, and he was beloved of his God, and God made him king over all Israel; nevertheless even (*gam*) him did the foreign women cause to sin. Shall we then (*ve*) hearken unto you to do all this great evil, to break faith with our God in marrying foreign women?" {**+s**} If king Solomon, who was not very weak, was weak enough to be brought to sin by foreign

women, then we, who are much weaker, are weak enough to be brought to sin by foreign women. (First found by R.)

2 Chronicles 6:18. Solomon: "But will God in very truth dwell with men on the earth? Behold (*hine*), heaven and the heaven of heavens cannot contain Thee; how much less (*af ki*): in the case of this house that I have builded?" {**-s**} If the heavens are not big enough to contain God, then an earthly house is not big enough to do so. (First found by ZHK.) This is the same narrative as 1 Kings 8:27.

2 Chronicles 32:15. Sennacherib, king of Assyria (through his messengers) says: "For (*ki*): no god of any nation or kingdom was able to deliver his people out of my hand, and out of the hand of my fathers; likewise therefore (*af ki*) shall your God [presumed by the speaker as no different from other gods] not be able to deliver you out of my hand." {**-s**} If other national gods were not powerful enough to deliver their respective nations, then the God of Judah is not powerful enough to deliver his nation. (This of course wrongly equates God with non-gods, but it is how the Assyrian king thinks.) (First found by AS.)

4. MISHNA and 2 TALMUDS

There may be mentions of some Biblical a fortiori arguments in the **Mishna** (redacted and closed ca. 200 CE by R. Judah HaNasi, abbrev. M) – but so far, I have not looked into the matter (nor has anyone else, to my knowledge).

I published ten years ago[162] a list of 46 a fortiori arguments in the Mishna, based largely on work done by a Prof. Alexander Samely, with a few (minor) modifications by me. Looking now at this list, I see no passage that is of interest to the present research – that is, none that mentions Biblical a fortiori argument. The arguments listed are a fortiori argument, all right, but not Biblical; they are all Mishnaic. It is still possible that elsewhere in the Mishna there is reference to some Biblical verses as having a fortiori intent. If so, we need to eventually find them and place them first in our chronological listing.

Regarding the Mishnaic era, the 2nd Cent. CE Tanna **R. Eliezer ben R. Jose HaGelili**[163] (abbrev. EbJ) deserves mention. In a *baraita* on the 32 hermeneutic rules of Biblical interpretation he distinguishes between explicit (*meforash*) and implicit (*satum*) *qal vachomer* arguments (rules 5 and 6). He gives two Biblical examples of each[164]; namely, for the explicit, **Jeremiah 12:5 and Esther 9:12**, and for the implicit, Psalms 15:4-5. The two explicit cases, much later, appear in the *Genesis Rabbah* list of ten (see below), and the implicit cases are, later still, commented on by Rashi (see below).

It is notable that one of the two examples of 'explicit' a fortiori argument given here is Esther 9:12, which is arguably more credibly 'implicit'. In this case, the term *meh* (what) is interpreted as saying 'how much more', although the simplest reading is that it is merely intended as a question mark (i.e. *how many* more anti-Semites were slain?), without prejudice as to whether the answer is more

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[162] See AFL, appendix 2 (see also chapter 23).

[163] HaGelili means the Galilean.

[164] I got this information from L. Jacobs, who draws attention to the mention of these Biblical a fortiori arguments in the said *baraita* in his essay "The Qal Va-Homer Argument in the Old Testament," (in his book *Rabbinic Thought in the Talmud*, on p. 111). Note however that the given examples might not be part of the original *baraita*, but part of a later commentary added to it. See: https://jewishencyclopedia.com/articles/2500-baraita-of-the-thirty-two-rules. In Heb. hermeneutic rules are known as *midot*.

or less (were slain). The fact that an alternative reading of the text is possible is sufficient, in my view, to classify it as implicit rather than explicit *qal vachomer*. Nevertheless, I have kept this case in the list of explicit cases because it is so very widely accepted as such in rabbinic tradition.

Next chronologically is probably the Midrash *Genesis Rabbah* (abbrev. GR). It was probably started by the Amora R. Oshaya ben Hama, who flourished in the 3rd Cent. CE, including Tannaitic material with writings by that author, and then expanded over time with input by other editors and authors, until it was effectively closed in ca. 300-500 CE[165].

As regards the two Talmuds, to date I know of only one or two Biblical a fortiori arguments mentioned in each of these documents. Further research is of course needed to draw up exhaustive lists of such mentions in those two documents (and indeed, in others of that era).

The **Jerusalem Talmud** (Heb. Talmud *Yerushalmi*, abbrev. JT), in tractate *Sanhedrin* 10:1 (27d), mentions[166] the *qal vachomer* of **Genesis 4:24**. Going there, I see it says: "One argument *de minore ad majus*: For Cain would be avenged sevenfold, etc."[167].

The **Babylonian Talmud** (Heb. Talmud *Bavli*, abbrev. BT). We have found to date two cases. In BT tractate *Sanhedrin* 108a, a *qal vachomer* is perceived in **Genesis 6:9**. The relevant passage there is as follows[168]:

> "These are the generations of Noah: Noah was a just man, and perfect in his generations. R. Johanan said: In his generations, but not in other generations[169]. Resh Lakish maintained: [Even] in his generations — how much more so in other generations[170]."[171]

I analyze this a fortiori argument further on in the present essay, in the section (chapter 6) on Rashi[172]. I there assess it as implicit, not explicit.

Tractate *Baba Qama* 25a mentions the *qal vachomer* of **Numbers 12:14**, in relation to the Mishna *Baba Qama* 2:5, which introduces the concept and principle of *dayo* (sufficiency) for rabbinic a fortiori inferences. BT comments on this Mishna as follows, citing the said Torah passage:

> "Does R. Tarfon really ignore the principle of *dayo*? Is not *dayo* of Biblical origin? As taught: How does the rule of *qal vachomer* work? And the Lord said unto Moses: 'If her father had but spit in her face, should she not be ashamed seven days?' How much the more so then in the case of divine [reproof] should she be ashamed fourteen days? Yet the number of days remains seven, for it is sufficient if the law in respect of the thing inferred be equivalent to that from which it is derived!"

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165 https://jewishencyclopedia.com/articles/3056-bereshit-rabbah.

166 This is pointed out by Louis Jacobs in fn. 3 of his said essay.

167 https://www.sefaria.org/Jerusalem_Talmud_Sanhedrin.10.1.10?lang=bi&with=all&lang2=en.

168 English Texts based on the Soncino Talmud Edition, online at Halakhah.com.

169 Footnote in the Soncino ed.: "So Noah: by comparison with the rest of his generation, who were exceptionally wicked, he stood out as a righteous man; in other generations he would not have been superior to the average person."

170 Footnote in the Soncino ed.: "Thus, if Noah was righteous even when his entire surroundings were evil, how much more so had he lived amongst righteous men!"

171 The passage continues: "R. Hanina said: As an illustration of R. Johanan's view, to what may this be compared? To a barrel of wine lying in a vault of acid: in its place, its odour is fragrant [by comparison with the acid]; elsewhere, its odour will not be fragrant. R. Oshaia said: As an illustration of Resh Lakish's view, to what may this be compared? To a phial of spikenard oil lying amidst refuse: [if] it is fragrant where it is, how much more so amidst spices!"

172 We found this case through Rashi, and the Rashi commentary through Hirschensohn. The English translation of the Rashi commentary in Chabad.org cites, as well as San. 108a, Gen. Rabbah 30:9 and Tan. Noach 5.

The Gemara goes on at length on this matter. I have analyzed its discourse in great detail, and very critically, in my work *A Fortiori Logic* (AFL, chapters 7 and 8), and need not repeat myself here. Briefly put, this commentary fallaciously (contrary to logical theory and to actual practice in the rest of the Talmud) assumes that *all* a fortiori argument is a crescendo in form; moreover, it thus refers to only one of the two arguments formulated by R. Tarfon in the said Mishna (blithely ignoring the second, which is definitely not 'proportional'). Evidently, the Gemara is here trying to ground the Mishna's *dayo* in the said Biblical passage; but that is logically impossible because the *dayo* principle concerns inference from a Torah/Divine law to a rabbinic/human one, whereas the Numbers narrative is entirely Divine.

To repeat, there may be other mentions of Biblical a fortiori arguments in these two massive documents, but we have yet to look for them systematically.

Gen. 4:24 and Num. 12:14, found in JT and BT respectively, also appear in GR (though only the latter one as part of the list of ten). Gabay thinks the GR list probably came first, in which case it may have influenced the Talmuds. It is possible, however, since the Talmuds were completed (according to historians) respectively in ca. 350-400 CE and ca. 500 CE, but took centuries to develop (as of the Mishna's completion in 200 CE), that parts of them preceded GR, or overlapped with it. In that event, the authors of GR may have had privileged access to early drafts of the Talmuds, or perhaps heard of them by word of mouth. Indeed, it is even possible that some of the authors of GR were among the authors of JT and BT. So, it is very difficult for us to hypothesize the chronological order of these documents. In consequence, I am treating GR as the earlier document, but also treating findings in JT and BT as found independently of it.

It should be relatively easy for us to find cases of Biblical a fortiori argument in the Mishna and the two Talmuds if there exists *an index* with such information. In such case, we could for a start immediately check out, for each of the 72 Biblical verses already known to us, whether it is indexed in any of these three documents. We would then examine the Mishnaic or Talmudic passage concerned, and see whether it explicitly mentions, or even merely alludes to, the a fortiori character of the indexed Biblical verse. After that, we might pursue the matter further, by painstakingly checking out every other index entry (there may be hundreds of them) looking for additional cases of Biblical *qal vachomer* not yet known to us.

5. GENESIS RABBAH and after

In the Midrash *Genesis Rabbah* (Heb. *Bereshith Rabbah*, "Major Exegeses of Genesis" abbrev. GR)[173], which dates from ca. 300-500 CE, R. Ishmael (2nd Cent. CE), is quoted (GR 92:7) as saying that **Genesis 44:8** is "one of the ten *qal vachomer* arguments given in the Torah" (the term 'the Torah' being here understood as 'the Tanakh', as becomes evident). Then nine more instances are listed in the received text, namely: **Exodus 6:12, Numbers 12:14, Deuteronomy 31:27, 1 Samuel 23:3, Jeremiah 12:5 (2 instances), Ezekiel 15:5, Proverbs 11:31, and Esther 9:12**. This is the oldest known claim concerning the number of a fortiori arguments in the Jewish Bible. We may and will assume that all subsequent commentators on Biblical a fortiori argument were aware of this list (even if some partly disagreed with it).

[173] https://en.wikipedia.org/wiki/Genesis_Rabbah. The traditionally presumed author (or maybe just initiator) of this book is R. Oshaya ben Hama. The Heb. text can be downloaded at https://hebrewbooks.org/14385.

I have analyzed these ten classic cases of *qal vachomer*, and demonstrated their a fortiori status, in my previous works on this subject[174]; they are, furthermore, listed and briefly clarified in chapter 3 of the present essay. I will not go into detail regarding these, or indeed other, cases listed in the present paper if I have already sufficiently dealt with them in my past works; the purpose of the present essay is to deal with 'new' cases (that is, cases only brought to my attention in the course of the present study).

This list of ten cases is put in doubt by some commentators, as we shall see further on. For a start, some have contested the equal antiquity of the nine additional cases (besides the first mentioned Gen. 44:8), suggesting that they may have been added afterwards in the way of an editorial, or even as a later gloss[175], since R. Ishmael is not explicitly identified as their author. Furthermore, many commentators have realized that there are, in fact, more than ten instances of *qal vachomer* to be found in the Tanakh; and some have sought to explain this surprising yet undeniable fact, somehow.

It is important to note that R. Ishmael is quoted as saying *without qualification* that there are ten (i.e. precisely ten) instances of *qal vachomer* in the Tanakh. He does not suggest that the ten he has in mind (whichever set of ten is ascribed to him) are, in some way, the most significant or the most informative; and imply thereby that he is aware of other instances – whether a few or many more. Surely, if he (or whoever wrote this passage of GR in his name) was aware of additional instances, he would naturally have said so, and either mentioned their quantity, or listed them, or explained (however briefly) why he did not do either. For this reason, all attempts to claim that the number ten was intended as selective in some way, and that in fact R. Ishmael (or whoever) knew of all other instances, is engaged in anachronistic projection of hypothetical information into the given text – i.e. in fiction-writing, not in serious scholarship.

It is clear, then, that the number ten was originally meant as exhaustive; moreover, the number ten was apparently considered as exact throughout antiquity and well into the medieval period. As we shall see, Rashi, in his running commentary to the Tanakh, also mentions the number ten, even as he elsewhere explicitly draws attention to some more cases. Evidently, the rabbis were emotionally attached to the round number of ten from the beginning and very reluctant to deviate from it[176].

And yet... two other a fortiori arguments are mentioned in GR, namely **Genesis 4:24** and Genesis 17:20-21[177]. Thus, GR apparently mentions 12 cases in all. Assuming a single author is responsible for the whole work, this contradicts the GR 92:7 claim to only ten a fortiori arguments in all. Granting that these two cases were not among the original list of ten, we can well ask why they were excluded from it. It seems that although Gen. 4:24 is technically a valid and explicit case, the fact that the argument is spoken by an unsavory character (viz. Lemekh) and may be an attempt to somewhat justify an evil act (viz. killing) disqualified it on ethical grounds (in rabbinical eyes) from inclusion in the listing[178]. As for Gen. 17:20-21 its exclusion, at whatever stage, was probably due to the recognition that it is implicit rather than explicit.

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174 Namely in JL chapters 5-6, and in AFL, appendix 1. See also my essay "The 46 *Qal vachomer* Arguments in the Tanakh" (which was drawn from AFL in 2023).

175 See Jacobs's previously cited essay; in fn.2, he refers to a lengthy note by Theodor-Albeck suggesting that the given list is a gloss.

176 They could equally well have focused on seven cases or thirteen cases, or any other number popular in their midst; and we would have been asking the same questions!

177 I base this information on Jacobs. See footnotes 3 and 4 in his essay.

178 Allen Wiseman (in his doctoral 2010 thesis, *A Contemporary Examination of the A Fortiori Argument Involving Jewish Traditions,* pp. 174-6) suggests that this case falls under a rabbinical category of "evil" *qal vachomer*.

We should therefore focus on Gen. 4:24, and ask the obvious question: why was it not included in the classic list of ten? Many answers are conceivable. GR may have been a compilation of different documents by the same or different authors; and the editor(s) who put them together may not have noticed the inconsistency, or perhaps did not want to tamper with the merged text even if such abstinence meant that an inconsistency would remain. Even a single author might have first found and listed the ten cases; and then at a later time discovered the eleventh case but was loathe to rewrite the list. (Remember, authors in those days did not have word processors like we do!) Or the author may have known the Gen. 4:24 case before drawing up his list of ten, but then accidentally overlooked it when composing his list; or maybe he left it out intentionally, so as to have a list of exactly ten cases (a nice, round number). Still other explanations might be offered.

Genesis 4:24 reads: "If (*ki*): Cain shall be avenged sevenfold, then (*ve*): Lemekh [shall be avenged] seventy and seven-fold." Lemekh is apparently saying: An intentional killer is more abhorrent than an unintentional one (tacit major premise); whence it follows that if an intentional killer is not abhorrent enough to be punished immediately, then an unintentional killer will remain unpunished for a much longer time. This is *a negative subjectal a crescendo argument* (**-s&**); and it is reasonably explicit (one could hardly read it otherwise).

Genesis 17:20-21 reads: "And as for Ishmael, I have heard thee; behold (*hine*), I have blessed him, and will make him fruitful, and will multiply him exceedingly; twelve princes shall he beget, and I will make him a great nation. But (*ve*) My covenant will I establish with Isaac, whom Sarah shall bear unto thee at this set time in the next year."

Gabay notes, concerning these verses, the need to see Rashi in the name of Midrash Rabbah (GR 47:5), and Maharzu on the Midrash "that the reason why a KV is evident is because there is a repetition here of what was already said in v19," which reads: "And God said, 'Indeed (*aval*), your wife Sarah will bear you a son, and you shall name him Isaac, and I will establish My covenant with him as an everlasting covenant for his seed after him."

Rashi mentions the Midrash Rabbah passage (47:5) with the *qal vachomer* reading for v. 21 in his commentary to v. 19:

> "'And My covenant.' Why is this written? Is it not already written (verse 9): 'And you shall keep My covenant, you and your seed, etc.?' But because He said (verse 7): 'And I will establish, etc.,' one might think that the sons of Ishmael and the sons of Keturah are included in the establishment [of the covenant]. Therefore, Scripture states: 'And I will establish My covenant with him,' and not with others. Now, why does it say [again in verse 21]: 'But My covenant I will establish with Isaac?' This teaches us that he was holy from the womb. Another explanation [for the repetition of verse 19]: Said Rabbi Abba: Scripture here derives an a fortiori (*qal vachomer*) conclusion regarding the son of the mistress from [what is written regarding] the son of the handmaid. It is written here: 'Behold I have blessed him, and I will make him fruitful, and I will multiply him.' This refers to Ishmael. How much more so (*qal vachomer*), 'But My covenant I will establish with Isaac!' (*Gen. Rabbah* 47:5)."

The a fortiori argument proposed in GR is thus: If Ishmael, the son of the handmaid, will be blessed with fruitfulness and multiplication (minor premise), then *qal vachomer* Isaac, the son of her

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He does not list other members of this class, but it is easy to suggest some; for example, 2 Chronicles 32:15 would be one. Needless to say, even if an a fortiori argument is "evil" in this sense, it retains its logical status as a valid a fortiori argument. However distasteful or incredible the content, it is the form that counts not the content.

mistress, will be blessed equally much, and moreover with My covenant (conclusion). The tacit major premise here is: Isaac (P), the son of Sarah, is more worthy (R) than Ishmael (Q), the son of Hagar; and the tacit premise of proportionality is that the more worthy one is, the more blessed (S). So, the argument form is positive subjectal a crescendo (+**s&**).

The purpose of the a fortiori reading is, first, to argue that Isaac is, just as Ishmael is (in v. 20), being blessed with descendants; second, to argue that Isaac is being blessed (in v. 21) more than Ishmael. But the inference that Isaac will have descendants is clearly not needed here, since v. 19 already *explicitly* informed us that Isaac will have "seed after him" who will also be subject to the "everlasting" covenant. The intensification of the pure conclusion into an a crescendo conclusion is also clearly not needed here since Isaac is already *explicitly* blessed with God's covenant in v. 19 and again in v. 21, whereas Ishmael is not (there is no mention of him in these two verses or elsewhere in this regard).

So, there is no need for the proposed inferences since they are textually given. Therefore, contrary to the above-mentioned comment of Maharzu (i.e. Einhorn), v. 19 does not make the *qal vachomer* interpretation "evident," but *redundant*! What is not necessary for the comprehension of a text is a useless embellishment. Why then is Isaac's blessing with the covenant mentioned twice here? We can refer to the Midrash for answers to this question. According to it, v. 19 serves to exclude Ishmael and the sons of Keturah from it (by a *davka* reading): "'And I will establish My covenant with him,' and not with others;" and v. 21 serves to explain why the blessing is said before Isaac was born, viz. because: "he was holy from the womb."

That there are authoritative alternative interpretations to the a fortiori reading by R. Abba confirms my reasoned contention that this reading is implicit (*satum*) rather than explicit (*meforash*). It is noteworthy that the author of the GR 92:7 list of ten *qal vachomer* arguments in the Tanakh also saw the Gen. 17:20-21 *qal vachomer* proposed in GR 47:5 as implicit, since he did not include it in his listing. Note also that *Avot de-Rabbi Nathan* does not count Gen. 17:20-21 in its list of five *qal vachomer* in the Torah, even though this case is mentioned in the earlier document (i.e. in GR). These absences show that I am not alone in my contention[179].

It should be noted, however, that ***Avot de-Rabbi Nathan*** (abbrev. ARN), a haggadic work (author uncertain) dating from the geonic era (ca. 650-950 CE)[180], counts Gen. 4:24 as one of the five *qal vachomer* arguments in the Torah (here meaning the Pentateuch), together with the four instances listed in GR (shown above), whereas it does not count Gen. 17:20-21 in this list[181]. This confirms that, even in relatively ancient times, the former case was recognized as valid and explicit, while the latter was not so viewed. But the main historical significance of ARN is that it groups Gen. 4:24 with the four Torah instances mentioned in GR.

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[179] Note that I previously, in AFL 16:4, made the same judgment, arguing that, while the argument can indeed be read into the text, the text can also be read more simply. Note also that, as I there point out, Louis Jacobs regards this case as "extremely doubtful."

[180] https://en.wikipedia.org/wiki/Avot_of_Rabbi_Natan. https://jewishencyclopedia.com/articles/355-abot-de-rabbi-nathan.

[181] Jacobs gives as references: "(version B) 44; *Gen. Rabbah* 4:24 and Gen. 17:20-1 (ed. Theodor-Albeck, p. 225) and Jerusalem Talmud *Sanh*. 10:1 (27d)." Note also, in passing, that ARN does not mention three other Torah *qal vachomer* cases, viz. Gen. 3:22, 11:6-7, 17:17, later advocated by Hirschensohn (but rejected by Jacobs and me as at best implicit).

To complicate matters further, the much later Midrashic work called ***Yalkut Shimoni*** (abbrev. YS), which means "[The] Gathering of Simon," compiled (authorship uncertain) in the medieval era, sometime between the 11th and 14th centuries[182], says that there are ten instances of *qal vachomer* in the Tanakh, but it apparently lists only nine instances, leaving out Ezek. 15:5[183]. This deficient list by the YS has given rise to some controversy. Objectively, it could simply have been an unintentional omission by the author, or more likely a scribal error at some later time. These are the most reasonable explanations of the omission. Since the *qal vachomer* implied by this verse is technically flawless, there was no logical reason to reject it.

But some commentators[184] have interpreted this omission as an *intentional rejection* of the Ezek. 15:5 case from the GR list of ten by the author of YS (implying it to have been a late addition); and even proposed a replacement for it. Personally, I do not agree with the latter opinion. In my view, if the YS author had been critical of the inclusion of Ezek. 15:5, he would have naturally said so and explained his rejection of the case (and maybe suggested another case in its stead); he would not just have ignored the case and effectively expected others to follow suit on his authority alone, without justification. Was such an explanation made and in time mislaid? It is easier to assume that the Ezekiel case itself was accidentally left out of the *Yalkut Shimoni* list, than to assume (as one would otherwise have to) that the required and missing explanation for its absence was what was lost accidentally!

The *Yalkut Shimoni*, by the way, is one of the commentaries that sought to explain why GR only listed ten cases of *qal vachomer* in the Tanakh, even though there are evidently very many more. Gabay informed me that YS proposes that each of the ten cases listed teaches us some 'novelty' (*chiddush*), some lesson that would have remained unknown without the given *qal vachomer*. For examples: but for Exodus 6:12 we might have supposed that Pharoah would listen to Moses; from Numbers 12:14 we learn the *dayo* principle; and so forth.

To my mind, this explanation is far too undefined and unsubstantiated. To be credible, every case included in the GR list of ten would have to be clearly and convincingly explained in the proposed manner; and moreover, every case *not* included in that list would have to be shown *not* to have this didactic property. Since the differentia claimed is very vaguely identified (effectively, as 'some sort of lesson'), the task is in fact impossible to apply in practice[185]. That this research method was not used by the author of that commentary is obvious anyway from the fact that he does not propose (as he would need to) a complete list of all the cases of *qal vachomer* in the Tanakh, let alone a detailed analysis of all cases (both the included and the excluded) proving his farfetched thesis.

Clearly, this theory was just formulated in the way of a speculative trial balloon, a mere act of faith and hope. It was just an expression of the author's wish to find some solution to the problem at hand, one which would confirm the putative infallibility and omniscience of the ancient rabbinical author of *Genesis Rabbah* concerning *qal vachomer* arguments in the Tanakh (albeit his short listing of only ten cases). The truth is that nothing really distinguishes the ten cases listed in GR 92:7 from all other cases of *qal vachomer* in the Tanakh. Neither logical form, nor language used,

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[182] https://en.wikipedia.org/wiki/Yalkut_Shimoni. https://jewishencyclopedia.com/articles/15057-yalkut#anchor4.

[183] https://www.sefaria.org/Yalkut_Shimoni_on_Torah?tab=contents. In section 132, a commentary on 1 Sam. 23:3.

[184] Notably, the 19th Cent. commentator Zeev Wolf Einhorn, as will be seen further on in the section devoted to him.

[185] In truth, every Biblical verse teaches something; so, such a vague criterion is useless. A more precise criterion has not been specified by YS.

nor content, nor utility, can be pointed to as distinctive of these ten cases, and as requiring all ten cases. It follows that we can only honestly and safely assume that the author of GR only knew of the cases he actually listed (at the time he listed them).

A thought occurs to me: maybe this has something to do with the Talmud? Maybe the list of *qal vachomer* arguments in GR is a list of Biblical instances of a fortiori mentioned as such in the Talmud? We have seen above that each of the two Talmuds mentions at least one Biblical a fortiori argument. Maybe there are other such mentions in JT and/or BT; maybe all ten of those in GR. It is possible, assuming chronological order does not preclude this hypothesis, that the GR listing is intended to reflect Talmudic information. This issue is certainly worth investigating.

It would anyway (as already suggested) be interesting to research, for its own sake, all mentions, explicitly or by allusion, of Biblical *qal vachomer* in both Talmuds. There may be less or more of them than the 10 or 12 cases mentioned in GR. We might even learn of new cases, cases which no one has rediscovered since then....

Some of the assumptions of the above discussion will be considerably overturned later in the course of the present study, when we examine (in chapter 11) the work of Chaim Hirschensohn, because he apparently had the genial idea of reading R. Ishmael's statement (in GR 92:7) that there are "ten *qal vachomer* arguments given in the Torah" as meaning literally in *the Torah* (the *Chumash*), and not as everyone else has assumed for centuries as meaning in *the Tanakh*. Moreover, Hirschensohn managed to muster ten Torah instances (and indeed more than ten), thus confirming his interpretation. We will look into and discuss this reset in more detail later in this volume, when we have more data at hand.

6. R. Shlomo Yitzchaki (RASHI)

R. Shlomo Yitzchaki, better known by his acronym Rashi (abbrev. R, b. 1040 in Troyes, France, d. 1105)[186], wrote a running commentary on the Tanakh[187] in which he occasionally draws attention to a fortiori discourse, using the Hebrew expressions *qal vachomer* and/or *kol sheken* as flags.

I do not know whether anyone has previously made a list of all the cases he flags; but it occurred to me that I could, at least, check whether any or all of the 67 cases we identified thus far through other authors in the present study (listed in chapter 3) were flagged by him earlier. This approach would not necessarily reveal all the cases known to Rashi, since he might have spotted cases not on our list (and so not yet known to us). Moreover, he might also elsewhere, in his running commentary on the Talmud or some other work, have revealed some cases unknown to us. But we could at least by this means identify which cases *among those we found* should be attributed as 'historic firsts' to Rashi, since he lived and wrote long before the other writers we have investigated.

And indeed, it turns out that Rashi discovered at least **14 instances** of *qal vachomer* in the Tanakh before anyone else we know about (14 instances besides the 11 instances given in Midrash GR, of course). Namely: **Genesis 6:3; 2 Kings 5:13; Ezekiel 14:13-21, 33:24; Proverbs 15:11, 17:7,**

[186] https://en.wikipedia.org/wiki/Rashi. https://jewishencyclopedia.com/articles/12585-rashi-solomon-bar-isaac.

[187] https://www.chabad.org/library/bible_cdo/aid/63255/jewish/The-Bible-with-Rashi.htm.

19:7, 19:10, 21:27; Job 9:13-14, 15:15-16, 35:13-14; Daniel 2:9; Nehemiah 13:26-27. These, then, are all to be categorized as 'historic firsts'. Rashi flags verses as a fortiori argument by use of the terms *qal vachomer* (or *q"v*) and/or *kol sheken* (or *k"sh*)[188], usually only on the basis of the verse concerned involving a typical expression like *af ki*.

Note that the listed Job 35:13-14 case was not one of the 67 cases found by later authors, but was found by me in the course of the present research (see below).[189]

In most cases, Rashi is content to flag arguments in this way, without analyzing them further; that is, without sufficiently identifying the terms and propositions involved, some of which may be located in verses other than the one he is directly commenting on. Sometimes, these missing details are obvious enough; but sometimes, a more detailed presentation would have been didactically useful. This shows that Rashi's comprehension of a fortiori argument was intuitive rather than formal (which of course is not surprising, as formal studies of a fortiori argument developed much later historically in Jewish circles).

Note that Rashi proposes three more cases as *qal vachomer*, but these I reject for reasons given below.

Derived cases. All 14 cases of *qal vachomer* above attributed to Rashi can be regarded as 'historic firsts' until and unless some earlier commentator is found to have mentioned them. On the other hand, the 11 cases mentioned in GR can all be regarded as 'derived', i.e. as learned by Rashi from that (or some other) earlier source.

Concerning the latter cases, the following is worth noting. (a) Rashi does not offer any comment on three of those cases, namely: Deut. 31:27, Jer. 12:5b, and Esth.9:12; nevertheless, it is safe to assume that he knew and approved of these GR cases. (b) Rashi does in his commentary explicitly flag as a fortiori seven cases, viz.: Gen, 4:24, Gen. 44:8, Ex. 6:12, Num. 12:14, Jer. 12:5a, Ezek.15:5, and Prov. 11:31. The remaining case of 1 Sam. 23:3 is flagged indirectly: there is no comment *ad loc*., but the comment to Ezek. 14:21 refers back to it[190] as also a fortiori.

(c) In one case, namely Gen. 44:8, Rashi explicitly states: "This is one of the ten *qal vachomer* [arguments] spoken in the Torah, and all of them are enumerated in *Bereshith Rabbah*;" thus showing his acquaintance with GR, and indeed repeating its primary focus on Gen. 44:8 and its peculiar use of the word 'Torah' in the sense of 'Tanakh'. Rashi's comment to Ex. 6:12 reads only: "This is one of the ten *qal vachomer* [arguments] that are in the Torah;" without mentioning GR (although there is a reference to GR 92 in brackets, which I suspect is merely an editorial gloss). In the remaining nine GR cases, there is neither mention of GR nor of the number ten. Thus, Rashi clearly specifies only two cases as being among the ten listed in GR (and both are Torah cases).

Rashi offers more lengthy comments on the contents of some of the 11 GR cases, namely on Gen. 4:24, Ex. 6:12, Num. 12:14, Jer.12:5 (a &b), Ezek. 15:5, and Prov. 11:31. Some of his detailed a fortiori readings differ somewhat, but not radically, from mine. For example, he interprets in Jer. 12:5a as saying: "If even your brethren the priests come to kill you, surely the princes of Judah will rise up against you to kill you," thus giving more specific meanings to the terms used in the text.

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[188] We can, I think safely assume that Rashi means the same by both expressions, even if in his comment to Ezek. 14:21 he uses a conjunction of both for some reason (supposedly just to emphasize).

[189] Another case I discovered recently was Gen. 18:12. I attributed this case to Hirschensohn, because it was inspired by one of his findings.

[190] It also refers forward to Ezek. 15:5, though the latter also has a direct comment to same effect.

One of his fuller readings is, however, especially noteworthy, being very different from mine. It is that regarding Num. 12:14, because in it he reiterates the interpretation of this Torah narrative found in BT *Baba Qama* 25a, which has the a fortiori argument going a crescendo from a confinement of 7 days to 14 days, whereafter the penalty is reduced back to 7 days by application of the *dayo* principle. This claim, as I have explained in detail in AFL, is not a literal (*pshat*) reading of the verse, but a fanciful interpolation open to much criticism. See my brief comment on this case in chapter 4 (above).

It is interesting to note that Rashi does not reject Ezek. 15:5 as a genuine a fortiori (and thus, by implication, as not a true member of the list of ten cases in GR). This deserves notice, because (as we have mentioned earlier) the YS does not mention this case as one of the GR ten (listing only nine cases in all) and this omission has led some commentators (notably Einhorn, as we shall see) to doubt its credibility as a fortiori discourse. Rashi's treatment of this case without any suggestion that it is abnormal effectively contradicts that skeptical thesis.

Since the YS is considered by historians as dating from some time in the 11th to 14th centuries, and Rashi is dated specifically as 1040-1105, we can safely assume that Rashi antedated the YS and never read it. Unless it is discovered that Rashi does clearly mention YS somewhere, in which case historians will have to date the YS more narrowly, to before or during Rashi's lifetime[191]. But in the latter case, we would expect Rashi to give his opinion on the absence of Ezek. 15:5 in the YS listing, which (unsurprisingly) he does not do.[192]

Furthermore, we should take note of the fact that Rashi nowhere (to my knowledge) questions the claim by GR that there are (only) ten *qal vachomer* instances in the Torah (Tanakh), even though he himself has cheerfully discovered many more instances! Indeed, commenting on Gen. 44:8, he states: "This is one of the ten *qal vachomer* [arguments] spoken in the Torah, and all of them are enumerated in *Bereshith Rabbah*," giving the impression that he agrees unreservedly with the claim made in GR. Yet, paradoxically, he advocates at least 14 more valid explicit cases, 3 more implicit ones, and one invalid one. It would be surprising that someone so perceptive and intelligent as Rashi, a thinker willing when necessary to express criticism, would fail to notice the discrepancy, and at least mention it, if not offer some explanation for it. Maybe he does so somewhere, unbeknownst to me. This is an open question.

Historic firsts. Let us now look more closely at the 14 valid and explicit cases of Biblical a fortiori first discovered by Rashi. I will briefly note and sometimes comment on Rashi's remarks. All but three of these cases have already been analyzed by me in my 2013 list of 47 cases (in AFL), reproduced in the present essay (in chapter 3). The three cases new to me, viz. Genesis 6:3, Proverbs 17:7, and Job 35:13-14, will here be analyzed by me for the first time and therefore in a bit more detail. Nevertheless, I shall also review the other 11 cases so as to relate them to Rashi's commentary.

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[191] There is an editorial suggestion to that effect in the online edition I have used, in the English commentary to Gen.4:24. Where Rashi says "So did Rabbi Tanchuma explain it," there is a comment in square brackets saying: "This does not appear in extant editions of Tanchuma, but in Yalkut Shim'oni it is quoted from Tanchuma." However, to my mind, this editorial suggestion should not be taken as advocating that Rashi read the YS, but rather more simply as saying that there were editions of Tanchuma at the time the YS was written, and therefore also in Rashi's probably earlier lifetime, which included the explanation concerned.

[192] In https://jewishencyclopedia.com/articles/15057-yalkut#anchor4, it is clearly stated that Rashi nowhere mentions the YS.

Genesis 6:3. "And the Lord said: 'My spirit shall not debate concerning man forever, in that also (*beshagam*) he is flesh; and (*ve*) his days shall be a hundred and twenty years.'"

Our attention was drawn to this case through Hirschensohn's listing. Gabay reports that Hirschensohn refers to Rashi's reading of this verse as *qal vachomer*. He (i.e. presumably Gabay) summarizes Rashi's interpretation as follows: "And the Lord said: My spirit shall not contend [in Me] forever [i.e., interminably] concerning man, [whether or not to destroy him,] in that [this,] too, [is in him, that] he is flesh, [and not a hard substance — and even so he does not humble himself before Me!] and his days shall be one hundred and twenty years, [at which time, if he has not repented, I shall bring a flood.]"

Looking at Rashi's actual commentary, the a fortiori perceived by Rashi relates to the sentence בְּשַׁגַּם, הוּא בָשָׂר, with Rashi taking the first word as בְּשֶׁגַּם (changing a vowel). This sentence is translated (at the Chabad.org site online) as "because this is also in him that he is [only] flesh." Rashi adds the comment: "and nevertheless (*af al pi*), he does not subordinate himself before Me. What if (*umah im*) he were fire or a hard substance?" Which the translator explains in square brackets as meaning: "[i.e., How much greater would his insubordination be!]."

"My spirit shall not quarrel (*yadon*) concerning (*ba*) man forever" signifies that God has come to a decision concerning man's fate. "In that also he is flesh" involves the expression בשגם, which can be viewed as the sum of: in (*be*) + that (*she*) + also (*gam*). The words 'in that' can be rendered as 'because'; but what is the intent here of the added word 'also'? I think *gam* here rather means 'especially', and thus read the sentence as: "especially because he is flesh." This reading implies that if man was *not* made of flesh, God's decision might be in some way different. The underlying reference to a creature 'not of flesh' justifies Rashi's introduction of 'fire or a hard substance' as a descriptive antonym.

"And his days (*yamav*) shall be one hundred and twenty years." This spells out the decision God has taken. The first word of this sentence is the conjunction *ve*, generally meaning 'and', but in the present context obviously meaning 'therefore', because it provides a needed link to the preceding sentence, which does not yet specify what God's verdict is. Moreover, the 'especially' (*gam*) in the preceding sentence suggests that the inference implied by the 'therefore' here is made through an a fortiori argument. That is, 'especially' can be read as 'all the more'. This seems to be Rashi's cue for formulating a *qal vachomer*. Note that this is the only place in the whole Tanakh where the expression בשגם (*b-sh-g-m*) is used.

A simple (*pshat*) reading of the term 'his days' would be that the life expectancy of mankind is henceforth to be 120 years (presumably at most, though perhaps occasionally less). But Rashi insists on a more complex interpretation, viz. "Until a hundred and twenty years I will delay My wrath towards them, but if they do not repent, I will bring a flood upon them...." In that case, 'his days' refers to the survival of humanity as a whole (except for Noah and his family, as it turns out). This sentence, then, is a call for repentance and a threat of punishment if that call remains unheeded.

One could argue in favor of the simpler explanation by saying that man's sinfulness is due to his feeling invulnerable in view of his expecting enjoyment of a long life, so that reducing human lifespan drastically will limit his *chutzpah*, his hubris. But the same argument could be made in favor of Rashi's thesis by saying that the threat of a deadly flood within the specified delay might cause people to repent. Rashi's interpretation of the 120 years has the advantage of mentioning the flood as the essence of God's decision, whereas the simpler interpretation does not so much as allude to the flood even though it is contextually the main subject-matter.

In view of the above interpretations, it is clear that if we want to cast the Biblical narrative in a fortiori form, we must use *lèse-majesté* against God (the motive for God's decision) as the middle term (R) and destruction of humanity within 120 years (God's decision, introduced by *ve*) as the subsidiary term (S). Unrepenting sin must be mentioned in the major and minor terms, as the condition for the said eventual penalty. Finally, man made of flesh must appear in the conclusion (since it is about him that God's decision is made); and therefore, the creature made of fire or some hard substance, more invulnerable than man and thereby closer to divine, which is hypothetically proposed by Rashi, must appear in the minor premise.

So, the argument runs as follows: Unrepentant sinning, by a high powerful creature made of fire or some hard substance, (Q) is insubordinate to God (R) enough to merit annihilation in 120 years (S) (minor premise); all the more, unrepentant sinning, by a lowly weak creature made like man of mere flesh, (P) is insubordinate to God enough to merit annihilation in 120 years (conclusion). The tacit major premise here is: unrepentant sinning by a lowly weak creature, made like man of mere (*beshagam*) flesh is more insubordinate to God than unrepentant sinning by a high powerful creature, made of fire or some hard substance. The argument is positive subjectal a fortiori argument (**+s**).

However, the *qal vachomer* here formulated, though based on Rashi's commentary, is not the one Rashi had in mind. He has[193]: "because this is also in him that he is [only] flesh, and nevertheless, he does not subordinate himself before Me. What if he were fire or a hard substance? [i.e., How much greater would his insubordination be!]." This *qal vachomer* is inadequate because it does not reflect the full meaning of Gen. 6:3, but only seeks to understand the phrase "in that also he is flesh;" it does not mention the judgment God is consequently making. Moreover, it argues a fortiori wrongly from a creature of 'flesh' being insubordinate to one of 'fire and hard substance' being more so, whereas the argument should be in the opposite direction, because obviously God seeks to draw a conclusion about man and not about some hypothetical more invulnerable creature that man will never become.

For these reasons, Rashi's *qal vachomer* reading of Gen. 6:3, though significant, can only be admitted as implicit (*satum*). An explicit (*meforash*) a fortiori reading is nevertheless possible, as I have shown above; this should be regarded as the full and accurate rendition of the given verse. Rashi may be said to have drawn attention to it, but not to have fully verbalized it.[194]

2 Kings 5:13. Naaman's servants: "Had the prophet bid thee do some great thing, wouldst thou not (*ha-lo*) have done it? how much more when (*ve-af ki*) he [merely] saith to thee: wash and be clean [you should do it]!" (brackets mine).

This is a positive subjectal (**+s**) a fortiori argument. My paraphrase: If the prophet making some difficult request would have seemed powerful enough in your eyes to succeed in healing you, causing you to obey him, then his making an easy request suggests he may be more powerful than you expected and indeed powerful enough to heal you, and this should cause you to obey him. Rashi's analysis: "Would you not do it even if he ordered you to do something requiring exertion? *Qal vachomer* (a fortiori), since he said to you to do an easy thing, [viz.] immerse yourself and become clean."

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[193] The explanatory additions in square brackets are given by the translator in the Chabad.org edition.

[194] This is why, in the general list of Biblical a fortiori arguments given above in chapter 3, I identify this case as "First found by R/AS."

Ezekiel 14:13-21. God: "Son of man, when (*ki*) a land sinneth against Me by trespassing grievously, and I stretch out My hand upon it… and send famine upon it… [Or] if I cause evil beasts to pass through the land… Or if I bring a sword upon that land… Or if I send a pestilence into that land… though Noah, Daniel, and Job, were in it, as I live, saith the Lord God, they shall deliver neither son nor daughter; they shall but deliver their own souls by their righteousness… How much more (*af ki*) when I send My four sore judgments against Jerusalem, the sword, and the famine, and the evil beasts, and the pestilence, to cut off from it man and beast" (brackets mine).

This is a negative predicatal (**-p**) a fortiori argument. My paraphrase: If such holy men lack sufficient spiritual credit to prevent the execution of each of the four negative decrees (penalties for sins committed) separately, then they lack enough to stop all four of these decrees together. Rashi's commentary states that *af ki* is "*lashon qal vachomer ve-k"sh*" (language of a fortiori and all the more so); but it does not point out verses 13-20 as premises for the conclusion in v. 21.

Ezekiel 33:24. God: "They that inhabit those waste places in the land of Israel speak, saying: Avraham was one, and he inherited the land; but (*va*) we are many; the land is given us for inheritance."

This is a positive subjectal (**+s**) a fortiori argument. My paraphrase: If one man is important enough to inherit the land, then many men are important enough to inherit the land. (Obviously, though God is reporting this argument, He is not its author. It is not very credible, and rightly rebutted in the verses 25 and 26: it is not numbers but moral worth that makes possible inheritance of the land.) Rashi's interpretation is notably different: he reports the reading of R. Shimon ben Yochai as "Avraham, who was commanded with only one commandment [namely circumcision] inherited the land. We, then, who have been commanded with many commandments should surely have the land given to us for an inheritance (*Tosefta Sotah*, 6,7)." He states that this reading differs from that of R. Akiva, but he does not spell out the latter's version. Rashi then reads v. 25-26 as the prophet's reply: which he paraphrases as "Though you were commanded [many commandments] you do not keep [them].

Proverbs 15:11. Solomon: "Hell and destruction are before the Lord; how much more (*af ki*): the hearts of the children of men [are before the Lord]?" (brackets mine).

This is a positive predicatal (**+p**) a fortiori argument. My paraphrase: If God is powerful enough to look into hell and destruction, then He is powerful enough to look into people's hearts. Rashi just says that *af ki* is indicative of *q"v* (*qal vachomer*).

Proverbs 17:7. "Overbearing speech becometh not a churl; much less (*af ki*) do lying lips a prince."

This is a positive predicatal (**+p**) a fortiori argument (at least). My paraphrase: More unbecoming behavior is expected of a churl than of a prince (tacit major premise); whence it follows that if wrongful speech is unbecoming enough in a churl (minor premise), then it is unbecoming enough in a prince (conclusion). Note that I have here used the term 'wrongful speech' as the common ground of the terms 'overbearing speech' and 'lying lips'. If we regard lying lips as more unbecoming than overbearing speech, as seems intended, the *qal vachomer* here would be a crescendo (**+p&**). Rashi reads *af ki* as meaning *k"sh* (*kol sheken*). Note that *nadiv*, above translated as 'prince' can also be understood more broadly as referring to anyone of 'princely' character.[195]

[195] This is another case that I somehow missed in my past research, albeit its use of the key expression *af ki*.

Proverbs 19:7. Solomon: "All the brethren of the poor do hate him, how much more (*af ki*): do his friends go far from him?"

This is a positive predicatal (**+p**) a fortiori argument. My paraphrase: If the poor man is disliked enough that his brothers avoid him, then he is disliked enough that his friends avoid him. We can infer from the friends' 'distancing' that the brethren's 'hatred' is also expressed by avoidance. There is not necessarily an intent to say that the distancing is a worse reaction than the hatred; i.e. the argument need not be read as a crescendo (**&**). Rashi reads the expression *af ki* as meaning *kol sheken* (all the more so).

Proverbs 19:10. Solomon: "It is not seemly for a fool to have luxury; how much less (*af ki*): for a servant to have rule over princes [would be seemly]" (brackets mine).

This is a positive subjectal (**+s**) a fortiori argument. My paraphrase: If for a fool to have luxury is inappropriate enough to be unseemly, then for a servant to have rule over princes is inappropriate enough to be unseemly. Rashi reads the expression *af ki* as meaning *q"v* (*qal vachomer*, a fortiori).

Proverbs 21:27. Solomon: "If [even brought with a good intention] the sacrifice of the wicked is an abomination; how much more (*af ki*): brought with a bad intention [is it abomination]?" (brackets mine).

This is a positive subjectal (**+s**) a fortiori argument. My paraphrase: If the sacrifice of the wicked brought with a sincere intent is abominable enough to be rejected, then the sacrifice of the wicked brought with an insincere intent is abominable enough to be rejected. Notice the 'mirror' effect used to convey information briefly: 'the sacrifice of the wicked' and its 'abomination' are mentioned in the first half of the dictum, but not the second; whereas the quality of 'intention' is mentioned in the second half (bad), but not the first (good). Rashi reads the expression *af ki* as meaning *k"sh* (*kol sheken*, all the more so).

Job 9:13-14. Job: "God will not withdraw His anger; the helpers of Rahab did stoop under Him. How much less (*af ki*) shall I answer Him, and choose out my arguments with Him?"

This is a negative subjectal (**-s**) a fortiori argument. My paraphrase: If Rahab's helpers were not worthy enough to argue with God, then Job is not worthy enough to do so. Rashi's comment is just to read the expression *af*[196] in v. 14 as *k"sh* (short for *kol sheken*), i.e. as 'all the more'; but he does not explicitly relate this to v. 13.

Job 15:15-16. Eliphaz the Temanite: "Behold (*hen*): He puts no trust in His holy ones; and (*ve*) the heavens are not clean in His sight. How much less (*af ki*): one who is abominable and filthy, man, who drinks iniquity like water [does He trust or consider clean]!" (brackets mine).

This is a positive predicatal (**+p**) a fortiori argument. My paraphrase: If God is demanding enough to judge His holy ones as untrustworthy and the heavens as unclean, then He is demanding enough to judge mere human beings as untrustworthy and unclean. Rashi's commentary to v. 16 states that *af ki* means *kol sheken* (all the more so); but he does not explicitly mention this verse's relation to v. 15.

Job 35:13-14. "Indeed (*akh*) God will not hear vanity, neither will the Almighty regard it. Yea, when (*af ki*) thou sayest thou canst not see Him – the cause is before Him; therefore, wait thou for Him."

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[196] No mention of *ki* in the Chabad.org rendition of Rashi.

This a positive subjectal (**+s**) a fortiori argument (or possibly an a crescendo one, **&**). The speaker is Elihu, note. The intended meaning, following Rashi's brief explanations, seems to be as follows: God is not immediately (*akh*) moved by the expectations of vain persons, still less (*af ki*) by the demands of persons who doubt Him; nevertheless, He does in due course respond to all causes presented to Him for judgment if one is patient. The a fortiori argument, then, would be roughly: Doubters of God who make demands of Him are more annoying to Him than vain persons with sundry expectations (tacit major premise); whence it follows that if God ignores the vain ones for a while (minor premise), then He will ignore the doubters for as much (or maybe more) time (conclusion). Rashi flags this case, with reference to the expression *af ki* in it, as *k"sh* (*kol sheken*).[197]

Daniel 2:9. Nebuchadnezzar: "Thus (*lahen*): tell me the dream, and (*ve*): I shall know that you can declare its interpretation to me [since it is more difficult to tell it than to interpret it]" (brackets mine).

This is a positive predicatal (**+p**) a fortiori argument. My paraphrase: If Daniel is powerful enough to tell the dream, then he is powerful enough to interpret it. Rashi reads the expression *lahen* as meaning *k"sh* (*kol sheken*, all the more so).

Nehemiah 13:26-27. Nehemiah: "Did not Solomon king of Israel sin by these things? yet among many nations was there no king like him, and he was beloved of his God, and God made him king over all Israel; nevertheless even (*gam*) him did the foreign women cause to sin. Shall we then (*ve*) hearken unto you to do all this great evil, to break faith with our God in marrying foreign women?"

This is a positive subjectal (**+s**) a fortiori argument. My paraphrase: We are morally weaker than the great king Solomon (tacit major premise); whence it follows that if even (*gam*) a man of Solomon's spiritual caliber was weak enough to be brought to sin by foreign women (minor premise), then we who are spiritually at a lower level are weak enough to be brought to sin by foreign women (conclusion); therefore, to avoid sinning and thus breaking faith with our God, we must not marry foreign women (implication of the conclusion). Rashi reads the expression *gam* as meaning *kol sheken* (all the more so).

Influences on later commentators. Rashi discovered 5 cases before Ashkenazi, namely: Proverbs 15:11, 19:10, 21:27; Job 9:13-14, 15:15-16. Rashi discovered 3 cases before Katzenellenbogen, namely: Ezekiel 33:24; Proverbs 17:7, 19:7. Rashi discovered 1 case before Einhorn, namely: Nehemiah 13:26-27. Rashi discovered 1 case before Strashun, namely: 2 Kings 5:13. Rashi discovered 1 case before Hirschensohn, namely: Genesis 6:3. Rashi discovered 2 cases before me (1 in JL and 1 in AFL), namely: Ezekiel 14:13-21; Daniel 2:9.

Because Rashi's Biblical a fortiori discoveries are scattered throughout his running commentary, and maybe elsewhere too, and there is no listing of all of them together: (a) one cannot suppose that someone who lists one of Rashi's cases learned it from Rashi (unless, of course, he mentions having done so); and (b) one cannot infer from the fact that someone (by his own admission) learned one or more cases from Rashi that he knew of all Rashi's other cases; and indeed, (c) one cannot infer from the fact that someone lists some but *not* all of Rashi's case that he did *not* learn any case from Rashi.

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[197] I found this case during the present research effort by looking for Biblical verses involving the key phrase *af ki*. I somehow missed it in my past research, even though I looked for cases involving that expression. Most likely, it didn't look a fortiori to me at the time; it is only through Rashi's explanation that I now perceive it as such.

As a matter of fact, looking at the results of the present research, no commentator (until the present) has shown acquaintance with all of Rashi's 14 cases. Ashkenazi's list has 5 cases found in Rashi, but lacks 9 others. Katzenellenbogen's list has 7 cases found in Rashi, but lacks 7 others. Einhorn has 1 case found in Rashi, but lacks 13 others. Strashun's list has 10 cases found in Rashi, but lacks 4 others. My 1995 list has 7 cases found in Rashi, but lacks 7 others. Hirschensohn's list has 8 cases found in Rashi, but lacks 6 others. Jacob's list has 8 cases found in Rashi, but lacks 6 others. My 2013 list has 11 cases found in Rashi, but lacks 3 others. We can conclude that Rashi has not been studied as a source of Biblical a fortiori cases, at least not in a systematic manner, by any of the commentators here investigated (myself included).

Rejects. Besides the above-mentioned 14 valid and explicit cases of Biblical a fortiori, Rashi apparently proposed another 5 cases – which, however, I do not recognize as valid and explicit, and therefore have deliberately excluded from our listing. These are: **Genesis 6:9, Ezekiel 23:39-40, Habakkuk 2:4-5, and Psalms 15:4, 15:5**. I will now analyze them at length and explain why they deserve rejection (the second one as not a fortiori, and the other four as at best implicit a fortiori).

Genesis 6:9. "These are the generations of Noah. Noah was in his generations a man righteous and whole-hearted; Noah walked with God."

Gabay reports that Hirschensohn cites Rashi, who in his *ad loc.* commentary writes, concerning the term 'in his generations' (*bedorotav*):

> "Some of our Sages interpret it [this term] favorably: How much more so (*kol sheken*) if he had lived in a generation of righteous people, he would have been even more righteous. Others interpret it derogatorily: In comparison with his generation, he was righteous; but if he had been in Avraham's generation, he would not have been considered of any importance [Sanh. 108a, Gen. Rabbah 30:9, Tan. Noach 5]."

Rashi is referring to BT *Sanhedrin* 108a. Note that Rashi has the positive reading before the negative, whereas the Talmud has the negative one before the positive. Note also that the Talmud explicitly points to a *qal vachomer* in the positive thesis, saying "how much more," but does no such thing for the negative one. The relevant passage there is as follows[198]:

> "These are the generations of Noah: Noah was a just man, and perfect in his generations. R. Johanan said: In his generations, but not in other generations[199]. Resh Lakish maintained: [Even] in his generations — how much more so in other generations[200]."

The Biblical text states that Noah is righteous etc. 'in his generations', implying a *qualification* of some sort of the attribute of righteousness. This qualification cannot be ignored; its intention must be clarified; so, the commentaries by the Talmud and Rashi are quite justified. The word 'in' (*be*) is taken to mean 'compared to others in'. Note also that the term 'his generations' is plural, presumably to suggest that Noah spans two or more generations.

The positive reading is: since Noah was righteous among unrighteous people, then a fortiori he would be righteous even in a generation of righteous people, and indeed more righteous in that

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198 English Texts based on the Soncino Talmud Edition, online at Halakhah.com.

199 Footnote in the Soncino ed.: "So Noah: by comparison with the rest of his generation, who were exceptionally wicked, he stood out as a righteous man; in other generations he would not have been superior to the average person."

200 Footnote in the Soncino ed.: "Thus, if Noah was righteous even when his entire surroundings were evil, how much more so had he lived amongst righteous men!"

hypothetical environment than in his actual environment. The negative reading is: Noah was counted as righteous only in comparison to other people in his generations, but if compared to people like Avraham he would not merit such high status.

First question: what exactly is the *qal vachomer* proposed in the positive scenario? It is that if Noah (S) made a moral effort (R) enough to be righteous while living in a vicious environment (P) (minor premise), then he would perforce have made a moral effort enough to be righteous living in a virtuous environment (Q), and even more so (conclusion). The tacit major premise here is that more moral effort is required to be righteous in the midst of a vicious environment (where temptations to vice are widespread) than to be righteous in the midst of a virtuous environment (where the good example of others facilitates virtue). This is positive predicatal a fortiori reasoning (**+p**). The argument is not a crescendo, even though environmental influences affect the scale, as well as the potential, of vice or virtue, because the subsidiary term (S), viz. Noah, remains per se unchanged.

Next question: what form of reasoning is involved in the proposed negative scenario? It is argued that though Noah was worthy enough to be counted as righteous while living in a vicious environment, *it does not follow that* he was worthy enough to be counted as (particularly) righteous living in a virtuous environment. The assumption here is that the attribute of 'righteousness' is not absolute, but only relative. Given that someone is worthy to a certain extent, he may appear as 'righteous' if he is compared to very unworthy people, and yet appear as not (particularly) 'righteous' if he is compared to very worthy people.

In the positive formulation, the fact of righteousness is regarded as more difficult to attain in the vicious environment than in the virtuous one, whereas in the negative formulation, the appearance of righteousness is regarded as more difficult to earn in the virtuous environment than in the vicious one. These rival theses point in opposite directions, so they are in conflict somewhat. And yet they both seem reasonable. They are not in strict contradiction because the term 'righteousness' is treated differently in them. Nevertheless, the negative thesis dampens the enthusiasm of the positive one, calling for a more critical assessment.

The objection raised is that appearances may be illusory. The moral effort required by someone to appear righteous among the unrighteous may not in fact be sufficient to make that person stand out as (particularly) righteous in a righteous society. *The same amount* of moral effort may give an appearance of righteousness in a relatively morally poor context and not do that in a relatively morally rich context. Indeed, one may even exercise less virtue in the former case, and seem righteous, and more virtue in the latter case, and not seem (very) righteous. This insight does not deny the truth of the previous argument's major premise, but it calls for a more subtle approach.

Thus, if we tried to build an a fortiori argument for the negative assessment, using the middle term 'seem righteous' instead of 'righteous', we would obtain the following: More moral effort (R) is required to appear comparatively righteous in a virtuous environment (P) than in a vicious one (Q) (major premise in opposite direction). Noah (S) made enough moral effort to seem righteous in a vicious environment (similar minor premise); but not enough, *ceteris paribus*, to seem righteous in a virtuous environment (dissimilar 'conclusion'). Clearly, this attempt is a failure because the argument is positive predicatal and so cannot go from the minor term (vicious) to the major term (virtuous).

However, to repeat, it remains possible that Noah would generate a different amount of moral effort in the two environments, and thus indeed seem to be, and indeed be, virtuous, or vicious, in

both. That is why the two theses are not strictly contradictory: the positive one assumes *variable* effort in the two milieux, whereas the negative one assumes a *like* effort in them.

Clearly, the interjection "in his generations" requires explanation; but it is not clear which of the two explanations offered by the Sages is to be preferred. The purpose of the first is to heap praise on Noah, the virtuous father of post-deluge humanity; while that of the second is (dixit Rashi) to salute Avraham, the virtuous patriarch of the Jewish people. Both suggestions being authored by the Sages, they are both intended as true.

In conclusion, while the Biblical verse hints at a valid a fortiori argument, a credible counterargument is also compatible with it. This implies that the stated a fortiori argument is only implicit (*satum*); it cannot be taken as explicit (*meforash*) because, as Rashi himself (and the Talmud) informs us, it is not the only possible reading of the given text. The equivocation inherent in the given text means that neither reading can be declared with certainty as the exclusive interpretation.

Ezekiel 23:39-40. 39 For when they had slain their children to their idols, then they came the same day into My sanctuary to profane it; and, lo, (*ve-hine*) thus have they done in the midst of My house. **40** And furthermore (*ve af ki*) ye have sent for men that come from far; unto whom a messenger was sent, and, lo, (*ve-hine*) they came; for whom thou didst wash thyself, paint thine eyes, and deck thyself with ornaments...

I do not see any a fortiori argument in these two verses, explicit or implicit, valid or invalid, even taking their contexts (verses before and after) into account. What would be the assumed a fortiori argument? The people's behavior decried in v. 40 is implied to be *worse than* that decried in v. 39. This could be used as a major premise; but I see no argument whatever intended, no further discourse leading us from a minor premise to a conclusion. Yet, Rashi reads the expression *ve af ki* as here equivalent to *kol sheken* (all the more), implying that an a fortiori argument is intended[201]. But he does not clarify what he considers as the premises and conclusion of that argument.

The English translation of Rashi's commentary here has "And surely, worst of all is that they would send to bring to them [invite] some of the princes of the nations, their lovers, and these see the profanation with which they profane My sanctuary." This typically translates *kol sheken* as "surely." But all English translations of the verse itself that I have seen[202], including the one shown above, translate *ve af ki* here more vaguely as "And furthermore" or as "Moreover." This translation is, in my opinion, inadequate, because it suggests mere conjunction whereas the context suggests that an intensification is occurring. Thus, *ve af ki* here should rather be read as "And worse still." But this expression, in itself, carries no connotation of a fortiori argument; it merely implies progress from bad to worse.

Rashi rightly often interprets the use of *af ki* as indicative of *qal vachomer*, namely in the following 12 cases: 1 Samuel 23:3; Ezek. 14:13-21, 15:5; Prov. 11:31, 15:11, 17:7, 19:7, 19:10, 21:27; Job 9:13-14, 15:15-16, 35:13-14. I can name 9 more cases which he does not mention, where *af ki* signals a fortiori discourse: Deut. 31:27; 1 Sam. 14:29-30, 21:6; 2 Sam. 4:10-11, 16:11; 1 Kgs 8:27; Job 25:5-6; 2 Chr. 6:18, 32:15. But Rashi goes further, in his comment to Ezek. 14:13-21,

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[201] Gabay drew my attention to the commentary of *Metzudot David* (by R. David Altschuler of Prague, 1687-1769) having the same reading. Later, Strashun (19th Cent., see below) offers the same a fortiori interpretation. Perhaps these two were merely following Rashi's claim.

[202] Jerusalem Bible in print; and online: Chabad, Mechon-Mamre, JPS, and Sefaria.

and overgeneralizes by saying *af ki* that "it is the language of *qal vachomer* and *k"sh* [i.e. a fortiori and all the more] *throughout Scripture*" (my italics).

Yet, clearly, the 21 examples just listed do not prove Rashi's general claim, for the simple reason that there are at least two Biblical verses where *af ki* is used and there is clearly no a fortiori intent. Indeed, Rashi comments on both of them, without suggesting that any *qal vachomer* is intended. The cases are Genesis 3:1, where Rashi takes *af ki* to mean "even though" (*af al pi*, with no suggestion of a fortiori intent); and Nehemiah 9:18, where Rashi makes no comment (and the Eng. translation rightly reads *af ki* as "although").

It is therefore quite legitimate for me to say that Ezek. 23:39-40 does not contain an a fortiori argument even though it uses the *af ki* expression which is admittedly often indicative of such argument. It is clear to me that Rashi interpreted *af ki* as meaning *kol sheken* in this case a bit hastily; it is, in this instance, inaccurate.

I asked Gabay to find some rationale for Rashi's position, and he pointed out to me that several commentators[203] focused on the larger picture and suggested that v. 46 (among others), which describes the horrific consequences of the grave sins listed in v. 39-40 (and earlier), could be the conclusion of the putative *qal vachomer* argument. This later verse in Ezek. 23 reads:

46 For thus saith the Lord God: An assembly shall be brought up against them, and they shall be made a horror and a spoil.

Based on Gabay's response, the intended *qal vachomer* could be formulated as follows: Inviting people to watch the profanation of God's sanctuary, etc. (v. 40) is a greater sin than only profaning the sanctuary, etc. (v. 39 and earlier) (tacit major premise); whence it follows that if only profaning the sanctuary, etc. is sinful enough to bring about the severe punishments spelled out (in v. 46 and on) (minor premise), then inviting outsiders to watch such profanation is sinful enough for these same punishments or worse (conclusion). This would be a positive subjectal (+**s**) a fortiori argument, or possibly an a crescendo one (+**s&**).

However, looking at this proposal, I am still not convinced. I would like to follow suit but cannot honestly do so. It is clear enough that we are told that publicizing the desecration is worse than mere desecration (v. 39-40), and that heavy punishments are promised (v. 46). But nowhere in the received text is there a hint that the penalty for the publicity is *inferred* (whether in equal or greater measure) from the penalty for the desecration. Even if the list of misdeeds is clearly progressive from bad to worse (v. 39-40), it is clear that the prescribed penalty (v. 46) is intended for *all* the misdeeds *lumped together*. If the intent of the text had been otherwise, it would have indicated that the stated punishments (v. 46) are due *specifically for the earlier, lesser sins* (v. 39), and left it for us *to infer* similar or greater punishments for the later, greater sins (v. 40). The given text clearly makes no such division.

A comparative proposition does not by itself necessarily give rise to an a fortiori argument. More information is needed for that task. We are indeed given what could have been a major premise; but we are not also given a minor premise from which to draw the putative conclusion, or a conclusion from which to back-engineer the minor premise. I therefore maintain that there is no *qal vachomer* argument intent in this Biblical passage, whether explicit or implicit; that is, Ezek. 23:39-40(, 46) does not belong in our listing.

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[203] He mentioned David Kimchi (Radak, 1160-1235), Isaac Abarbanel (1437-1508), and Meir Lob Ben Yehiel Michal (Malbim, 1809-1879), and Mattityahu Strashun (1817-1885). I have not seen what they say precisely.

Let me now add a comment regarding Habakkuk 2:5. Even though it involves the expression *af ki*, and so might (following Rashi's usual assumption) thereby be intended as *qal vachomer*, it was not in fact flagged as such by Rashi. Nevertheless, it is worth examining more closely.

Habakkuk 2:4-5. 4 Behold (*hine*), his soul is puffed up, it is not upright in him; but the righteous shall live by his faith. **5** Yea, moreover (*ve af ki*), wine is a treacherous dealer; the haughty man abideth not; he who enlargeth his desire as the nether-world, and is as death, and cannot be satisfied, but gathereth unto him all nations, and heapeth unto him all peoples.

As already mentioned, Rashi's commentary on v. 5 does not flag it *qal vachomer* (or *kol sheken*). That in itself does not prove that Rashi did not view it as such; but the assumption that he did so would be speculative. If we look at what he does say, we see that he relates the narrative to historical persons and events. I do not however, for my part, perceive any a fortiori intent in his analysis. But if there was such intent, we could only at best say that he was construing an implicit a fortiori argument, because the terms it would involve (Belshazzar, drinking wine with the vessels of the Temple, etc.) are not explicitly given in the verse. Surely, the wording of the verse is rather vague.

Looking for some sort of a fortiori argument in this passage, I would construe the following more literal – and more general (ethical rather than historical) – reading, using v. 4 as well as v. 5: 'Certainly (*hine*), the puffed-up soul, lacking inner uprightness, [shall not survive,] unlike the righteous who by his faith shall live; all the more so (*ve af ki*), a haughty man, afflicted by drunkenness, and full of unlimited, insatiable desires, will not abide'. If this interpretation is correct, then the intended *qal vachomer* would be as follows: A man who is haughty etc. (P) is more wicked (R) than one who is puffed-up etc. (Q) (tacit major premise); whence it follows that if the puffed-up man is wicked enough to deserve a shortened life (S) (minor premise), then the haughty man is wicked enough to deserve a shortened life (conclusion). That would be a positive subjectal (+**s**) a fortiori argument. Rashi's commentary would then seem to be an application of this general argument to the historical circumstances he specifies.

Should we then count this case (in its proposed more general format) as an explicit *qal vachomer*? If so, it would not be attributable to Rashi, as he does not explicitly flag it as a fortiori. Although, as we have seen, he has made the sweeping claim that all Biblical verses involving the expression *af ki* are a fortiori, we cannot assume that he had in mind this case since he does not identify it as such *ad loc*., and moreover we have found other such cases that he clearly did not consider as a fortiori. In that event, I would be the discoverer of this new case.

I hesitate to claim it, however, because one could also read the two verses as separate statements, with *af ki* merely serving as an intensifier[204] rather than as a signal of inference. That is, because the terms used (puffed-up, haughty, etc.) are very vague, the two verses might be referring to one and the same foolish man, at the same time or over time, with v. 4 describing some (bad) aspects of him, and v. 5 adding some more (even worse) aspects of him[205]. In that case, I could not claim an explicit (*meforash*) *qal vachomer* intent, but only at best an implicit (*satum*) one.

Next, let us consider the Ps. 15:4 and 15:5 cases. As already mentioned (in chapter 4, above), these two cases are presented as examples of implicit (*satum*) *qal vachomer* in the *baraita* of R. Eliezer ben R. Jose HaGelili (or at least in the commentary attached to that *baraita*). So, these two cases

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[204] Notice, in support of this thesis, the above translation of *ve af ki* as "yea, moreover."

[205] Or the foolishness in the first verse might be more generic and that in the second more specific, i.e. the former might be inclusive of the latter.

cannot be attributed to Rashi. But because he uses the expression *q"v* (*qal vachomer*) in his commentary to both verses, but does not flag them as implicit (*satum*), it is appropriate for me to analyze them here.[206]

Psalms 15:4. He swears to his own hurt and does not retract (his oath).

Rashi comments: How much more (*q"v*) does he not retract if it concerns something that is not to his hurt![207] His a fortiori reading is thus: avoiding hurt to oneself is normally more pressing psychologically than doing oneself good (tacit major premise); whence it follows that, for a virtuous man, if avoiding hurt is not pressing enough to make him go back on his word (minor premise), then doing himself good is not pressing enough to make him go back on his word (conclusion). The argument form is negative subjectal (**-s**).

Why is this classified as implicit rather than explicit? Because, while it is true that if once having sworn an oath a good man does not revoke it even when it to his disadvantage, it is all the more true that he will not do so when it is to his advantage, nevertheless the verse is not making this larger statement. The verse is not intended to present an a fortiori argument aimed at the said implicit 'conclusion'; it is only concerned with making the explicit statement which we have used as 'minor premise'. The context shows that it is describing the various virtues *specific* to a good man[208]. The act of 'not revoking an advantageous oath' is found in bad men too; so, it is not of interest to this list of a good man's specific virtues.

Thus, Ps. 15:4 is not meant to communicate a *qal vachomer* to us. So, Rashi's *q"v* comment must be assumed to refer to the implicit *qal vachomer* which can eventually be read into it. The presentation of this verse by R. Eliezer as an example of implicit *qal vachomer* confirms this assessment.

Psalms 15:5. Nor did he accept a bribe against (*al*) the innocent.

Rashi's commentary reads: (Nor did he accept a bribe) "against a poor man, to condemn him in judgment by judging perversely. Our Sages explained it further to mean that he would not accept a bribe to exonerate him in judgment, and he a fortiori (*q"v*) will not take a bribe to pervert the judgment." So, the proposed *qal vachomer* here is: a bribe to condemn an innocent in judgment is more unethical than a bribe to exonerate an innocent in judgment (tacit major premise); whence it follows that if a judge views a bribe offered to him to exonerate an innocent in judgment unethical enough to refuse it (minor premise), then he will view a bribe offered to him to condemn an innocent in judgment unethical enough to refuse it (conclusion). The argument form is negative subjectal (**+s**).

The above translation reads the word *al* as 'against'. Louis Jacobs uses another as translation[209]: "Nor taketh a bribe to side with (*al*) the innocent;" and he takes this to mean: "if he refuses to take a bribe to support the innocent how much more will he refuse to take a bribe to support the guilty." Here, as Jacobs points out, the word *al* is interpreted as meaning 'on behalf of', rather than as

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[206] Note that I already analyzed these two cases in AFL, chapter 16, because Louis Jacobs drew attention to them in his previously cited essay. Jacobs rejects them, and I agree with him. He mentions the said *baraita* as their source, but he does not mention Rashi.

[207] In his previously cited essay, Jacobs translates the verse as: "He sweareth to his own hurt and changeth not;" and interprets it as meaning "if he 'changeth not' (i.e. does not go back on his word) where it is to his own hurt how much more will he not change where it is to his own good." This looks like a paraphrase of Rashi's comment.

[208] Defined in v. 1 as one "who shall sojourn in [God's] tabernacle, who shall dwell upon [His] holy mountain."

[209] In his aforementioned essay.

'against'. In Jacobs's interpretation, the minor term (support an innocent) is the same, while the major term (support the guilty) is slightly changed, suggesting a conflict between two parties. Obviously, if a corrupt judge took a bribe to exonerate (or be more lenient to) the truly guilty party, he would effectively be condemning (or being more severe to) the innocent party. So, the a fortiori argument is essentially unchanged (still +**s**).

This verse, however the expression *al* is read, may be classified as an implicit rather than explicit *qal vachomer* for the same reason that the preceding one was. There is evidently no intent to build an argument here – there is just a single statement. If an argument were intended, an additional statement would have been made (the major premise or the conclusion for that minor premise) in order to stimulate an inference, a reasoning process. *If a single statement were sufficient to claim an a fortiori argument, then every single statement could be turned into such an argument, and there would be no end to it*.

Here again, the context shows that our verse is merely descriptive, part of a list of the virtues specific to a good man. Such a man will not accept a bribe, whether in support of or in opposition to the innocent, or for that matter the guilty. The statement "Nor taketh a bribe to side with or to oppose (*al*) the innocent" is meant as all-inclusive. This is in implied contrast to the bad man, who is easily tempted for one or the other of these nefarious purposes (even if he might regard one or the other of them as easier or harder, so as to pretend to himself that he has a working conscience).

Here again, then, since Ps. 15:5 is not meant to communicate a *qal vachomer* to us, Rashi's *q"v* comment must be assumed to refer to the implicit *qal vachomer* which can eventually be read into it. The presentation of this verse by R. Eliezer as an example of implicit *qal vachomer* confirms this assessment.

Incidental a fortiori discourse. Besides Rashi flagging cases of Biblical a fortiori discourse, he frequently resorts to such argument in his running commentary, for homiletic or halakhic purposes. In such cases, it is not his intent to draw attention to an a fortiori intention in the Biblical narrative, by rather to discuss some tangential issue. Although he sometimes spins such argument by himself, he often draws on Midrashic or Talmudic sources where such argument has been used. There are very many examples of this, as can easily be ascertained by using, in search facility of the online edition of his commentary to the Tanakh, the following search strings: קַל וָחֹמֶר; ק"ו; כָּל שֶׁכֵּן; כ"ש.

To give one example: commenting on Gen. 1:12, Rashi claims, in the name of the *haggadah* of Tractate *Chulin* (60a), that although the herbs were not commanded by the Creator to grow "according to [their] kind," when they heard that the trees had been so commanded, they applied a *qal vachomer* argument to themselves, i.e. considered themselves as subject to the same obligation. Here, there is no *qal vachomer* intent in the Biblical verse itself, but it is used as a springboard for an additional claim.

Sometimes there are more than one such incidental a fortiori arguments in a chapter, or even within a single verse; so, it is necessary to carefully follow every lead returned by the search engine to make sure one sees all the cases involved. It must be said, however, that there may well be among these numerous incidental cases of a fortiori discourse, one or more cases of direct *qal vachomer*, i.e. of Biblical verses having themselves clear a fortiori intent, which we have not so far spotted and recorded in the present study. So, it is a research task worth pursuing, if not mandatory, to look at all cases mechanically found. But I have not found the time to do this arduous work myself.

7. R. Shmuel Yaffe ASHKENAZI

R. Shmuel Yaffe Ashkenazi (SYA, b. 1525, in Bursa, Ottoman Turkey, d. 1595)[210] wrote ***Yefeh Toar*** ("Beautiful of Form," abbrev. YT), published in Venice, 1597 (composed ca. 1560-80)[211]. Note that the late 16th Cent. is already, intellectually, the 'modern' era; this is not an 'ancient' or 'medieval' listing.

Gabay reports that **thirteen instances** of *qal vachomer* in the Tanakh are listed in this work (besides the 11 instances given in Midrash GR). These 13 are: **1 Kings 8:27[212]; 2 Kings 10:4; Isaiah 20:6; Jeremiah 25:29, 45:4-5, 49:12; Proverbs 15:11, 19:10, 21:27; Job 4:18-19, 9:13-14, 15:15-16, 25:5-6**.

It appears that *8 of these cases are 'historic firsts'*, viz. 1 Kings 8:27; 2 Kings 10:4; Isaiah 20:6; Jeremiah 25:29, 45:4-5, 49:12; Job 4:18-19, 25:5-6. The remaining 5, namely Proverbs 15:11, 19:10, 21:27, and Job 9:13-14, 15:15-16, may be considered as 'independent' finds. The latter 5 are all found in Rachi's Torah commentary; but since there are 8 other cases listed in it that are *not* listed in YT, we cannot infer that Ashkenazi based his list on the scattered comments of Rashi. To be sure, Ashkenazi may have derived one or more of these cases from Rashi; but unless he specifically mentioned doing so, we cannot suppose he did so.

My list of 2013 (reproduced above, in chapter 3) includes (of course, long after) and analyzes all the YT cases, except Isaiah 20:6; so, there is no need for me to here analyze any case other than the latter. Isaiah 20:6 is not at first sight clearly a fortiori, which is no doubt why some listings (my own and that of Jacobs) failed to include it. However, Ashkenazi, and (as we shall see) others after him, did include it. To grasp its a fortiori intent, one needs to look at the narrative context, i.e. the preceding five verses[213]. We then obtain the following reading; notice the language suggestive of Biblical a fortiori argument (*hine – ve-ekh*)[214]:

Isaiah 20:6. And the inhabitant of this coastland [around Ashdod, cf. v. 1] shall say in that day: "Behold (*hine*), such is our expectation [viz. to be led away captive like Egypt and Ethiopia, cf. v. 4-5], whither we fled for help to be delivered from the king of Assyria; and how (*ve-ekh*) shall we [Judeans, who are weaker than them,] escape?" (brackets mine).

My paraphrase: God is predicting how the Judeans will argue: If Egypt and Ethiopia, nations to which we Judeans are wont to flee for help, were not strong enough to avoid defeat from the king of Assyria, then we are surely not strong enough to do so. More formally put: Egypt and Ethiopia (P) are stronger (R) than Judea (Q) (tacit major premise); whence it follows that if Egypt and Ethiopia were not strong enough to avoid defeat (S) (minor premise), then Israel is not strong enough to avoid defeat (conclusion). This is a negative subjectal (**-s**) a fortiori argument.

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[210] https://www.jewishvirtuallibrary.org/jaffe-**s**amuel-ben-isaac-ashkenazi.

[211] https://beta.hebrewbooks.org/42241.

[212] Interestingly, 2 Chronicles 6:18 (a repetition of 1 Kings 8:27, but for one letter *heh*) is *not* included in the YS list. It only begins to appear in the Katzenellenbogen's listing (see next section).

[213] Gabay clarified the text to me on the basis of the commentaries of David Altschuler (aka *Metsudat David*, 1687-1769) and Meir Leibush ben Yehiel Michel Wisser (aka *Malbim*, 1809-1879).

[214] Compare the very similar argument of 2 Kings 10:4. In view of its language, I think I must have seen this case back in 1995, but evidently I failed to see its a fortiori character.

8. R. Zvi Hirsch KATZENELLENBOGEN

R. Zvi Hirsch Katzenellenbogen (ZHK, b. 1796, in Vilna, in Russian Lithuania, d. 1868)[215], wrote ***Netivot Olam*** ("Ways of the World," abbrev. 'NO'), published in Vilna, 1822[216]. Note that we are here already in the early 19th Cent.[217]

Gabay reported that **twenty instances** of *qal vachomer* in the Tanakh are listed (p. 30) in this work (besides the 11 instances given in Midrash GR). Initially, he found 16 cases, viz. **1 Samuel 14:29-30; 2 Samuel 4:10-11, 12:21, 16:11; 1 Kings 8:27; 2 Kings 10:4; Ezekiel 33:24; Proverbs 15:11, 17:7, 19:7, 19:10, 21:27; Job 4:18-19, 15:15-16, 25:5-6; 2 Chronicles 6:18** (repeat of 1K8:27). But later, after finding them mentioned in Hirschensohn's work, he found four more cases, viz. **Deuteronomy 32:39; 2 Samuel 12:18; Psalms 25:8-9, 78:20**.

Note that these four additional passages were not initially reported by Gabay, when he first investigated Katzenellenbogen's work. But he later reported that Hirschensohn listed these passages as instances of *qal vachomer* found in Katzenellenbogen's work. I asked him therefore to look again at Katzenellenbogen's listing, and he confirmed he had previously missed them and now saw them there. But when I asked him to tell me how Katzenellenbogen or Hirschensohn formulated the supposed a fortiori arguments, I got no further reply.[218]

It appears that *9 of the 20 valid cases are 'historic firsts'*, viz. Deuteronomy 32:39; 1 Samuel 14:29-30; 2 Samuel 4:10-11, 12:18, 12:21, 16:11; 2 Chronicles 6:18; Psalms 25:8-9, 78:20. The remaining 11 cases may be considered as 'independent' finds. Of those, 3 cases were previously flagged by Rashi but not by Ashkenazi, namely, Ezekiel 33:24, and Proverbs 17:7, 19:7; 4 cases were previously flagged by Rashi and later by Ashkenazi, namely, Proverbs 15:11, 19:10, 21:27, and Job 15:15-16; and 4 cases were previously flagged by Ashkenazi but not by Rashi, namely, 1 Kings 8:27, 2 Kings 10:4, and Job 4:18-19, 25:5-6. One or more of the said 7 cases attributed to Rashi might have been derived from his scattered Torah commentary by Katzenellenbogen; but there is no way for us to know it if he did not mention the fact. As for the said 8 cases previously flagged by Ashkenazi, we cannot assume that Katzenellenbogen derived them from his listing, since there are 4 other cases listed in YT that are *not* listed in 'NO', namely Jeremiah 25:29, 45:4-5, 49:12, and Job 9:13-14. For these reasons, we must consider the said 11 cases as 'independent' finds by Katzenellenbogen (unless, of course, information is eventually found to the contrary).

Note that Katzenellenbogen's discovery of Deut. 32:39 is the first finding of an additional *qal vachomer* in the Torah proper (i.e. the *Chumash*) since Rashi discovered Gen. 6:3 in the 11th Cent. Apparently[219], Katzenellenbogen was not aware of the Gen. 6:3 case.

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215 https://yivoencyclopedia.org/article.aspx/Katzenellenbogen_Tsevi_Hirsh.

216 https://beta.hebrewbooks.org/24940.

217 This about the same time as the 'Science of Judaism' movement was taking shape in Berlin.

218 I am assuming here that Gabay cited the four cases correctly, although he has occasionally made errors of inattention or typing. I have no way to personally verify the data he transmitted to me. Note that he additionally mentioned three other passages as Katzenellenbogen *qal vachomer* findings, namely: Proverbs 10:17, Job 28:5-6, Daniel 14:16. But he omitted Proverbs 10:17 from a later listing, presumably intentionally (but maybe unintentionally). Job 28:5-6 was probably a typing error, intending Job 25:5-6. And Daniel 14:16 must have been a typing error, as the book of Daniel has only 12 chapters! I looked at verses 14-16 in all chapters which have verses so numbered but spotted nothing resembling a fortiori argument there.

219 Assuming the list of cases reported by Gabay was correct.

My list of 2013 (reproduced above, in chapter 3) includes (of course, long after) and analyzes all the 'NO' cases, except Deuteronomy 32:39, 2 Samuel 12:21, and Psalms 25:8-9; so, there is no need for me to here analyze any case other than those three.

Deuteronomy 32:39. "See now (*reu ata*) that (*ki*) I, even I, am He, and there is no god with Me; I kill, and I make alive; I have wounded, and I heal; and (*ve*) there is none that can deliver out of My hand."

The proposed a fortiori reading is presumably: If no one other than God (S) is powerful (R) enough to kill and revive, to wound and heal (like He does, alone) (P); then surely, no one other than He is powerful enough to deliver out of His hand (i.e. to stop Him killing and reviving, Him wounding and healing, at will) (Q). The tacit major premise is that the acts of killing and reviving, wounding and healing, require more power than (or at least as much power as) deliverance from these acts does. This is a positive predicatal a fortiori argument (**+p**). This can be counted as an explicit, as well as valid, case.

2 Samuel 12:21. "Then said his servants unto him [David]: 'What (*mah*) is this thing that thou hast done? Thou didst fast and weep for the child, while it was alive; but when (*ve-ka-asher*) the child was dead, thou didst rise and eat bread?'" (brackets mine).

We can infer from the text what/how king David's servants were thinking. After the death of a child, one's behavior should be sadder (R) than while the child lived (tacit major premise); whence it follows that if David while the child still lived (Q) was sad enough to fast and weep (S) (minor premise), then David after the child died (should be sad enough to do same and even more (>S) (conclusion). This is a positive subjectal (**+s&**) a crescendo argument (which may be characterized as explicit, although it refers to an inferred thought, because there is no other explanation of their statement). But to the servants' surprise David's behavior was less sad. In v. 22, he explains to them why (thus rebutting their major premise).

Psalms 25:8-9. "Good and upright is the Lord; therefore (*al ken*) doth He instruct sinners in the way. He guideth the humble in justice; and He teacheth the humble His way."

The proposed a fortiori reading seems to be: If God (S) is good and upright (R) enough to instruct sinners in the way (P); then He is good and upright enough to guide the humble in justice and teach him His way (Q). The tacit major premise is that instructing sinners in the way requires more goodness and uprightness than guiding the humble in justice and teach him God's way. This is a positive predicatal a fortiori argument (**+p**). This can be counted as an explicit, as well as valid, case.

Rejects. Katzenellenbogen reportedly claims as *qal vachomer* at least two other Tanakh passages, namely: **Leviticus 10:19**; **Proverbs 6:30**. I must reject these cases, the former as merely implicit and the latter as not a fortiori.

Leviticus 10:19. "And Aaron spoke unto Moses: 'Behold (*hen*), this day have they offered their sin-offering and their burnt-offering before the Lord? And (*ve*) there have befallen me such [tragic events] as these; and (*ve*) if I had eaten the sin-offering today, would it have been pleasing in the sight of the Lord?'" (brackets mine).

Rashi offers a lengthy explanation of this passage and its context (verses 16-20). It has to do with laws of sacrifice and laws of mourning for priests. If I understood correctly, the following are the main points of Rashi's commentary. Two of Aaron's sons, Nadab and Abihu, died that day (v. 2), making Aaron and his surviving sons, Eleazar and Ithamar, *onanim* ("mourners for a close relative on the day of that relative's demise"). Moses reproves them for burning (v. 16) and not eating (v.

17) the sin-offering. Aaron retorts that neither his sons nor he could legally eat it. If the surviving sons, being ordinary *kohanim* (priests) in mourning that day, had sprinkled the blood of the sacrifice they would have invalidated it, and therefore could not eat it. But it was not they who performed the sin-offering, it was Aaron himself, as *Kohen Gadol* (High Priest), who did so. The law is different for him: he could ordinarily have performed the sacrifice and eaten it, except that the sin-offering under discussion was special, relating to *Rosh Chodesh* (the New Month), and in such case Aaron was legally forbidden to eat of the sacrifice during the day he performed it (though he could after nightfall).

Katzenellenbogen reportedly proposes a *qal vachomer* reading; but I have not seen it. However, based on Rashi's commentary I assume that the proposed reading was as follows. Performing the sin-offering by Aaron's sons (Q), would for various reasons have been legally inappropriate (R) enough to stop them eating of it (S) (minor premise); just as surely, performing the sin-offering by Aaron (P) was for various reasons legally inappropriate enough to stop him eating of it (conclusion). The tacit major premise would be that, in the given circumstances, even though the reasons for inappropriateness are different in the two cases, Aaron offering this sacrifice is as legally inappropriate as his sons' offering it. This would constitute a positive subjectal a fortiori argument (**+s**). Notice that I have cast it as egalitarian (saying, 'just as surely'), because the law is just as strict in both cases, even though the reasons for it are different.

Even so, the proposed a fortiori argument seems to me contrived, because we are not really engaged in inference from the said minor premise to the said conclusion, because the latter is known to be true independently of the former. The major premise, which is needed to justify the inference, is only known to us because we are already given (as Rashi's explanations make clear) both the minor premise and the conclusion. So, while the proposed argument is formally valid, it is rather artificial and can hardly be considered explicit (*meforash*). I would therefore declare it merely implicit (*satum*) – assuming I got it right, i.e. assuming that was indeed the *qal vachomer* that Katzenellenbogen had in mind and advocated.

I can reinforce this assessment as follows. Let's step back a moment and ask: what is the narrative about? It is a discussion between Moses and Aaron. The former criticizes the behavior of the priests, and the latter justifies it. Both are referring to the same body of laws, but apparently (so Rashi remarks regarding v. 29) Moses forgot some laws and Aaron reminds him of them. Aaron's retort is twofold. First, he exonerates his sons, second, himself, by showing that the law in either case forbids them, albeit for different reasons, from eating of the sin-offering in the specified circumstances, contrary to what Moses assumed. Aaron is not arguing from the prohibition applicable to his sons to that applicable to him. He is reminding Moses of two *separate, unrelated* sets of law; both so complex that neither of them can be deduced from the other. This is probably the most accurate rendition of the narrative. An a fortiori argument might be constructed (as above done by me or some other way), but the very fact that a reading without a fortiori argument is possible means that any proposed a fortiori reading is bound to be merely implicit. It could be explicit only if it was the only possible reading.

Proverbs 6:30. "Men do not despise a thief, if (*ki*) he steal to satisfy his soul when (*ki*) he is hungry."

Rashi comments at length on this verse, placing it in context (verses 29-35). The context is adultery, and a hungry thief is mentioned in this context to provide a contrast: whereas a hungry man stealing food can be forgiven and could eventually compensate the victim by repayment, an adulterer has no excuse for his act and cannot make up for it ex post facto. So, the former is not as

despised as the latter. But I do not see what *qal vachomer* argument could be constructed on the basis of this statement; I would call the discourse a contrario rather than a fortiori. Maybe Katzenellenbogen has succeeded, but until I see what he wrote, I cannot confirm that it is both valid and explicit. So, I must for now reject this case.

9. R. Zeev Wolf EINHORN

R. Zeev Wolf Einhorn (ZWE, a.k.a. *Maharzu* or *Maharzav*, b. 1813, in Grodno, in Russian Lithuania, d. 1862)[220]; wrote ***Midrash Tannaim*** ("Exegeses of the Tannaim," abbrev. MT), published in Vilna, 1839[221].

The following is a translation by Gabay of the relevant passage of that book (KV here is short for *qal vachomer*).

> "This is one of ten KVs. This teaching is a mystery, why are only ten KVs enumerated, while many more can be found in the books of the Prophets and the Writings, I have compiled a list which I have written in a *kontras* (a notebook) of about forty more KVs besides these ten.
>
> I reasoned that the main intention of the Sages who enumerated these KVs was that we would learn the principle of the KV how to reason from a minor premise to a major premise and from a major one to a minor one as a convincing argument. This is unique to these ten enumerated here, while the many other examples are just to reinforce the subject and not intended to convince and cannot be used as a learning model.
>
> After having written this I discovered that all the above is already contained in the words of the *Yefeh Toar*, therefore I am keeping my remarks brief. My only difficulty with *Yefeh Toar*'s explanation is the verse "If the wood of a grapevine has little use when it is whole, it obviously has no value when it is charred!" (Ezekiel 15:5), which is obviously just to reinforce the subject rather than to be convincing (since we see it is charred, it obviously of no value) and it is therefore not similar to the nine other examples.
>
> It seems that *Yefeh Toar* is right that this was never part of the original Midrash but rather a later addition from one of the scholars. I believe, after some research, that the tenth KV to complete the list is from the end of Nehemiah 13:26-27 "Did not Solomon king of Israel sin…shall we then hearken to you…" which would seem to be an argument designed to convince."

I now comment on the above. The "teaching" referred to above is the claim in the Midrash *Genesis Rabba* 92:7 that there are 10 *qal vachomer* arguments in the Tanakh. In ancient times, it was apparently always assumed that the number of instances is literally *only* ten; significantly, no larger number is mentioned in any known text from that era. It is only in more recent times that commentators have noticed other instances of *qal vachomer* than those initially listed in the Midrash and begun to wonder why the Midrash did not include them. That question is, of course, very pertinent. Einhorn tries to answer it by claiming without any written evidence whatsoever

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[220] https://he-m-wikipedia-org.translate.goog/wiki/%D7%96%D7%90%D7%91_%D7%95%D7%95%D7%9C%D7%A3_%D7%90%D7%99%D7%99%D7%A0%D7%94%D7%95%D7%A8%D7%9F?_x_tr_sl=auto&_x_tr_tl=en&_x_tr_hl=en-US.

[221] https://www.hebrewbooks.org/7058.

that the authors of GR knew that there are more than ten cases, but intentionally limited their list to ten.

Let us first address Einhorn's boast of having himself found about 40 instances of Biblical a fortiori argument, in addition to the ten given in GR. Note that this claim was made in the first half of the 19th Cent. Unfortunately, Gabay did not find a list of those alleged forty cases; presumably it was never published; or if it was, the relevant text is now lost. In the absence of a written list, we cannot reasonably take for granted that Einhorn did indeed find forty more cases. Keep in mind that we cannot assume offhand that Einhorn's alleged forty new cases were all indeed formally valid and explicit a fortiori arguments; some may have been merely implicit, and some may have been invalid. Why does he say "about forty" rather than give an exact number? This is suspicious, suggesting he did not carefully consider the apparent cases but only glanced at them superficially. We need to see them before we can admit them. An unverifiable claim is an irrelevant claim.

Looking at the rest of Einhorn's commentary, if we read his statement that "all the above is already contained in the words of the *Yefeh Toar*" as an explanation of why he did not publicly propose a list of 40 additional *qal vachomer* cases, i.e. as a suggestion that these cases were all already listed by YT, we find it factually wrong. The YT list known to us includes only 13 extra cases, far from the 40 he claims to know. We can assume from his discourse that he did indeed read YT (published over two centuries earlier). If he really had 40 cases listed in his notebook, he would have surely hastened to inform his readers of the 27 cases not found in YT. His not doing so is itself "a mystery."

Note in passing that there is no evidence that Einhorn knew of the list published relatively recently by his older contemporary and compatriot Katzenellenbogen (in *Netivot Olam*, 'NO'), which included 5 new cases not listed in YT. Maybe he did, since he lived in Vilna simultaneously for many years (from 1830 till his death), and since the two writers published their work there (respectively, in 1822 and 1839); but he does not apparently (to my knowledge) mention it anywhere. In any event, even if he did mention it somewhere, it is still a mystery why he did not make public his list of 22 additional cases (40 minus 13 of YT minus 5 of 'NO'). I mean, it would have only taken him a few minutes and a few lines to add this valuable information to his book.

Perhaps, then, Einhorn's "all the above" does not refer to the statistical information, but only concerns his reflections concerning "the main intention of the Sages who enumerated these KVs," i.e. the ten cases in GR. Einhorn claims (perhaps he was following and improving on the suggestion made centuries before in *Yalkut Shimoni*) that these ten are "unique" in their ability to teach us "how to reason" by means of a fortiori argument. Other examples just serve "to reinforce the subject" and were "not intended to convince and cannot be used as a learning model." These claims are all bunkum, sorry to say so bluntly. He certainly does not clarify them or demonstrate them. It is clear from them that he did not fully understand the nature of a fortiori argument.

It is additionally clear from Einhorn's description of a fortiori argument[222], as reasoning "from a minor premise to a major premise and from a major one to a minor one," that he did not really understand it. This misunderstanding on Einhorn's part raises doubt regarding his ability to judge what is a fortiori and what is not. To be precise, a fortiori inference proceeds from a major premise and a minor premise to a conclusion. Such inference may be, for a given major premise, from the minor term in the minor premise to the major term in the conclusion, or from the major term in the minor premise to the minor term in the conclusion.

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222 Assuming Gabay's translation accurate.

Einhorn's proposed explanation why the Midrash lists only ten instances of *qal vachomer* is that these ten were necessary and sufficient to teach a fortiori argument, while those not enumerated were useless or at least unnecessary in this regard. I would criticize that explanation as follows. On the surface, it is mere apologetics, designed to defend the myth of infallibility and omniscience of the rabbis of antiquity; but its practical motive is to allow Einhorn to look for and find new cases without seeming to contradict traditional belief in just ten cases. Whatever its motive, it is methodologically mere conjecture on Einhorn's part that the past rabbis (whoever among them authored GR) knew of his putative forty additional cases; he cites no ancient text in support of that claim, and without such evidence it is gratuitous.

Moreover, analysis of the ten cases, *with regard both to logical form and to language used*, does not confirm the special status of didactic models that Einhorn claims they have. In my 1995 book *Judaic Logic* (JL), chapter 3, I show that there are only four forms of logically valid a fortiori argument: the positive and negative subjectal forms (in which the major and minor terms are subjects) and the positive and negative predicatal forms (in which the major and minor terms are predicates). The fact is that the GR list of ten cases (luckily) does indeed include samples of all four valid forms (viz. **+s**, **-s**, **+p**, **-p**). However, if the purpose of listing ten was a logical teaching, four cases would have sufficed and six cases were obviously redundant.

In JL, chapter 5, I engage in linguistic analysis of the ten cases. The language used in them includes various if-then expressions like *hen* or *ki* with *ve-ekh*, *hen* or *hine* with *af ki* or *ve-af-ki*, and *ve* with *ha-lo*. I there used these expressions to find additional cases of *qal vachomer* argument, so they were indeed useful as models in my research. However, some of these expressions are repeated in the list of ten (e.g. *hen/ve-ekh*), so the ten cases were not all needed. Furthermore, I found many additional cases which did *not* involve the language used in the ten, so that list was *not* linguistically comprehensive and capable of pointing us to all other cases.

Therefore, Einhorn's claim that the given ten cases were intended and needed as models is spurious.

As regards Einhorn's claim that cases of Biblical *qal vachomer* not included among the ten of GR just serve "to reinforce the subject" and were "not intended to convince" or to be "used as a learning model" – it is quite meaningless. What does "reinforce the subject" mean? What subject is he referring to? And don't the ten cases "reinforce the subject," whatever that means, too? What does "not intended to convince" mean? Don't all arguments intend to convince? Also, in what way are the features of "reinforcing the subject" and "not intended to convince" tied together? Why are only the ten cases able to be "used as a learning model"? What is missing in the others for that didactic purpose? Bizarre claims like those are just make-believe, just fanciful nonsense. They were invented by Einhorn *ad hoc* in order to give a false impression (to the uninitiated) that abstract logical criteria were applied by him.

Note well that nowhere does Einhorn actually show how his alleged categories (viz. reinforcing subject, not intending to convince, not useable as learning models) would apply literally and exclusively to all cases of Biblical *qal vachomer* not included in the ten. He applies them to only one case, namely Ezek. 15:5. Clearly, he made them up and tailored them only for that specific purpose. To clarify and truly prove his theory, he would have had to actually list literally all cases, or at least the 40 he claimed to know; and then shown the said categories to be applicable to them all. This he evidently never did. Thus, not only were his categories meaningless, but his theory was not properly tested and verified.

In conclusion, so far as we know from the extant published evidence, Einhorn can only claim a single Biblical a fortiori finding of his own, viz. the **Nehemiah 13:26-27** case. However, though he apparently thought this was an original finding of his (which, however, as we shall soon see, he paradoxically tried projecting into the ancient GR list of ten), because it was not included in the list of his predecessor Ashkenazi (which he knew of – nor for that matter in that of Katzenellenbogen, which he did not apparently know of), he was *not* in fact the 'historic first' to mention it. The first commentator to ever mention it was none other than Rashi, as we have seen (in chapter 6, above). Since he apparently (so far as I know) does not mention Rashi as his source for this case, we can reasonably regard it as an 'independent' finding of his (unless information to the contrary emerges in time).

Let us now consider Einhorn's treatment of Ezek. 15:5. As we have seen earlier, the 11th-14th Cent. Midrashic work called *Yalkut Shimoni* (YS) declares that there are ten *qal vachomer* arguments in the Tanakh, but then lists only nine of those given in GR, leaving out Ezek. 15:5. As already pointed out, this omission may have been quite unintentional and most probably was. But Einhorn pounces on it, apparently following a suggestion in *Yefeh Toar* (YT) that this case might have been an addendum by some later scholar. He pushes the speculation further, claiming the omission by YS was intentional and pretending to know precisely why. In his opinion, this case is "obviously" distinct from the other nine and should on this basis be dropped out and replaced by another. Here is the verse in question:

Ezekiel 15:5. "Behold (*hine*): when (*be*) it [the vine-tree] was whole, it was not meet for any work; how much less (*af*): when (*ki*) the fire hath devoured it and (*ve*) it is burned, shall it then (*ve*) yet be meet for any work?" (brackets mine).

My paraphrase: The speaker is God and He is forewarning the inhabitants of Jerusalem (here symbolized by the vine-tree) of coming destruction. He says: A whole vine-tree (P) is in better condition (R) than a thoroughly burnt one (Q) (tacit major premise); whence if when still whole the vine-tree was not in good condition enough to be useful (S) (minor premise), it follows that now, when thoroughly burnt, it is not in good condition enough to be useful (conclusion).

This is a formally valid and sufficiently explicit negative subjectal (**-s**) a fortiori argument. Note that the argument does not seem intended as an a crescendo one, judging by the Hebrew original; that is to say, the wood was just as useless (not meet) in the minor premise (when the wood was still whole) as it becomes (rhetorically put, yet meet?) in the conclusion (when the wood is burned out) – i.e. the subsidiary term involved remains the same. But even if we take the argument as a crescendo, as Einhorn seems to, and assume that the whole wood still had some utility, whereas the burned wood has none left, there is no logical basis for rejecting the argument. It remains valid (**-s&**).

The fact that the wood is finally *valueless* does not make the argument less formally valid – it is merely a material issue. The form is not affected by the content, contrary to what Einhorn seems to imagine. The zero value of the end result does not cause the subject to be "reinforce[d]," whatever that might mean (he does not say), and the argument is no less "convincing" than it would be if there was some residual value. Einhorn's attempted differentiation of this case from the others is pure fantasy. Moreover, there is one other negative subjectal (**-s**) case in the GR list of ten cases – namely Ex. 6:12. In this case, the conclusion is that Pharaoh will not 'listen' to Moses, either not much or not at all. Here, too, as in Ezek. 15:5, the result could be viewed as valueless; yet Einhorn does not reject this formally similar case.

In truth, every valid a fortiori argument is equally informative and forceful. As I fully demonstrate in my detailed 2013 study of a fortiori logic (AFL), this form of reasoning allows for any value whatever in the conclusion (as indeed in the minor premise) – a positive value, a zero value, and even a negative value[223]. All that matters logically is that the comparison declared in the major premise (which may even be egalitarian, of course) is adhered to in the minor premise and conclusion. There is no formal control over the material content. Einhorn evidently did not realize that.

The motive behind Einhorn's elaborate spin concerning Ezek. 15:5, the real reason for his attempt to eject it from the GR list of ten, is that he wants to replace it with the one case he believes he discovered, namely Nehem. 13:26-27. His downgrading of the former is merely a pretext to make possible his upholding of the latter. He claims that "after some research" he came to believe that this new case ("which would seem to be an argument designed to convince") was the most fitting replacement for the one he (willfully) ejected. What "research" exactly? He does not say. What sort of "research" could conceivably result in such a conclusion? He does not say.

Moreover, he does not state why he chose this one particular argument, i.e. Nehem. 13:26-27, as a fitting substitute for Ezek. 15:5, *rather than any of the other 39 or so arguments* he claims (without providing evidence) to have found. He does not say in what way this pet *qal vachomer* of his is to be viewed as superior to them all, and thus more worthy of inclusion in this ancient list of ten than them. All he says to justify his choice is that it was made: "after some research" – but such a vague statement is obviously insufficient. His proposal must therefore be regarded as arbitrary and without credibility.

And of course, we must question Einhorn's right to modify the list of ten given in a Midrash (GR) written a millennium and a half before he was born! Earlier on, he was seemingly defending the received GR text; now he is trying to put it in doubt and correct it, effectively denying an ancient tradition, basing his initiative on very tenuous grounds.

It is also worth noting the fact that Einhorn does not just propose the Nehemiah case as an eleventh case, but tries to insert it as one of the original ten. This shows incidentally that he is aware that the ancient tradition of ten was a tradition of *davka* ten, i.e. 'only ten' – and not 'ten and more'. Moreover, since, as far as Einhorn knew, the Nehemiah case was an original discovery of his (indeed the only one he explicitly presented), because predecessors known to him did not mention it before him, he should have reasoned that it was very unlikely that this case would have ever been part of the ancient list of ten!

A better candidate, surely, would be Gen. 4:24, which was known centuries before the *Yalkut Shimoni* omission, having been mentioned in GR outside the list of ten, then in the *Avot de-Rabbi Nathan* (ARN) as one of five instances of *qal vachomer* in the Pentateuch, and in a commentary *ad loc.* by Rashi. But the safest course, I suggest, is to leave well enough alone and accept Ezek. 15:5 as one of the ten cases. The reason cited for wanting to eject this case is simply too flimsy.

One more note concerning Einhorn. In my 2013 list of Biblical a fortiori argument (in AFL), I wrongly attributed all the cases listed by Louis Jacobs (other than those of GR) to him, labeling them all as WE (for Wolf Einhorn). This error was due to my misreading Jacobs's essay on this

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[223] "An (inclusive) range R may have any value from minus infinity through zero to plus infinity" (AFL, chapter 1.4). R here refers to the middle term, which in the major premise relates the major (P) and minor (Q) terms, and in the minor premise and conclusion relate these two terms, through sufficiency of the middle, to the subsidiary term (S).

subject (see my fuller explanation further on, in the section devoted to Jacobs). I relabel the cases concerned as LJ in the present essay.

10. R. Mattityahu STRASHUN

R. Mattityahu Strashun (MS, b. 1817, in Vilna, in Russian Lithuania, d. 1885)[224] wrote ***Mattat Yah*** ("Gift of God," abbrev. MY), published posthumously in Vilna, 1892 (composed 1838-78)[225]. This author can be credited as the one, among those here considered, who has listed the most Biblical a fortiori arguments.

Gabay reports that **twenty-three instances** of *qal vachomer* in the Tanakh are listed (p. 21) in this work (besides the 11 instances given in Midrash GR), and so can be attributed to Strashun. But another twenty instances can be added to his list, namely all those included in Katzenellenbogen's list (see chapter 8, above), because we know that Strashun read and commented on the latter's work.

The 23 claimed by Strashun are: **Judges 14:16; 1 Samuel 14:39; 2 Samuel 11:11; 2 Kings 5:12, 5:13, 18:23-24, 18:35; Isaiah 1:3, 10:11, 20:6, 36:8-9 (repeat of 2K18:23-24[226]), 36:20 (repeat of 2K18:35[227]); Jeremiah 2:11, 8:7, 25:29, 45:4-5, 49:12; Ezekiel 3:4-7,** 23:39-40**; Jonah 4:10-11; Job 9:13-14,** 28:17**; Nehemiah 13:26-27**. However, I do not recognize as valid and explicit two of these cases, namely: Ezekiel 23:39-40 and Job 28:17; so, the real count for Strashun is only 21 cases. To which, of course, we can add Katzenellenbogen's 20 cases, making 41 in all. Add to that the 11 of GR, the grand total is 52 cases.

With regard to the 21 cases, it appears that *14 of them are 'historic firsts'*, namely: Judges 14:16; 1 Samuel 14:39; 2 Samuel 11:11; 2 Kings 5:12, 18:23-24, 18:35; Isaiah 1:3, 10:11, 36:8-9, 36:20; Jeremiah 2:11, 8:7; Ezekiel 3:4-7; Jonah 4:10-11. Another 7 are possible 'independent finds' by Strashun, namely: 2 Kings 5:13, Isaiah 20:6; Jeremiah 25:29, 45:4-5, 49:12; Job 9:13-14; Nehemiah 13:26-27; because they were not listed by Katzenellenbogen. The first and last of these were first flagged by Rashi; and the other five were previously listed by Ashkenazi, but since we cannot yet determine whether Strashun was aware of those earlier discoveries, we must categorize them as 'independent'[228]. Additionally, as already mentioned, we can attribute to Strashun, under the heading of 'derived', the 11 cases of GR and the 20 cases of Katzenellenbogen.

We now need to look more closely at the above-mentioned 14 'historic first' cases. My list of 2013 (reproduced above, in chapter 3) includes (of course, long after) and analyzes 4 of them, namely: Judges 14:16, 2 Kings 18:23-24, Isaiah 36:8-9, and Jonah 4:10-11; so, these need not be examined

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[224] https://www.encyclopedia.com/religion/encyclopedias-almanacs-transcripts-and-maps/strashun-mathias. Also, https://publishup.uni-potsdam.de/opus4-ubp/frontdoor/deliver/index/docId/41778/file/pardes24_S.27-45.pdf.

[225] https://beta.hebrewbooks.org/6310.

[226] Word for word, except that Isaiah has *ha-melekh*, where Kings has *et-melekh.*

[227] Word for word, except that Isaiah has an extra word, *ha-eleh*.

[228] Unless Strashun has somewhere written having learned one or both of the said 2 cases from Rashi's Torah commentary, we must assume that he did not. Likewise, regarding the 5 cases previously found in Ashkenazi's work, we do not know at this stage whether or not Strashun knew the work of Ashkenazi. If we find that he did, then these 5 cases should be counted as derivative; but if we find that he did not, they should be counted as independent. I have assumed the latter until the matter is settled. Note that all cases found in Ashkenazi are found in Strashun, so it may well be that the latter got them from the former; but it may also be a coincidence. With regard to the Nehemiah case, note that Strashun may have learned it from Einhorn, since both lived for many years in Vilna and published there, and Einhorn published well before Strashun.

again here. This leaves us with 10 new cases to examine here, namely: 1 Samuel 14:39; 2 Samuel 11:11; 2 Kings 5:12, 18:35; Isaiah 1:3, 10:11, 36:20; Jeremiah 2:11, 8:7; and Ezekiel 3:4-7.

1 Samuel 14:39. "Though (*ki*) it [the crime of eating despite the king's prohibition (in v. 24)] be in Jonathan my son, (*ki*) he shall surely die" (brackets mine).

My paraphrase: Saul is apparently saying: My son (P) is to me worth (R) more than anyone else (Q) (tacit major premise); whence it follows that if my son is not worth enough to me to escape my killing him if he ate (S) (minor premise), then no one else is worth enough to me to do so (conclusion). This is a negative subjectal (**-s**) a fortiori argument. Note that, according to Gabay, Strashun views it as "*satum*" (only implicit); but I disagree, because there is no alternative interpretation of Saul's words (his mention of Jonathan) – so, I take it as explicit.

2 Samuel 11:11. "And Uriah said unto David: 'The ark, and Israel, and Judah, abide in booths; and my lord Joab, and the servants of my lord, are encamped in the open field; shall (*va*) I then go into my house, to eat and to drink, and to lie with my wife?'"

My paraphrase: Uriah is saying: The rest of the army etc. (P) are more worthy (R) than me (Q) (tacit major premise); whence it follows that if they are not worthy enough to merit listed pleasures (S) (minor premise), then I am not worthy enough to merit such pleasures (conclusion). This is a negative subjectal (**-s**) a fortiori argument.

2 Kings 5:12. "'Are not Amanah and Pharpar, the rivers of Damascus, better than all the waters of Israel? may I not (*ha-lo*) wash in them, and (*ve*) be clean?' So, he turned, and went away in a rage."

My paraphrase: Naaman is saying: The rivers of Damascus (P) are better (R) than all the waters of Israel (Q) (major premise); whence it follows that if Damascus rivers, in which I have often bathed, were not good enough to heal me (S) (minor premise), the waters of Israel cannot be good enough to do it (tacit conclusion, implied by his turning away angrily without bathing). This is a negative subjectal (**-s**) a fortiori argument. Note the use of the key word *ha-lo*, often found in Biblical *qal vachomer* argument.

This reading is based on that of the 10th Cent. commentator ibn Janakh, in *Sefer Harikma,* p. 21, which Gabay has drawn my attention to and clarified[229]. At first sight, Naaman's statement seems to imply that he will (future tense) go back to Damascus and try bathing there, instead of in Israel's Jordan river. That is, he is saying (with the same major premise): If the waters of Israel are good enough to heal me (tacit minor premise), then Damascus water is good enough for that (conclusion); so, I prefer to go there. This would make it a positive subjectal (**+s**) a fortiori argument. However, Ibn Janakh understands[230] Naaman as rather saying that he has (past tense) often before bathed in Damascus waters without getting healed and therefore does not believe bathing in the Jordan, whose water he believes less powerful, would heal him. I agree that this interpretation makes more sense. As ibn Janakh points out, Naaman at this stage imagines the promised healing is a physical power of water; although later when he bathes in the Jordan and is healed, he clearly realizes the cure was a miracle from the God of Israel.

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[229] Gabay also draws attention to Redak's comment on this verse, which presumably goes in the same direction.

[230] Ibn Janakh takes the letter *vav* (the word 'and' in our translation) as indicative here of a fortiori argument, i.e. as meaning 'all the more so' (*kol sheken*). That is, the if-then couple are here *ha-lo/ve*.

2 Kings 18:35. "Who are they among all the gods of the countries, that (*asher*) have delivered their country out of my (the king of Assyria's) hand, that (*ki*) the LORD should deliver Jerusalem out of my hand?'" (brackets mine).

My paraphrase: Rabshakeh, in the name of the king of Assyria: The gods of the already-conquered other countries (P) are stronger than or as strong as Israel's God (Q); whence, if the gods of the already-conquered other countries were not strong (R) enough to prevent my conquests (S), then Israel's God is not strong enough for that. This is a negative subjectal (**-s**) a fortiori argument. The tacit major premise of course reflects the Assyrian's belief system.

Isaiah 1:3. "The ox knoweth his owner, and the ass his master's crib; but Israel doth not know, My people doth not consider."

My paraphrase: God is saying: The people of Israel (P) are (i.e. should theoretically be) smarter (R) than lowly animals like oxen or asses (Q) (tacit major premise); whence it follows that if the animals are smart enough to obey their masters (S), the people should (in principle) be smart enough to obey their master (God). This is a positive subjectal (**+s**) a fortiori argument, which can be regarded as explicit because only thus can the mention of ox and ass be explained.

God goes on, saying effectively that since His people are not smart enough to obey Him, they are less smart than animals. That is, by denying the conclusion of the *qal vachomer*, He refutes the major premise of the theoretical argument initially put forward. This is not an additional a fortiori argument, but only the neutralization of one. The purpose of such discourse is to shame the people for their bad behavior, and entice them to improve their ways.

Isaiah 10:11. "Shall I not (*ha-lo*), as (*ka-asher*) I have done unto Samaria and her idols, so (*ken*) do to Jerusalem and her idols?"

My paraphrase[231]: according to God, Sennacherib (the Assyrian king) is thinking that if Samaria's idols (Q) were weak (R) enough to allow its and their destruction (S) (minor premise), then Jerusalem's idols (P) will be weak enough for the same outcome (conclusion). The tacit major premise is that Jerusalem's idols are the same as Samaria's (and therefore they are equally weak). This is a positive subjectal (**+s**) a fortiori argument. Notice that the proposed major premise is *egalitarian*, based on Rashi's commentary (to v. 10) that the graven images of the regional nations were "from Jerusalem and from Samaria;" which is why the king reasoned: "since the worshippers of the graven images of Samaria and Jerusalem fell into my hands, and their graven images did not save them [i.e. the nations], so will Samaria and Jerusalem not be saved." Samaria having fallen, Jerusalem was bound to likewise fall. Rashi does not flag v. 11 as a fortiori discourse. Note in passing the use of the keyword *ha-lo*.

Isaiah 36:20. "Who are they among all the gods of these countries, that (*asher*) have delivered their country out of my hand, that (*ki*) the Lord should deliver Jerusalem out of my hand?"

My paraphrase: This verse is the same as 2 Kings 18:35, and in both Rabshakeh effectively says, in the name of the king of Assyria: The gods of other countries are, in their powers, equal to or greater than the God of Jerusalem (tacit major premise); whence it follows that if the other gods (P) were not strong (R) enough to prevent my conquests (S) (minor premise), then Jerusalem's God (Q) is not strong enough for that (conclusion). This is a negative subjectal (**-s**) a fortiori argument.

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[231] Gabay told me that the *qal vachomer* involved is clarified in the commentaries Redak and *Metzudot David* to v. 10. But I have not seen these commentaries so far.

Note additionally, in this context, 2 Kings 19:17-18: “Of a truth, Lord, the kings of Assyria have laid waste the nations and their lands, and have cast their gods into the fire; for they were no gods, but the work of men's hands, wood and stone; therefore, they have destroyed them.” This statement by Hezekiah, while admitting the minor premise of the preceding a fortiori argument, refutes its tacit major premise, so as to reject its conclusion. Needless to say, the earlier argument was formally valid, even if it involved false content; Hezekiah is not making an a fortiori argument, but only neutralizing one.

Jeremiah 2:11. “Hath a nation changed its gods, which yet are no gods? But (*ve*) My people hath changed its glory for that which doth not profit.”

My paraphrase: God is saying: Israel’s God (P) has more credibility and worth (R) than the gods of all other nations (Q) (tacit major premise); whence it follows that if the gods of other nations are credible and worthy enough to remain unchanged by their nations (S) (minor premise), then Israel’s God is (i.e. should be) credible and worthy enough to remain unchanged by His nation (conclusion). This is a positive subjectal (+**s**) a fortiori argument. The explicit a fortiori intent of this verse is evident from the comparisons it makes.

God pursues his argument by observing that His people have abandoned Him for lesser values. This denies the conclusion of the *qal vachomer*, and thus refutes the major premise initially put forward, thereby implying that Israel does not value its God as the other nations value their gods. This is not an additional a fortiori argument, but only the neutralization of one. The purpose of such discourse is to shame the people for their bad behavior, and entice them to improve their ways.

Jeremiah 8:7. “Yea (*gam*), the stork in the heaven knoweth her appointed times; and the turtle and the swallow and the crane observe the time of their coming; but (*ve*) My people know not the ordinance of the Lord.”

My paraphrase: God is arguing: His people (P) is, in principle, more cognitively efficacious (R) than animals like the stork, the turtle or the crane (Q) (tacit major premise); whence it follows that if the said animals are cognitively efficacious enough to know certain facts relevant to them (S) (minor premise), then God’s people are (i.e. should be) cognitively efficacious enough to know certain facts relevant to them (conclusion). This is a positive subjectal (+**s**) a fortiori argument.[232]

God goes on to point out that His people in fact do not know certain facts relevant to them (denial of conclusion), and thus puts in doubt the theoretical claim that they are intellectually superior to animals (refutation of major premise). This is not an additional a fortiori argument, but only the neutralization of one. The purpose of such discourse is to shame the people for their bad behavior, and entice them to improve their ways.

Ezekiel 3:4-7. “**4** And He said unto me: 'Son of man, go, get thee unto the house of Israel, and speak with My words unto them. **5** For (*ki*) thou art not sent to a people of an unintelligible speech and of a slow tongue, but to the house of Israel; **6** not to many peoples of an unintelligible speech and of a slow tongue, whose words thou canst not understand. Surely, if I sent thee to them, they would hearken unto thee. **7** But (*u*) the house of Israel will not consent to hearken unto thee; for they consent not to hearken unto Me; for all the house of Israel are of a hard forehead and of a stiff heart.”

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[232] Compare Isaiah 1:3 and other similar arguments.

My paraphrase: God is saying[233]: The house of Israel (P) is more intelligent (R) than certain other peoples (Q) (major premise, in v. 5); whence it follows that if these other peoples are intelligent enough to listen to the words I send through you (S) (minor premise, in v. 6), the house of Israel is (i.e. should be) intelligent enough to listen to My words (tacit conclusion). This is a positive subjectal (+**s**) a fortiori argument.

God goes on to predict that His people in fact will not obey Him (denial of conclusion, in v. 7), and thus puts in doubt the theoretical claim that they are more intelligent (refutation of major premise). This is not an additional a fortiori argument, but only the neutralization of one. The purpose of such discourse is to shame the people for their bad behavior, and entice them to improve their ways.

Now, regarding the two cases in Strashun's list that I have **rejected**:

Ezekiel 23:39-40. "For when they had slain their children to their idols, then they came the same day into My sanctuary to profane it; and, lo, (*ve-hine*) thus have they done in the midst of My house. And furthermore (*ve af ki*) ye have sent for men that come from far; unto whom a messenger was sent, and, lo, (*ve-hine*) they came; for whom thou didst wash thyself, paint thine eyes, and deck thyself with ornaments..."

This case was previously advocated by Rashi. I have already shown in the section concerning him (chapter 6, above) that it does not qualify as a fortiori argument, explicit or implicit.

Job 28:17. "Gold and glass cannot equal it [i.e. wisdom]; neither shall the exchange thereof be vessels of fine gold" (brackets mine).

My analysis: This verse is stating that gold and glass *are inferior in value* to wisdom, and moreover[234] that wisdom *cannot be purchased with* vessels of fine gold. I do not see an a fortiori argument in that. Simply read, in the first half wisdom is estimated as worth more than gold and glass, and in the second half it is declared that wisdom cannot be obtained by means of vessels of pure gold. The two halves of the verse are not in opposition, but merely reiterate the same thought in different ways, namely that wisdom is not a material commodity. There is no indication here that 'gold and glass' are somehow inferior to 'vessels of fine gold', leading to some sort of a fortiori argument with these terms in the major premise[235]. The verse does not aim to compare these two terms, and then formulate an a fortiori argument, but only to variously weigh them against wisdom.

The context of this verse is all poetic praise of wisdom, in a way typical of Hebrew poetry (saying something and emphasizing it with another similar statement in other words, and in some cases with a superlative). The form and message of the adjacent verses, before and after it, seem no different: "It cannot be gotten for gold, neither shall silver be weighed for the price thereof. It cannot be valued with the gold of Ophir, with the precious onyx, or the sapphire... No mention shall be made of coral or of crystal; yea, the price of wisdom is above rubies. The topaz of Ethiopia

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233 Note that Strashun has only 3:6-7; but I think that 3:4-5 are necessary to fully grasp the argument.

234 Note that the translation applies the same initial negative, *lo*, to both halves of the verse. It is not repeated in the original Hebrew but (credibly) taken as tacitly intended.

235 We could say that the proposition 'wisdom cannot be bought with vessels of fine gold' implies the proposition 'vessels of fine gold are less valuable than wisdom'. Or we could say that 'gold and glass are less valuable than wisdom' implies 'wisdom cannot be bought with gold and glass'. In either event, the two halves of our verse become comparable. But since 'gold' is a generic term and 'vessels of fine gold' is a specific one, they still cannot be placed as respectively minor and major terms. We would have to take 'gold and glass' together as the minor term – but what does this compound term signify? Clearly, any attempt to formulate a major premise with these terms seems incredible.

shall not equal it, neither shall it be valued with pure gold." In short: wisdom is priceless. If v. 17 is spun as an a fortiori argument, then so might verses 15, 16, 18, and 19, be. Certainly, anyway, if an a fortiori argument was intended, it could only at best be classified as 'implicit' (*satum*), in view of the possibility of interpreting the text very differently; that is, as simply not a fortiori.

I asked Gabay to respond to these doubts of mine. He brought to bear several possible explanations given in commentaries and some of his own. However, as I replied to him in detail, none of these suggestions succeed in credibly turning v. 17 into an a fortiori argument. I do not include the full conversation here, because it is rather long-winded and finally not very important. Every attempt to force this verse into some sort of a fortiori format is artificial. People tried and tried, apparently because someone first suggested this verse was *qal vachomer*, but it is a silly quest. The natural reading is poetic, as mentioned above. To conclude: I do not accept Job 28:17 as an explicit *qal vachomer*.

11. R. Chaim HIRSCHENSOHN

R. Chaim Hirschensohn (HH, b. 1857 in Safed, d. 1935)[236] wrote ***Berure HaMidot*** ("Clarifications of the Hermeneutic Principles," abbrev. BM), published in Jerusalem, 1928[237]. This seems to be the earliest 20th Cent. author on this subject.

Gabay reports that Hirschensohn's work (on p. 41) lists **19 instances** of *qal vachomer* **in the Torah**. However, as I show in detail further on, this number is misleading. Five cases were already known since antiquity (GR, ARN), namely: **Genesis 4:24, 44:8; Exodus 6:12; Numbers 12:14; Deuteronomy 31:27**. Two other cases were derived from Rashi, viz. **Genesis 6:3** and 6:9, the former being valid and explicit, but the latter rejected by me as only implicit. Two other cases were derived from Katzenellenbogen, viz, Leviticus 10:19 and **Deuteronomy 32:39**, the former rejected by me as at best implicit, while the latter was admitted by me as valid and explicit. Another three cases were upon examination rejected by me: one (mentioned long ago in GR), viz. Genesis 17:20-21, because only implicit; and two (newly proposed by Hirschensohn), viz. Genesis 3:1 and 27:37, because I could not see any a fortiori intent in them.

Consequently, in my estimate, *only 7 cases* can be attributed, wholly or partly, to Hirschensohn as '*historic firsts*', namely: **Genesis 3:22-23**, **4:14, 11:6, 14:23, 17:17, 39:8-9,** and **Exodus 6:30**. Note that all these cases are from the Torah proper, not from the Tanakh. This is not accidental. As Gabay observed: "H is trying to find 10 KV in the Torah itself – not in the Prophets and writings, so he does not expend that much energy on Nakh." We shall return to this central feature of Hirschensohn's research further on.

However, the above attribution of seven new cases to Hirschensohn is a generous assessment that needs to be qualified considerably. As I show below, only three of these cases are entirely attributable to him, viz. Genesis 14:23, 17:7, 39:8-9. Four other cases, viz. Genesis 3:22-23, 4:14, 11:6, and Exodus 6:30, were rightly flagged as *qal vachomer* by Hirschensohn, but wrongly formulated by him and had to be rectified by me to be made credible.[238]

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[236] https://en.wikipedia.org/wiki/Chaim_Hirschensohn.

[237] https://hebrewbooks.org/3094.

[238] For these reasons, in the general list of Biblical a fortiori arguments given above in chapter 3, I identify the former three cases as "First found by HH" and the latter four as "First found by HH/AS."

I would add to Hirschensohn's list of '*historic first*' Torah cases, *an 8th case*, viz. **Genesis 18:12**. Although it was *not* (to my knowledge) spotted by him, I propose to add it to his list because it is thanks to his discovery of Genesis 17:17 that I noticed the analogous argument in 18:12.[239]

It should additionally be noted that, until we learned from Hirschensohn of the *qal vachomer* in Genesis 6:3, we were not aware that Rashi had spotted this case. Therefore, although Hirschensohn did not discover this case, he should still be credited with drawing our attention to it (in Rashi's name). This case falls under the category of 'derived' for Hirschensohn, of course. It should be noted that Hirschensohn evidently did not systematically look for all the cases flagged by Rashi (see chapter 6), and so missed many he could have learned from him. However, the explanation may be that Rashi did not find any new Torah cases, and these were the cases of principal interest to Hirschensohn.

The majority of other cases mentioned by Hirschensohn were derived from Ashkenazi and Katzenellenbogen; surprisingly, however, he does not mention all the cases flagged by these two authors (see chapters 7 and 8), which suggests that he did not fully peruse their work (assuming Gabay's research was thorough). He also refers to Einhorn, with much admiration, but it is not clear what he learned from him since Einhorn has not published a list (see chapter 9). Surprisingly, Hirschensohn nowhere mentions Strashun's work, so he does not seem to have known of this author's many discoveries (see chapter 10)[240].

As already mentioned, Hirschensohn did not apparently search for new instances of *qal vachomer* **in the Nakh** (the Biblical books after the Torah), because he was especially interested in the Torah proper. Instead, he referred briefly to work in this field by Ashkenazi and Katzenellenbogen. According to Gabay, he only mentioned (on pp. 44-45) the **22 cases** found in these two authors' works listed below. The fact that these are not all the cases listed by these authors means that we cannot assume that Hirschensohn was aware of those not specifically listed.

Gabay reported that Hirschensohn cited 5 cases found *in Ashkenazi's work*, namely: **Isaiah 20:6; Jeremiah 25:29, 45:4-5, 49:12; Job 9:13-14**. These are all cases already known to us and validated. One of them, viz. Job 9:13, was found by Rashi before Ashkenazi; and the other four were newly found by Ashkenazi. It is interesting to note that these five cases in Ashkenazi, and only them, were not later mentioned by Katzenellenbogen[241].

Gabay also reported that Hirschensohn cited 18 cases found *in Katzenellenbogen's work*, namely: **Deuteronomy 32:39; 1 Samuel 14:29-30; 2 Samuel** 10:16, **12:18, 16:11; 1 Kings 8:27; 2 Kings 10:4; Ezekiel 33:24; Psalms 25:8-9, 78:20; Proverbs 15:11, 19:7a, 19:10, 21:27; Job 4:18-19, 15:15-16, 25:5-6**, plus one of the following two: initially, Gabay listed Proverbs 10:17 (without specifying its a fortiori intent) as the 18th case, but in a later listing he omitted it without explanation, and instead listed Proverbs 19:7b (again without specifying its a fortiori intent) as the 18th case. I asked him to clarify this inconsistency, but he did not reply[242].

Of the said 19 cases, I confirm 16 as valid. Two valid cases, viz. Deuteronomy 32:39 and Psalms 25:8-9, we did not have in our listing for Katzenellenbogen (or anyone else), and added them on

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239 I therefore, to be fair, label this case as "First found by HH/AS."

240 I specifically asked Gabay to double-check this fact and he confirmed it.

241 I asked Gabay to look for an explanation; he replied: "I have no idea why K missed these."

242 I must mention Gabay's non-reply to certain queries, to explain why there are some lacunae in my treatment of Hirschensohn. Unfortunately, after six months of friendly and fruitful cooperation, before the job we set out to do together was finished, R. Gabay suddenly stopped communicating, without any forewarning or apology, claiming (after I requested an explanation) to be too busy!

to it ex post facto. Two valid cases, viz. 2 Samuel 12:18 and Psalms 78:20, we did not have in our listing for Katzenellenbogen, but did have them for Strashun and me (in JL), respectively. Two more valid cases are already known to us as first appearing in Katzenellenbogen's listing, viz. 1 Samuel 14:29-30 and 2 Samuel 16:11. Ten more valid cases were already known to us as listed by Katzenellenbogen but not as the first: two preceded by Rashi only: Ezekiel 33:24 and Proverbs 19:7a; four preceded by Rashi and Ashkenazi: Proverbs 15:11, 19:10, 21:27; Job 15:15-16; and four preceded by Ashkenazi only: 1 Kings 8:27; 2 Kings 10:4; Job 4:18-19, 25:5-6. The three remaining cases, viz: 2 Samuel 10:16, Proverbs 10:17, and Proverbs 19:7b, which are new to us, I reject because I see no a fortiori intent in them.

As can be seen, Gabay did not previously include 4 cases in Katzenellenbogen's listing, viz. Deuteronomy 32:39, 2 Samuel 12:18, and Psalms 25:8-9, 78:20. I therefore ex post facto added them to it, as well as to Hirschensohn's list (and Strashun's and my accounts were accordingly corrected too, as appropriate). Also note that Hirschensohn did not always find the earliest mention of cases he listed. Thus, he did not notice various cases to have been previously known to Rashi only (2 cases), to both Rashi and Ashkenazi (4 cases), and to Ashkenazi only (4 cases). His missing out on 6 cases in Rashi shows that he did not always check out Rashi's commentaries; his missing out on 8 cases in Ashkenazi shows that he did not read his work very carefully and was content instead to accept Katzenellenbogen's claims without verification.

Summing up the above findings, we can say that Hirschensohn presented a list of 41 instances of Biblical a fortiori passages; of which, 19 were in the Torah and 22 were in the Nakh (5 in Ashkenazi and 17 in Katzenellenbogen). Hirschensohn regards these numbers as probably minima, since (Gabay told me) he says: "who can tell how many more *qal vachomer* can be found after more profound research?" (p. 44).

However, after careful scrutiny, I have recognized – as valid, explicit a fortiori argument – only the following cases. A total of only **15 cases in the Torah** proper, including the 5 from GR, 1 from Rashi, 1 from Katzenellenbogen, 7 historic-firsts by Hirschensohn, and **1** additional historic-first added on by me. To this we may add a total of only **26 cases in the Nakh**, including the 6 remaining cases from GR, 5 cases he found in Ashkenazi's work and 15 more he found in Katzenellenbogen's work. Thus, of the cases proposed by Hirschensohn, I have for one reason or another, *rejected 8 cases*, of which 5 Torah cases (1 from GR, 1 from Rashi, 1 from Katzenellenbogen, and 2 newly proposed) and 3 Nakh cases (also newly proposed). Thus, his valid list comprises only **40 cases** (not counting the **1 added** on by me) instead of 48 cases that he originally proposed (according to Gabay's reports).

The following is my **detailed analysis** – narrative, linguistic, and logical – of the above-mentioned eight historic-first cases of Biblical a fortiori argument in Hirschensohn's listing. Other cases mentioned by him, which I rejected for one reason or another, will be considered in equal detail after that.

Genesis 3:22-23. "Behold (*hen*), the man is become as one of us, to know good and evil; and now (*ve-atah*), lest he put forth his hand, and take also of the tree of life, and eat, and live forever. Therefore (*ve*), the Lord God sent him forth from the garden of Eden, to till the ground from whence he was taken."

Gabay reports that Hirschensohn, based on the Vilna Gaon's claim that use of the expression *hen* in a Biblical verse is indicative of *qal vachomer* intent, proposes the following reading of v. 22: "Behold, man has become like one of us, knowing good and evil, even before eating of the tree of

life – now, should he put forth his hand and take also of the tree of life – he will certainly become even more like us, and live forever."

The a fortiori argument proposed by Hirschensohn, more formally stated, is thus: Man prior to eating of the tree of life is godlike enough to know good and evil (minor premise); therefore, man after eating of that tree would be godlike enough to live forever (conclusion). The tacit major premise here is: Man after eating of the tree of life will be more godlike ("as one of us") than he is already before doing so; and the proportionality premise is that one's power may grow from 'knowing good and evil' to 'living forever' as one becomes increasingly godlike. This is a positive subjectal a crescendo argument (**+s&**).

However, as regards the language used in this passage, it is not *hen* alone, but the combination of *hen* and *ve-atah*, which together signal an if-then statement, that here indicate (if at all) an a fortiori discourse. Clearly, v. 22 cannot logically be comprehended without v. 23, because v. 22 says "and now (*ve-atah*)" implying some consequence, but leaves it unspecified, and it is only in v. 23 that the intended consequence is told. The sentence "lest (*pen*) he put forth his hand, etc." cannot be the intended follow-up of "and now (*ve-atah*)." So, there is an ellipsis at the end of v. 22, and v. 23 must be mentioned with it to make its intent explicit.

Note that Hirschensohn's *qal vachomer* reading does not mention or take into consideration v. 23. So, his interpretation is deficient and cannot be an accurate rendition of the narrative. We therefore need to formulate an a fortiori reading that integrates the whole narrative. But before proceeding further, let us take a look at the wider context.

The trees of life and of knowledge of good and evil are both first mentioned in Gen. 2:9. Then, in v. 16, man is allowed to eat of all trees; but in v. 17 eating of the tree of knowledge of good and evil is forbidden and the penalty for doing so is said to be death on that day. Note that the tree of life is not mentioned in this interdiction, so is presumably among the allowed trees. Gen. 3:1-5 tells the story of the temptation of Eve by the serpent, which ends in v. 6 with Eve and then Adam eating of the tree of knowledge of good and evil (this tree being alluded to v. 5 and in v. 6). Adam is then condemned to toil in verses 17-19. Finally, God in v. 22 reasons that if man, who has now acquired knowledge of good and evil, also eats of the tree of life, he will also life forever; so, in v. 23, man is expelled pre-emptively from Eden, and in v. 24 precautions are taken to prevent his return there.

I would infer from this narrative that the sentence of death (in 2:17) for eating of the tree of knowledge is in fact carried out by man's expulsion from Eden (in 3:23), to prevent him from eating (or continuing to eat) of the tree of life, making him mortal outside of Eden. Since eating of the tree of life was apparently not initially forbidden (being apparently allowed in the general statement of 2:16), it can be supposed that man would have eaten of the tree of life quite legally and lived forever if he had not eaten of the tree of knowledge. It is not stated whether man when he leaves Eden retains or loses the knowledge of good and evil illicitly acquired. And it is not made clear just what "knowledge of good and evil" means. But this lack of information need not concern us here.

In view of the above, we should interpret the narrative by means of the following a fortiori argument (instead of the one proposed by Hirschensohn). Minor premise: Man having already eaten of tree of knowledge of good and evil (Q) is excessively godlike (R) enough to have to be expelled from Eden (S) (in retribution). Conclusion: if man additionally now ate of the tree of life (P), he would be even more excessively godlike enough to have to be expelled from Eden (pre-emptively). The tacit major premise here is: Man after eating of both the tree of knowledge of good

and evil and the tree of life is more excessively godlike ("as one of us") than man after eating of the tree of knowledge of good and evil but before eating of the tree of life. This is a positive subjectal a fortiori argument (**+s**).

We could read it as a crescendo (**&**), if the appropriate proportionality premise were added, increasing the subsidiary term (S) from 'expulsion from Eden' to some still more severe penalty (say, for example, 'expulsion and homelessness'). But the text (v. 22) does not mention such increasing penalty. We can however note accessorily that whereas the penalty for eating of the tree of knowledge is retributive, that for additionally eating from the tree of life is pre-emptive. But the subsidiary term (S) remains essentially the same: expulsion from Eden (as decided in v. 23). Clearly, our reading here is far more accurate than Hirschensohn's, even though inspired by it[243].

In sum, I accept Hirschensohn's claim that there is an a fortiori intent in Gen. 3:22, but I beg to differ regarding its precise form because he fails to take v. 23 into consideration in his formulation. The a fortiori argument proposed by me may be counted as explicit, even though it is not immediately apparent and relatively complex, because it accurately rephrases the Torah passage in question and no other interpretation does so. Admittedly, I have in the past[244] assessed this verse as "at best implicit," following the opinion of Jacobs[245], who viewed it as "extremely doubtful." But that was before I was informed by Gabay of Hirschensohn's insightful interpretation. Note in passing that Rashi does not signal a *qal vachomer* here; so, it is not quite so obvious.

Incidentally, concerning the word *hen*, usually translated as 'behold', Gabay reports that the Gaon de Vilna says, in his book *Aderet Eliahu*, that *hen* is "an expression of KV" (i.e. of *qal vachomer*). While it is empirically evident, just by looking at a concordance, that *hen* is *often* associated with a fortiori discourse, it is equally evident that this is *not always* the case. For this reason, I wonder whether the Vilna Gaon actually claimed that there is *necessarily* an a fortiori intent in Biblical text wherever *hen* is used – it could be that Hirschensohn, or maybe Gabay, misunderstood him.

In fact, in the Pentateuch, there are 45 instances of *hen*; and in the rest of the Tanakh, another 273 instances[246]. Looking at these verses, it cannot by any stretch of the imagination be said that they all involve a fortiori discourse! One certainly cannot assume a priori, just because the Vilna Gaon said so, that if a verse contains the expression *hen*, it necessarily involves a *qal vachomer*. The Gaon de Vilna would have had to first show through ad hoc analysis that each and every one of these 318 instances of *hen* involves a fortiori discourse, *before* he had the right to make such a sweeping claim – and I very much doubt he ever did the required exhaustive research (surely Hirschensohn or Gabay would have mentioned it if he had). One may appeal to an authority if the person has evidently done the homework; but relying merely on someone's name and fame is *ad hominem* argument with zero weight of rational conviction.

Genesis 4:14. "Behold (*hen*), Thou hast driven me out this day from the face of the land; and (*ve*) from Thy face shall I be hid; and (*ve*) I shall be a fugitive and (*ve*) a wanderer in the earth; and it will come to pass (*ve-hayah*), that whosoever findeth me will slay me."

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243 It is interesting to note, in passing, the similarity in language between the verses Gen. 3:22-23 and Gen. 11:6; namely, the use of the expressions *hen/ve-atah* in both. Hirschensohn rightly spots a fortiori intent in both; but whereas he correctly formulates the argument in 11:6, he does not quite get it in 3:22, because he fails to see that *ve-atah* here refers to 3:23.

244 In my AFL, chapter 16:4.

245 In his previously mentioned essay, fn. 7.

246 See here: https://www.blueletterbible.org/lexicon/h2005/wlc/wlc/0-1/.

Gabay reports that Hirschensohn interprets the verse as: "If I am banished 'now', when You have already cursed me (v. 12: 'When you till the soil, it shall no longer give her strength to you'), *kol sheken* (a fortiori): if 'from Your face I am to be hidden' then I will be cursed 'I am to be unsettled and a wanderer on the earth; whoever finds me, will kill me'".

Note that Rashi does not flag this verse as *qal vachomer*, nor even at all comment on it. However, the verse does use language, viz. *hen/ve*, which is in some cases indicative of *qal vachomer*. This may well signal that an if-then statement is intended here, though its antecedent and consequent need to be identified. However, it is not clear exactly what Hirschensohn has in mind when claiming an a fortiori argument occurs here. What is 'the more' (the major term, P) and what is 'the less' (the minor term. Q), and what are they more or less *of* (the middle term, R), and what is the intended predicate for these subjects (the subsidiary term, S)?

We must ask: is the proposed a fortiori argument really intended, or even at least implied, by the given text? Or is it artificially read into the text? I think it is fair to say, looking at the context, that Cain is overwhelmed by all the curses befalling him as a result of his murdering Abel. The curses can well be perceived as going from bad to worse. But does a *progression* necessarily signify an a fortiori argument? No – unless an *inference* from one thing to another is arguably intended. I think in this case not only is a progression intended, but also indeed an inference.

The purpose of Cain's speech is to alert God to the likelihood that someone (though the Torah narrative does not mention the existence of other people yet, other than Adam and Eve) will eventually kill him, given the many ways that God has cursed him (in v. 14, banished, not looked after, fugitive, wandering). This is evident from the next verse (v. 15), which reads: "And the Lord said to him, 'Therefore (*lachen*), whoever kills Cain, vengeance will be wrought upon him sevenfold', and the Lord placed a mark on Cain [so] that no one who finds him slay him." This shows that God received Cain's complaint as an argument leading to a conclusion. God's response begins with a 'therefore', implying that He is following up on Cain's said plea.

Here, note well, the premise is indicated by the expression *hen* (behold) and the conclusion is indicated by the expression *ve-hayah* (and it will come to pass). This linguistically distinguishes this last item from the preceding list of four curses tied together by three *ve* (and) conjunctions. The last item (the danger of being killed) is thus not an added curse by God, but a predicted result of the preceding. God did not decree it, but Cain inferred it as a likely effect of God's decrees. He feared that as someone with no social status he would be in a very risky position.

What was Cain's argument, then? It was that God sentenced Cain to a number of curses; but these curses would likely produce an *additional* penalty (being killed by someone) that God had not explicitly decreed and therefore presumably did not intend; therefore, since the death of Cain (killed by some random perpetrator) was apparently not God's will (at least not immediately), Cain appealed for its prevention, and God duly responded by issuing a threat to potential killers. This would seem like a fair assessment of the reasoning involved.

Now the question is: can this discourse be cast into an a fortiori argument? There is certainly an element of sufficiency (which is essential to a fortiori reasoning) in this discourse: the four curses mentioned are perceived as punishment *enough* for Cain's crime; so that adding a fifth curse (namely, being killed off) would (according to Cain's appeal) amount to *too much* punishment. In other words, if the four said curses suffice to punish the crime, then allowing Cain to be killed would be an excessive further punishment.

This analysis suggests the following compound argument: The four curses listed plus the said threat to life (P) are a more severe a penalty (R) than the four curses without that endangerment

(Q); and these four curses alone (Q) are severe enough to fully punish Cain's crime of murder of Abel (S); therefore, the four curses plus endangerment (P) are severe enough to punish that crime and more (>S). Moreover, since what fully punishes is just, whereas what punishes more than that is unjust, it follows that adding threat to life to the four is unjust. Therefore, if God has not intended such endangerment as part of his sentence, He should somehow ensure its prevention.

Cain's appeal would thus consist of a positive subjectal a crescendo argument (**+s&**), corresponding to v. 14, followed by application of a couple of if-then principles to its result, the first to judge endangerment unjust, the second to prescribe its prevention, corresponding to v. 15. We can therefore admit Gen. 4:14 as involving an explicit a fortiori argument, in that the text cannot be correctly understood without such interpretation.

Now, let us return to Hirschensohn. He can be credited with having vaguely pointed to the presence of an a fortiori argument in this verse. But he cannot reasonably be said to have correctly formulated that a fortiori argument. It does not suffice simply to insert a *kol sheken* somewhere in the middle of the verse and hope for the best. The interpreter must be able to say exactly where that connective phrase belongs and why it belongs there. Looking at Hirschensohn's commentary, his interpretation is far from clear. It certainly, anyway, does not correspond to the correct interpretation proposed above.[247]

Genesis 11:6. "And the Lord said, 'Behold (*hen*), they are one people, and they have all one language; and this is what they begin to do; and now (*ve-atah*) will it not be withheld from them, all that they have planned to do?'"

Gabay reports that Hirschensohn, based again on the Vilna Gaon's guideline that use of the expression *hen* signals a *qal vachomer* intent, proposes the reading: "If what causes them to come together to rebel against Me is the fact that they feel a togetherness because 'they are one people, and all of them have one language' (v.6), then so much more so [would they be inclined to rebel] should they succeed in 'building for ourselves a city and a tower with its top in the heavens, and let us make a name for ourselves' (v.4) – therefore they should certainly 'be withheld from all that they scheme to do'.

The a fortiori argument proposed by Hirschensohn, more formally put, is thus: The people united only by a common language (Q) are rebellious (R) enough to be opposed by God (S) (minor premise); therefore, the people further united by building a city and tower (P) would be rebellious enough to be opposed by God (conclusion). The tacit major premise here is: People further united by a city and tower will be more rebellious than people united only by a common language. This is a positive subjectal a fortiori argument (**+s**). In my opinion, there is no need for an a crescendo reading here; it is conceivable that God's opposition to 'united by common language' (implied by the minor premise) alone would suffice to make Him scatter the people, in which case their 'building a city and tower' would be just added cause.

Note that, although the Vilna Gaon reportedly regards *hen* as a general indicator of *qal vachomer*, it is not *hen* alone which is indicative here of such argument, but *hen* combined with *ve-atah*. The expressions *hen/ve-atah* together signify an if/then statement; the sentence after *hen* is the antecedent, and the sentence after *ve-atah* is the consequent. We should additionally draw attention to the next verse (v. 7), although it is not part of the *qal vachomer* as such. This reads: "Come (*habah*), let us go down, and there confound their language, that they may not understand one another's speech." The expression *habah* amplifies the *ve-atah* consequent; the sentence after it

[247] For these reasons, I claim part of the credit for this case and label it HH/AS.

tells us more precisely how what the people's intentions will be countered – viz. by confounding their language.

While Hirschensohn's proposed a fortiori reading is credible, a better reading is possible, as I will now show. Although Rashi does not signal a *qal vachomer* here, his commentary is pertinent. He interprets[248] "they are one people, and they have all one language" as "goodness (*tovah*)," i.e. as something positive and therefore *not* disapproved of by God – whereas in Hirschensohn's reading (assuming Gabay has correctly rendered it) these factors are given a negative twist. Following Rashi's reading, the initial problem (the minor term, Q) is not the people being united and able to speak to each other, but rather "this is what they begin to do," with the word 'this' pointing to 'building a city and tower' (the plan mentioned in v. 4). The intensification of the problem (the major term, P), which God wants to forestall, is then 'succeeding to build a city and tower'.

Thus, the *qal vachomer* should rather be worded as follows: The people beginning to build a city and a tower (Q) are rebellious (R) enough to be opposed by God (S) (minor premise); therefore, the people succeeding to build a city and tower (P) would be rebellious enough to be opposed by God (conclusion). The tacit major premise here is: People succeeding to build a city and tower will be more rebellious than people beginning to build a city and a tower. This is a positive subjectal a fortiori argument (**+s**). The reading is not a crescendo because God is clearly determined to intervene (presumably in the specified way) already at the early stages of construction, without waiting for the completion of the projects.

This seems to me a more credible formulation, because of Rashi's said remark and because Hirschensohn's explanation does not take into consideration the words "and this is what they begin to do" in the verse. This reading, then, perceives God's anger as directed, not at the unity and monolingualism of the people of Babel, but at their building projects (which they have started but not yet completed). His statement "will it not be withheld from them, all that they have planned to do?"[249] tells us His decision to pre-empt these building projects, and the means for that is specified in the next verse (v. 7) to be to "confound their language," adding "[so] that they may not understand one another's speech." Regarding the latter addition, note that the people will be made to cease understanding each other as a *pragmatic* way to obstruct their building projects – not because (as suggested in Hirschensohn's reading) God disapproves *in principle* of monolingualism and unity.

To conclude, Hirschensohn's reading is formally acceptable, but this alternative reading based on Rashi's comments looks to me like a materially closer fit[250]. I fully agree with Hirschensohn's claim that an explicit a fortiori argument is to be found in Gen. 11:6, but I beg to differ regarding its precise terms. The fact that there are two possible *qal vachomer* readings should not lead us into viewing both as implicit, because whichever reading is adopted the verse will still *in fine* be characterized as a fortiori in intent. Note that I now accept the argument as explicit, even though in the past[251], before Gabay informed me of Hirschensohn's formulation, I assessed it as "at best implicit," following the opinion of Jacobs[252], who viewed it as "extremely doubtful." I am always open to correcting my errors on the basis of new information.

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248 Based on *Seder Eliyahu Rabbah*, chapter 31, according to the English translation in Chabad.org.

249 Rashi remarks that "will it *not* be withheld..." is intended as a question; this is of course correct, but the question is rhetorical; it means: "it *will* be withheld."

250 For these reasons, I claim for Rashi and myself parts of the credit for this case and label it HH/R/AS.

251 In my AFL, chapter 16:4.

252 In his previously mentioned essay, fn. 7.

Genesis 14:23. "I will not (*im*) take a thread nor a shoe-latchet nor aught that is thine, so that (*ve*) thou shouldest not say: I have made Avram rich."

Gabay reports that Hirschensohn proposes a *qal vachomer* reading, as follows: "The King of Sodom said to Avram (v. 21) 'give me the souls, the wealth you can have'. Hirschensohn understands that the king of Sodom did not just mean that Avram should keep the wealth that was the spoil of war – this is not a present but Avram's right as a victor. Rather the king wanted to offer him wealth that he had hidden in his storehouses in his kingdom that the four kings had not captured in battle – this is a true gift. Avram's response was that [since he] had not taken even a shoelace or a sandal strap from the spoil of battle, to which he was entitled, he would certainly not accept the further gift of wealth that the king wished to bestow on him."

I assess this claim as follows. The proposed a fortiori argument is: If I have not accepted from you the spoils of war to which I was entitled, then obviously I will not accept gifts from you which would make you seem to have enriched me. More precisely put: if Avram (S) wishes to avoid being indebted (R) enough to refuse what is due to him (Q) (minor premise), then perforce Avram wishes to avoid indebtedness enough to refuse what is not due to him (P) (conclusion). The tacit major premise here is: More willingness to be indebted is required to accept things not due to one than to accept things due to one. The a fortiori argument is negative predicatal (**-p**), and quite valid.

The question now is: is this a fortiori argument explicit (*meforash*) or implicit (*satum*)? Note first that Rashi, though he comments on this verse, does not flag it as *qal vachomer*. Second, note that there is no word or phrase in the Biblical verse suggestive of *qal vachomer*. However, these missing indices are not determining. The main issue is: is there any way to understand this verse other than through the proposed *qal vachomer* reading? I would say not.

The a fortiori argument clearly reflects Avram's thinking, albeit unspoken; it explains his de facto reluctance to accept anything from the king of Sodom, be it due or gifted; otherwise, there would be no explanation for his attitude. Therefore, we must admit Hirschensohn's novel insight of an explicit a fortiori argument here.

Genesis 17:17. "And Avraham fell upon his face, and he laughed. And he said in his heart: Shall (*ha-le*) a man a hundred years old beget? and shall (*ve-im*) Sarah, a woman of ninety, bear?"

Gabay explains Hirschensohn's inclusion of this case in his listing as follows: "This can also be read as a KV. Even had Avraham had a young wife [of childbearing age] it would be impossible for him at the age of one hundred to father a child. Now that Sarah was an older woman of ninety years it is certainly impossible that she could bear a child."

Thus, the proposed *qal vachomer* here is: If Avraham (a hundred-year-old man) had a younger wife (of naturally childbearing age) (P), it would be difficult (R) enough for him to beget (S) (minor premise); all the more so, with his actual wife Sarah (a ninety-year-old woman, way past childbearing age) (Q) it would be difficult enough, indeed even more difficult, for him to beget (conclusion). The tacit major premise here is that an older man with an older woman has more biological difficulty begetting than the same older man with a younger woman. This is a positive subjectal a fortiori argument (**+s**). The argument is not a crescendo, because the subsidiary term (S) remains the same, viz. Avraham begetting.

Rashi does not signal an a fortiori argument here, but he makes some interesting comments. He interprets Avraham's two statements differently: for himself, he wonders whether "such kindness was done to anyone else" by God; while for Sarah, he wonders whether she shall be "worthy of giving birth." Thus, for Rashi, the issue is not primarily biological (one of age), but rather one of

merit and kindness. Nevertheless, Rashi does draw attention to the biological aspect, when he points out that in past generations (according to the Bible) people were having children at much older ages, but by Avraham's time people had to hasten and beget much earlier.

There is no doubting the empirical truth of the three premises and of the conclusion of Hirschensohn's proposed *qal vachomer*. But we might well ask: was that *inference* Avraham's thinking process? Could he not have been having successive thoughts without intending to formulate such an inference? He could have simply thought to himself that it is unlikely for a hundred-year-old man to beget; and then, separately, reflected that it is unlikely for a ninety-year-old woman to bear a child. This would be a mere conjunction of two thoughts, not a process of inference. Note that no language commonly associated with a fortiori discourse is involved here.

I was at first sight skeptical of Hirschensohn's a fortiori reading, judging it a bit forced. But I was convinced by taking into consideration Avraham's 'laughter' at the announcement. Though Rashi interprets this as rejoicing, it can also be considered as an expression of surprise and wonder. We can easily imagine Avraham (as well as rejoicing) contemplating the enormity of what he is being promised, thinking: "What? Not only am I rather old for this at age 100 (even with a younger woman), but *on top of that* Sarah is also rather old for it at age 90!" Remember, Avraham had indeed begotten Ishmael with a younger woman, namely Hagar, at 86; but now he was 100. As for Sarah, she had until now, even when much younger, been unable to bear a child.

So, there is definitely a movement of thought from the unlikelihood of an old man like Avraham begetting to the *compounded* unlikelihood of such a man doing so with an old woman like Sarah. I therefore finally agree with Hirschensohn's original insight that there is an a fortiori argument here, and that it proceeds through comparison to a younger woman. This is to be counted as an explicit (*meforash*) case, since it takes more evidence into consideration than the simpler reading above described.

Note that Louis Jacobs assessed this case as "extremely doubtful"[253]. I followed suit judging it as "at best implicit"[254], but I did not at the time know Hirschensohn's proposed *qal vachomer*. Once informed of it, I saw its power of conviction, and changed my assessment accordingly.

Genesis 18:12. "And Sarah laughed within herself, saying: 'After I am waxed old, will (*hayta*) I have smooth flesh [i.e. get pregnant]? Also (*ve*) my lord being old?'"

I propose, after reflection, the following *qal vachomer* reading for this verse: Sarah (now ninety years old, past menopause) would, with Avraham (now one hundred years old) if he were currently younger than he is, (Q) have had difficulty (R) enough having a child (S) (minor premise); all the more so, she can expect, with him now in fact quite aged, (P) have difficulty enough, indeed even more difficulty, to have a child (conclusion). The tacit major premise here is that an older woman would have more biological difficulty having a child with her husband in his actual aged condition than she would with her husband had he hypothetically been younger. This is a positive subjectal a fortiori argument (**+s**). The argument is not a crescendo, because the subsidiary term (S) remains the same, viz. having a child.

Although Hirschensohn did not, to my knowledge, point out this verse as *qal vachomer*, it is clear that we can perceive it as such once we have, following his lead, perceived Genesis 17:17 as such, because it is *a mirror image* of it from the female point of view. Sarah seems to be thinking: "Not

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253 In his previously mentioned essay, fn. 7.

254 In my AFL, chapter 16:4.

only am I rather old for begetting at age 90, but *additionally* Avraham is also rather old for it at age 100!" Note that the previous verse (v. 11) reads "Now Avraham and Sarah were old, coming on in years; Sarah had ceased to have the way of the women."

However, note well, I have intentionally formulated this *qal vachomer* differently than the one in Gen. 17:17, for the following reason. I could have had, by analogy, in the minor premise, Sarah having a child with some younger man just as previously Avraham is presented as having a child with some younger woman. But obviously, this narrative would have invalidated the argument, since a virtuous and loving wife like Sarah *would surely not even for a moment imagine herself* coupling with a man (of whatever age) other than her husband Avraham. It was culturally possible and legitimate for Avraham to have thought of another woman, but incredible that Sarah would ever call to mind another man.

This may explain why Hirschensohn and perhaps others did not point to this verse as an a fortiori argument – if they at all spotted it. Sarah could not possibly have had the said immodest thought. So, if such thought was essential to the a fortiori interpretation, such reading had to be abandoned; and instead, Sarah's thinking would have to be described as a conjunction of two separate thoughts (as earlier first proposed for Gen. 17:17).

However, it occurred to me after a while that the a fortiori interpretation can still be upheld if we simply assume that Sarah is comparing reproductive performance with a contra-factually younger Avraham and the factually older Avraham. We know, of course, that they tried having children in the past, when both of them were younger than they are now, and that they unfortunately failed. But though this past performance lowers the chances that they today succeed, it does not reduce them to zero; so, the *qal vachomer* remains credible.

In sum, we have here, in Gen. 18:12, found an additional instance of Torah *qal vachomer*, which we can consider as explicit (*meforash*) since it is the best reading, indeed the only fully accurate reading, of the given text. This finding was inspired by Hirschensohn's genial interpretation of Gen. 17:17, which I could not see without his guidance; but I can fairly claim the case as mine since neither he nor anyone else (to my knowledge) discovered it before me[255].

Genesis 39:8-9. "But he refused, and said unto his master's wife: 'Behold (*hen*), my master, having me, knoweth not what is in the house, and he hath put all that he hath into my hand; he is not greater in this house than I; neither hath he kept back anything from me but thee, because thou art his wife. How then (*ve-ekh*) can I do this great wickedness, and sin against God?'"

Gabay reports Hirschensohn's a fortiori exegesis of this verse is as follows[256]:

> "Again, using the indicator Hen, following the Vilna Gaon, this could be read as follows: Behold, my lord has given me full confidence and responsibility for everything in the house, and all that he has he has placed in my hand, he has withheld nothing from me but you, in that you are his wife. So, if I were to take advantage of him and steal an item from the contents of the house, I would be committing a sin in the eyes of G-d, even though nothing in this house has been withheld from me. Kol sheken if I would take advantage of his wife, that he has indeed withheld from me, I would certainly be committing a grave sin in the eyes of G-d."

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255 I consequently label this case as HH/AS, giving both of us part of the credit.

256 This is presumably a paraphrase, rather than a quotation, of Hirschensohn's position by Gabay. Kol sheken means 'all the more', i.e. it is indicative of a fortiori argument.

Thus, Hirschensohn here proposes the following a fortiori argument: If Joseph stole any property that he was permitted to use (Q), he would be dishonest (R) enough to be committing a grave sin (S) (minor premise); then all the more if Joseph stole any property that he was *not* permitted to use (such as his master's wife) (P), he would be dishonest enough to be committing a grave sin (conclusion). Its tacit major premise is: "stealing something one is *not* permitted to use is more dishonest than stealing something one is permitted to use." This is a positive subjectal a fortiori argument (**+s**).

It is clear to me that Hirschensohn is right, and an a fortiori argument is intended here; I would even advocate an a crescendo reading (**&**), meaning that the gravity of the sin increases from the minor term (Q) to the major term (P) – except that Joseph's statement is not emphasizing such intensification. The language used in this verse, viz. *hen/ve-ekh*, does suggest a *qal vachomer*; *hen* introduces its minor premise and *ve-ekh* introduces its conclusion. The *qal vachomer* can safely be declared explicit (*meforash*), as no other interpretation is conceivable: Joseph is evidently presenting a reasoning process from things permitted to him to something not permitted to him, to explain to his master's wife his vexing resistance to her frequent advances (but, of course, she is not reasonable).

Exodus 6:30. "But Moses said before the Lord, 'Behold (*hen*), I am of uncircumcised lips; so how (*ve-ekh*) will Pharaoh hearken to me?'"

Gabay reports that Hirschensohn, in his notes on Katzenellenbogen, cites this verse [as *qal vachomer*] 'in the name of Rashash'[257]. Rashi equates this verse with v. 12, saying: "Scripture repeats it here because it had interrupted the topic, and this is customary, similar to a person who says, 'Let us return to the earlier [topic].'"

Ex. 6:12, remember, has Moses arguing that since the children of Israel did not hearken to him, Pharaoh will not do so either, he (Moses) being of closed lips. But the present verse (v. 30) has Moses arguing that since his lips are closed, Pharaoh will not hearken to him. It does not mention the Israelites, so it is not identical. Rashi's identification of the latter verse with the former is reasonable from a literary point of view, but from a logical point of view it is only hypothetical. This means that if we followed Rashi, we could postulate the same *qal vachomer* here as in v. 12 – but this *qal vachomer* would have to be considered as here only implicit (*satum*), rather than as it is in v.12 explicit (*meforash*).

In my reading of v. 12, Moses's speech impediment, though mentioned by him, is not the operative middle term. I there propose the (tacit) major premise as effectively: Someone who has much faith in God (as the Israelites do) will hearken to a prophet (here, Moses) more likely than someone who has little or no faith in God (viz. Pharaoh). Whence the reasoning, using degree of faith as the middle term: If the Israelites did not have enough faith to hearken to the prophet Moses, then Pharaoh will not have enough of it to do so. This constitutes a negative subjectal a fortiori argument (**-s**).

If v. 30 is intended as a repetition of v. 12, albeit in abridged form, then the *qal vachomer* in the latter could be considered present in the former. But I do not think Rashi's explanation here tells the whole story. The earlier verse, v. 12, mentions the Israelites primarily, and Moses's speech impediment only accessorily, as an additional excuse: whereas the later verse, v. 30, does not at all mention the Israelites, and focuses entirely on Moses's impediment. Whereas the impediment

[257] This presumably refers to Samuel ben Joseph Strashun (Vilna, 1794-1872), the father of Mattityahu Strashun. But it could also point to R. Shalom Sharabi (Yemen. 1720-1777). Both are known by the acronym Rashash.

was mentioned in passing before, but logically ignored as incidental; here Scripture returns to it presumably to draw attention to its previously overlooked significance. That is, v. 30 is not merely a literary prop, a reminder of v. 12, but adds something extra to the narrative.

Here, in v. 30, if we are to formulate an a fortiori argument – and the language used, *hen/ve-ekh*, does suggest a *qal vachomer* – we must focus directly on Moses's handicap, as follows: More patience (R) is required to pay attention to someone with speech difficulties (P) than to pay attention to someone without such a handicap (Q) (tacit major premise); Pharaoh (S) would have patience not enough to hearken to Moses were he not handicapped (tacit minor premise); all the more, Pharaoh will have patience not enough to hearken to Moses he being in fact handicapped (given conclusion). This is a negative predicatal a fortiori argument (**-p**). I believe we can view this *qal vachomer* as explicit (*meforash*), even if both its major and minor premises are tacit, because one can hardly interpret Moses's statement differently.

We could admittedly read v. 30 as a mere statement of fact in hypothetical (if-then) form. It is reasonable to suppose that if someone (here, Moses) has speech difficulties, then people (here, Pharoah) will not hearken to him – though they might possibly indeed hearken to him if he did *not* have speech difficulties. But the a fortiori reading goes further: its minor premise and conclusion together suggest (by dilemmatic reasoning) that Pharaoh would likely not listen to Moses *anyway* (i.e. categorically, with or without handicap); and this intensification makes sense considering that Moses knew Pharaoh to be an arrogant and busy monarch.

Note that, although neither Hirschensohn nor Rashi before him formulated the *qal vachomer* proposed here, but assumed that the one in Ex. 6:12 is just repeated in Ex. 6:30, I do not claim this case as entirely my finding, because without Hirschensohn's suggestion that there is an a fortiori here, I would not have formulated one.

Rejects. In addition to the above-mentioned eight cases of Torah *qal vachomer* that Hirschensohn was the first to propose, and are recognized by me as valid and explicit, we need to consider another two cases which he was the first to propose, but which I reject (as not a fortiori), namely: **Genesis 3:1** and **27:37**.

Genesis 3:1. "Now the serpent was more subtle than any beast of the field which the Lord God had made. And he said unto the woman: 'Yea, hath (*af ki*) God said: Ye shall not eat of any tree of the garden?'"

Commenting on the sentence with the *af ki* expression, Rashi writes: "Did He say to you, 'You shall not eat of any, etc.?' Even though he saw them eating of the other fruits, he spoke to her at length in order that she answer him and come to speak of that tree." Note that Rashi does not say (as is his wont in similar situations) that *af ki* here means *qal vachomer*, nor does his reading suggest such argument.

Gabay reports that Hirschensohn interprets the verse as: "If the tree whose fruit is pleasing to eat, Hashem did not permit you to eat of it, you may certainly (*af ki*, i.e. *qal vachomer*) not eat from the fruits of the other trees." Hirschensohn then comments that "obviously the *qal vachomer* is false, because the Tree of Knowledge had good and bad in it."[258] The question we need to answer here is: is Hirschensohn's proposed a fortiori argument explicit or implicit, or even at all present in the given text?

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[258] Gabay told me that Hirschensohn 'brings many proofs' that *af ki* signifies *qal vachomer* intent, without spelling out these 'proofs'; but presumably they are simply other contexts in which *af ki* signifies *qal vachomer*.

The proposed argument runs: If the (more pleasing) fruit of the Tree of Knowledge is not pleasing enough to be permitted by God (minor premise), then the (less pleasing) fruit of all other trees is not pleasing enough to be permitted by Him (conclusion). The (tacit) major premise is that the fruit of the Tree of Knowledge is more (or at least as much) pleasing to eat than the fruits of other trees. The form of the argument is negative subjectal (**-s**). Hirschensohn's refutation of it is that the major premise is false, since the Tree of Knowledge is defined as 'having' both good and evil (not only good) in it.

But this proposal is a gross misrepresentation of the Biblical narrative! This can be seen if we consider the rest of it, i.e. verses **3:2-5**. They read: "And the woman said unto the serpent: 'Of the fruit of the trees of the garden we may eat; but of the fruit of the tree which is in the midst of the garden, God hath said: Ye shall not eat of it, neither shall ye touch it, lest ye die.' And the serpent said unto the woman: 'Ye shall not surely die; for God doth know that in the day ye eat thereof, then your eyes shall be opened, and ye shall be as God, knowing good and evil.'"

In my 2013 book *A Fortiori Logic*, appendix 6, dealing with teachings of logic implied in the Torah, I comment on this passage as follows:

> In Genesis 3:1-5, the serpent tries to tempt Eve by means of the following argument (here paraphrased): "since God did not say 'you shall not eat of any tree of the garden', then you may eat of this tree;" to which Eve rightly retorts, briefly put: "He said we may eat of all trees except this one." We can discern in this a teaching of logic, namely that the serpent's inference from 'not all X are Y' (i.e. 'some X are not Y') to 'this one X is not Y' is fallacious, and learn from Eve's reply that a proposition may be general and exceptive, i.e. have the form 'all X except this one are Y'.

Clearly, I do not interpret the passage as a fortiori, because there is no hint of such reasoning in it. Rather, I explain argument as a fallacious implication (by the serpent) from a particular negative categorical proposition (viz. that not all fruit trees are forbidden) to a singular one (therefore, this fruit tree is not forbidden), to which the correct retort (by Eve, evidently not so easily fooled) is that all fruit trees were permitted, although only one (the specified one) was exceptionally forbidden. The serpent is not trying to manipulate Eve through a complicated a fortiori train of thought – the thrust of his argumentum is much simpler.

This shows that Hirschensohn's proposed a fortiori argument is just fantasy: it does not reflect what the serpent is saying. The serpent is not, in the Biblical text, talking about the good taste (or pleasantness to eat) of the fruits of diverse trees, nor suggesting that God's forbidding consumption of a specified tree's fruit relates to its taste. Also, Hirschensohn's alleged refutation is based on a misreading of Gen. 2:9, where the forbidden Tree is defined as "of Knowledge of good and evil," and not as 'having' both good and bad taste!

Furthermore, Hirschensohn does not make explicit the tacit implication of the serpent's alleged conclusion, which would be a *reductio ad absurdum*: since you obviously *are* permitted to eat of all other fruits (as per 2:16, contrary to the a fortiori conclusion that you are not), it follows that you *are* permitted to eat fruits of the Tree of Knowledge (contrary to the a fortiori minor premise, 2:17). Obviously, the serpent's goal is not to prevent Eve from eating fruits of permitted trees (as the a fortiori argument has it), but to get her to eat fruit from the forbidden tree (as the unstated *reductio* has it)!

But most important, the logical form of the serpent's argument is not at all a fortiori, as above explained. Therefore, we may not acknowledge Hirschensohn's reading of Genesis 3:1 as *qal vachomer*, nor even admit it as an implicit case. Should we count anyone's arbitrary projection of

a fortiori into a text as at least implicit? Surely, there has to be some sort of hint in the text in support of such interpretation; it cannot be mere spin.

Clearly, Hirschensohn regards this case, and all others which he proposes, as explicitly a fortiori; he does not to my knowledge identify any case as merely implicit. I asked Gabay to look into Hirschensohn's book and tell me whether he anywhere discusses the traditional distinction between explicit (*meforash*) and implicit (*satum*) *qal vachomer* and formulates some relevant criteria; but Gabay did not reply to this query, so I cannot say what Hirschensohn thought on this subject.

Genesis 27:37. "And Isaac answered and said unto Esau: 'Behold (*hen*), I have made him thy lord, and all his brethren have I given to him for servants; and with corn and wine have I sustained him; and for thee, then (*apo*), what shall I do, my son?'"

Gabay reports Hirschensohn's a fortiori exegesis of this verse is as follows[259]:

> "Using the indicator Hen, following the lead of the Vilna Gaon, this could be read as follows: If you Esau, that I love you dearly, and that you are my eldest, I have still made Jacob a master – Gevir – over you, kol sheken your other brothers [that are younger and less worthy than you] have I given to him as servants. H then analyses this KV that this seems to contradict the principle of Dayo, because the KV should read: if Jacob is a Gevir over you, he should certainly be a Gevir over your brothers, but why should they be servants; Esau was never said to be a servant of Jacob. Unless the reading is: If he is a Gevir over you, even though you are eldest, then the younger children, who are less worthy, must be worse off – they are to be servants. H explains that the simple solution in untenable – namely that the KV should be simply: if Jacob is a Gevir over you [who is a favourite], he should certainly be a Gevir over your brothers [who are less]. This cannot be because the blessing Heveh Gevir le'akhekha – means be a master over [all] your brothers (Akhekha is plural, as opposed to Akhikha in the singular). Therefore we must read the KV as above, that the other brothers are to be considered servants, and we will not invoke the principle of Dayo if the KV will be invalidated – lo amrinan dayo lemifrakh KV."

Thus, Hirschensohn first proposes the following a fortiori argument: "If Isaac has made Jacob a master over his beloved eldest son Esau, then all the more he has given over as servants to Jacob his other brothers, who are younger and less worthy." Then Hirschensohn reflects that this argument may contradict the sufficiency (*dayo*) principle, because the minor premise states that Jacob is master over Esau, but does not say that Esau is a servant of Jacob, whereas the conclusion does not mention that Jacob is master over the other brothers, but does declare the other brothers to be servants. He then proposes instead the following a crescendo argument: "If Isaac has made Jacob a master over his eldest son Esau, then he has made his other brothers, who are less worthy, worse off by making them his servants." Hirschensohn rejects the simple reading: "If Isaac has made Jacob a master over his favorite son Esau, then all the more he has made Jacob a master over his (lesser) other brothers," arguing that (earlier, in v. 29) Isaac blesses Jacob by saying "be master over thy bothers," using the plural, from which Hirschensohn concludes that the other brothers are to be considered as servants, as he proposed initially (as in v. 37), thus ignoring the *dayo* principle in this case (as applying it would 'invalidate' the said a fortiori argument).

To my mind, all this is *pilpul* based on bogus logic.

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[259] This is presumably a paraphrase, rather than a quotation, of Hirschensohn's position by Gabay. H refers to Hirschensohn and KV means *qal vachomer*. Kol sheken means 'all the more', i.e. it is indicative of a fortiori argument.

Firstly, because Hirschensohn sets up a quite artificial hierarchical distinction between having someone as one's master (or lord) and being someone's servant (or slave). The terms master and servant are obviously intended as correlative[260]. Rashi, in his commentary on this verse implies it clearly: "If you acquire property, it will be his, for I have made him a *master* over you, and whatever a *slave* acquires belongs to his master[261]."

This means that the 'simple' reading of v. 37 that Hirschensohn rejects, namely: "If Isaac has blessed Jacob enough made him a master over his favorite son Esau, then all the more he has blessed Jacob enough to make him a master over his lesser remaining brothers," is quite appropriate. Its tacit major premise is: "More blessing is required to be made master over a favorite son than over a lesser son." This is a positive predicatal a fortiori argument (**+p**).

Anyway, v. 29, where Isaac blesses Jacob by saying "be master over thy brothers," using the plural, clearly applies to all Jacob's brothers, including Esau, which confirms the said 'simple' reading from Esau to other brothers that Hirschensohn considers 'untenable'. It is hard to see, then, how Hirschensohn manages to infer that the other brothers are to be considered as servants (lowlier than having a master). Surely, this is muddle-headed thinking by Hirschensohn (assuming Gabay has correctly described his discourse).

Moreover, v. 29 can used to construct a syllogism: All brothers of Jacob will be under his mastery; Esau is a brother of Jacob; so, Esau will be under his mastery" (1/ARR). That is, if any inference is to be drawn between these terms, it is rather this earlier syllogism than the a fortiori argument suggested by Hirschensohn with reference to v. 37. Since this inference from brothers in general to Esau is obvious, there is no need to look for a questionable *qal vachomer* from Esau to other brothers.

And who are the "other brothers" referred to here? Is it not true that Isaac has only had two sons, namely Esau and Jacob? Yet, as Hirschensohn remarks, "over thy brothers" (v. 29) is plural; and here "all his brethren" is also plural – so, these terms cannot refer to Esau alone. I presume this question has already been asked and answered by knowledgeable commentators, but I do not know the answer. Perhaps 'brethren' is used in a large sense, as a reference to non-family members of Isaac's household. I mention this side issue in passing, but it does not affect the main issue at hand.

Hirschensohn brings up the *dayo* principle (found in the Mishna and elaborated in the Gemara), claiming that it is here apparently contradicted, and ends up by claiming that it may exceptionally be ignored in this case because (this is a subsidiary principle, formulated in the Gemara) invoking it would effectively 'annul' the a fortiori argument. I submit that both these claims are out of place in the present context (because it is not specifically one of inference of law from the Torah). It is clear that Hirschensohn has not adequately studied the two principles he cites. His referring to them at all in the present context is theatrical nonsense.

The *dayo* principle is based on Mishna *Baba Qama* 2:5; it is then examined in BT *Baba Qama* 25a. I examine these two texts in great detail in my 2013 work *A Fortiori Logic*, in chapters 7 and 8. This topic is far too complex to review briefly here; interested readers should read my detailed analyses and conclusions in the said work. What needs be said here is only the following. (a) A fortiori argument is not limited to pure (non-proportional) reasoning, but is also valid in some cases that are a crescendo (proportional) in form; this is clear from the said Mishna and from other Gemara passages (even though 25a erroneously excludes proportionality). (b) The view expressed

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260 That is, if X is master of Y, then Y is servant of X; and vice versa.

261 *Pesachim* 88b.

in the said Gemara that the *dayo* principle may be ignored conditionally, specifically "when it would defeat the purpose of the a fortiori," is based on gross misunderstanding of a fortiori logic and not to be taken seriously.

Hirschensohn's discussion here around the applicability or not of the *dayo* principle is due to ignorance of the full content of the said Mishna, and to further confusion induced by the said Gemara. What he is actually asking is whether the Biblical verse should be read as purely a fortiori or as a crescendo. In truth, both these forms are formally valid, the difference between them depending on the truth or falsehood of a (usually tacit) premise of proportionality concerning the subsidiary term; but he has not studied a fortiori argument enough to know that. His discussion is thus based on fake logic and is mere blah-blah.

Let us now turn to the bottom line: is there an a fortiori argument in Genesis 27:37; and if so, what form does it take? The language used in this verse, viz. *hen-apo*, is certainly suggestive of a *qal vachomer* (*apo* derives from *af*, note). If there is an a fortiori argument here, its antecedent and consequent must be determined with respect to that wording. That is, any interpretation must have *hen* as pointing to the premise, and *apo* as pointing to the conclusion. The sentence following the word *apo* should be viewed as a rhetorical question; put in assertoric form, Isaac is denying that he can do anything for Esau.

Our hypothetical (if-then) reading is thus: If (*hen*) I have made Jacob master and you Esau and all his brethren his servants, and given Jacob sustenance with corn and wine, then (*apo*) there is nothing much left for me to bless you with. However, I see no way to turn these antecedent and consequent propositions into the minor premise and conclusion of an a fortiori argument. Therefore, I am obliged to deny that there is any *qal vachomer* intent in this verse. Note that Rashi does not suggest one, though he does comment on this verse.

Notice that I make no distinction between Esau and 'all the brethren' – Esau is one of them, but is singled out here because he is the one being addressed; that is, 'all his brethren' simply refers to all *other* brethren, *because* Esau has already been mentioned. My reading is therefore very different from Hirschensohn's. Unlike him, I do not at all acknowledge an inference from Esau to the other brothers (whoever they might be), and therefore have no recourse to any eventual distinction between having a master and being a servant. My reading includes in the antecedent not only Jacob's mastery, but also his corn and wine sustenance which Hirschensohn blithely ignores. The consequent about Isaac lacking a leftover blessing for Esau is confirmed in the next verse, where he replies: "Hast thou but one blessing, my father? bless me, even me also, O my father."

In conclusion, I very much doubt that we should admit Hirschensohn's a fortiori interpretation(s) of Gen. 27:37. His readings, including inference 'from Esau to other brothers' and distinction between having a master and being a servant, are mere spin. The given text does not intend them; it can certainly be credibly understood without getting into them. They are not even implicit; they are artificially tagged on.

There are three more Torah passages that Hirschensohn considers as having a fortiori intent, but which I **reject**, namely: **Genesis 6:9, 17:20-21; Leviticus 10:19.** These cases were proposed by other commentators before Hirschensohn, as he admits. I have analyzed them where they previously occurred: the first of these in the chapter on Rashi, the second in that on Genesis Rabbah, and the third in that on Katzenellenbogen[262]. So, I need not say more about them here.

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[262] Gabay does not flag Katzenellenbogen's case Proverbs 6:30 as also being claimed by Hirschensohn. I do not know whether this was an oversight on Gabay's part, or Hirschensohn indeed did not mention this case even though

Additionally, according to Gabay, Hirschensohn considered **2 Samuel 10:16** as having a fortiori intent. Maybe also one or two other cases... initially Gabay listed **Proverbs 10:17** as a case; but in a later listing he omitted it without explanation, and instead listed **Proverbs 19:7b** as a case. I asked Gabay to clarify for me Hirschensohn's *qal vachomer* formulations of these verses, but got no reply from him. Looking at the said passages, I see no a fortiori intent in any of them, and therefore reject them.

Judge for yourself. The first reads: "And Hadadezer sent, and brought out the Arameans that were beyond the River; and they came to Helam, with Shobach the captain of the host of Hadadezer at their head." The second reads: "He is in the way of life that heedeth instruction; but he that forsaketh reproof erreth." The third reads: "All the brethren of the poor do hate him; how much more do his friends go far from him! He that pursueth words, they turn against him;" the first half of this verse, being already known to Rashi as *qal vachomer* and easy to validate, presents no problem – it is presumably the second half (whose meaning is far from clear) which is here newly proposed as a second *qal vachomer*.

Regarding Proverbs 19:7b, which reads "He that pursueth words, they turn against him," maybe the proposed *qal vachomer* is: A man who is poor but does not 'pursue words' is despised enough to be avoided; a man who is poor and 'pursues words' is despised enough to be 'turned against' (i.e. more than just avoided). That would be a positive subjectal a crescendo argument (**+s&**); but it would be at best (if at all a fortiori) implicit. Rashi does not flag this sentence as *qal vachomer*. There is no linguistic indicator of a fortiori argument. The meaning of the term 'pursuing words' is in any event unclear and variously interpreted, so that an a fortiori reading is at best conjectural.

Concerning the tradition of ten *qal vachomer*.

As we have seen in the course of this study, the number of *qal vachomer* instances in the Tanakh became an issue and a scholarly quest sometime after the Midrash *Genesis Rabbah* (GR) was first published in ca. 300-500 CE, because this document quotes R. Ishmael (2nd Cent. CE) as saying that Genesis 44:8 is "**one of the ten *qal vachomer* arguments given in the Torah**" (GR 92:7). Moreover, *because* this same document (or a common, extant version thereof) lists nine more instances of *qal vachomer* **in the Tanakh**, viz. Exodus 6:12, Numbers 12:14, Deuteronomy 31:27, 1 Samuel 23:3, Jeremiah 12:5 (2 instances), Ezekiel 15:5, Proverbs 11:31, and Esther 9:12, the statement by R. Ishmael mentioning ten cases 'in the Torah' has always been taken to mean 'in the Tanakh' (reading the word Torah in its larger, colloquial sense).

An important problem was the discovery, at least from the time of Rashi (11th Cent.), that there are in fact ***more than ten*** instances of *qal vachomer* in the Tanakh. This put in doubt the said authoritative statement in GR 92:7. Was the number ten to be taken as meaning 'some large number', or perhaps 'at least ten, maybe more'; or were the ten mentioned in this document deliberately selected based on some unspoken criterion or criteria? No one dared accuse GR of simply being in error, even if other passages of the GR document itself mention additional examples of *qal vachomer* in the Tanakh, because the implication of error by an author from the Talmudic era would be devastating (even if, as is the case here, it is merely haggadic, not halakhic).

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it is claimed by Katzenellenbogen together with another case, viz. Leviticus 10:19, that Hirschensohn does mention. I did not ask Gabay this question because I stopped expecting him to reply.

Hirschensohn showed awareness of this problem when he wrote (in BM, vol. 1, p. 40, translation by Gabay): "The interpreters struggled with this Midrash when they found in the Bible many more than the number of KV considered here," and went on to remind readers that Ashkenazi, Katzenellenbogen, and Einhorn found new cases (without mention of Strashun, as already noted).

As regards the apparent inconsistency in GR itself, this could be explained away (I suggest) by viewing GR as a compilation of work by different authors, or by the same author at different times. But the number ten specifically in GR 92:7 could not be dismissed so easily. On the positive side, the ten cases listed in that passage included only four cases from the Torah, the other six being in the Nakh, thus confirming the interpretation of R. Ishmael's statement "in the Torah" as meaning "in the Tanakh." But on the negative side, even this list of four cases in GR 92:7 was incomplete – since it did not include, for instance, Gen. 4:24 (which is mentioned elsewhere in GR and in the later (7th-10th Cent.) work *Avot de-Rabbi Nathan* (ARN)).

A much later Midrashic work, *Yalkut Shimoni* (YS), dating from the medieval era (11th-14th Cent.), reiterates the tradition of ten instances of *qal vachomer*, but it apparently lists only nine instances, leaving out Ezekiel 15:5 without any explanation. Note in passing that from the nine cases that it does list, corresponding to cases listed in GR, it is evident that the author of YS read Ishmael's "in the Torah" as meaning "in the Tanakh,". The omission of one case (in the editions of YS that have come down to us) could easily, I daresay, have been an error of inattention by the author or by a later scribe. There was nothing logically wrong in the a fortiori argument implied by Ezek. 15:5, no technical reason to omit it. But this became a minor '*cause célèbre*', giving rise to considerable controversy in some quarters.

It was suggested, in due course, especially in more recent times, that the list of nine cases in GR 92:7, following Ishmael's comment about Gen. 44:8 being one of the ten cases in the Torah, might just be a later commentary, a gloss appended by someone else to the original statement by Ishmael. That would leave open a possibility for controversy over what 'the other nine' cases might be, but it would not of course resolve the issue of why only ten cases in all were officially acknowledged when many more were in evidence. Nevertheless, it meant that if some appropriate criterion or criteria of selection could be found, both issues might be resolved by judicious choice of nine cases to make up the number ten, and ten only, with Gen. 44:8. However, since the criteria proposed were too vague and not particularly exclusive, the selections proposed were not very convincing (at least not to exacting observers like me).

As we have seen (in chapter 9) Einhorn strongly supported the thesis that Ezek. 15:5 was a later addition, and he claimed that Ashkenazi was of the same opinion. Note that this is different from saying that the listing of all nine additional *qal vachomer* cases is a gloss (as Theodor-Albeck later proposed[263]). Hirschensohn accepted the exceptional doubt placed on Ezek. 15:5, when he wrote[264]:

> "What is even more astonishing is that, according to this Midrash, it seems that one was added by one of the publishers to complete the list of ten. It seems that the list in the Midrash originally only had nine items, as can be seen in *Yalkut Shmuel* I, Chapter 23. See [in] the comment from the esteemed Baal *Yafeh Mar'eh* a note from Rabbi Kalonymus from Rome, who adds the tenth KV from Ezekiel 15."

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[263] Cited by Jacobs in his essay "The Qal Va-Homer Argument in the Old Testament," fn.2.

[264] Gabay clarified for me that *Yalkut Shmuel* refers to the *Yalkut Shimoni* (on Samuel) and that *Yafeh Mar'eh* refers to S. Y. Ashkenazi's *Yefeh Toar*. He reported that Hirschensohn identified (on p. 45) the R. Kalonymus here mentioned with "with the scholar who was a contemporary of Rashi and is quoted in Rashi on *Beitsa* 24b".

As we have also seen, Einhorn advocated replacing Ezek. 15:5 with Nehemiah 13:26-27. Hirschensohn, on the other hand, advocated replacing it too, but with Gen. 4:24, citing its mention in the Midrash Rabbah (23:5)[265] and in the Jerusalem Talmud (*Sanhedrin* 10:1). In my view, Hirschensohn's candidate is more credible than Einhorn's – but both are quite speculative.

The fact that Rashi's commentary opposite Ezek. 15:5 does not mention the YS list of GR cases as being deficient (i.e. mentioning only nine cases instead of the declared ten), nor mention the identification by his contemporary Kalonymus of the Ezekiel case as the missing one, suggests (granting the implied early date for YS) that this whole issue is overblown by certain later scholars (notably Einhorn).

The simplest interpretation of events is that (a) the Ezek. case was indeed included among the ten by the author of GR 92:7; or (b) it was possibly added by some anonymous editor soon after him, together with eight other cases to the Gen. 44:8 case originally mentioned by R. Ishmael; and that (c) centuries later, the author of YS accidentally omitted it; or (d) a scribe sometime after him did; and that (e) the said Kalonymus in some gloss corrected the latter error; (f) presumably based on some earlier source no longer extant; so that (g) Rashi subsequently regarded all the preceding as so obvious and insignificant that he felt no need to even mention this matter in his commentary on the case!

As we have seen, Hirschensohn's research focused on finding additional instances of *qal vachomer* in the Torah proper. According to Gabay's account, he was "trying to find 10 KV in the Torah itself – not in the Prophets and Writings." This was a revolutionary undertaking; it was something that no one had tried to achieve, let alone achieved, before. It appears that, consciously or subconsciously[266], Hirschensohn took Ishmael's statement about ten cases "in the Torah" as meaning in the Torah proper (i.e. the Five Books of Moses), not the Tanakh as a whole. This was *a brilliant, novel idea* that no one else, including me, had thought of before him or since (so far as I know). All of us have taken for granted that R. Ishmael's statement in GR that Gen. 44:8 is "one of the ten *qal vachomer* arguments given in the Torah" really means (as the list of nine more cases in GR clearly implies) "in the Tanakh."

Hirschensohn's attempt to find ten instances of *qal vachomer* in the Torah could thus be explained as an attempt to justify the hypothesis that Ishmael's statement was intended *literally*. And since five cases are already known since antiquity, and at least one more was found by Rashi and one more by Katzenellenbogen, he would only need to find three more cases to prove his point! And, to my amazement, he *did* succeed in finding three more. In fact, as I have above confirmed, he found *more than* three more! This was not so good, because Ishmael had apparently specified that there are only ten cases in the Torah. So, while the hypothesis that Ishmael's 'in the Torah' might mean 'in the Torah proper' was confirmed, the hypothesis that there are 'ten' cases in it was simultaneously refuted *anew*!

Actually, as we have seen, Hirschensohn listed four Torah cases, proposed by Rashi and Katzenellenbogen; not just two. So, he must have thought he needed just one more Torah case to make up the needed ten. However, by my reckoning, while Gen. 6:3 and Deut. 32:39 are valid explicit cases, Gen. 6:9 and Lev. 10:19 are only implicit; that is why I say he objectively needed

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[265] "Where," Gabay noted, "it is called 'a KV of darkness' because it is flawed," adding that Hirschensohn "goes on to explain the flaw in the KV is not a flaw in the logic of the KV, its logic being perfectly sound, rather there was a tradition that Cain would be punished seven generations after his murder of Hevel. The 'darkness' in this KV is not in its logical construction but in its conclusion which did not consider the element of Divine justice."

[266] I cannot say which, since Gabay does not mention it and has stopped replying to my queries.

three more cases to make the count ten. Also remember, in this context, that two of the cases that Hirschensohn himself proposed, Gen. 3:1 and 27:37, I have rejected. Nevertheless, he did propose seven new cases that I consider valid and to which I have added an analogous case, making eight; so, he did more than succeed in his quest.

Gabay reports that Hirschensohn "goes on for tens of pages with a theory why only 10 in the Midrash were selected (out of at least 40)." I asked him to try and write a brief summary of how Hirschensohn explicates this discrepancy, but he did not respond. I do not, therefore, know Hirschensohn's thoughts on this matter. He has managed to surprise me thus far, by noticing seven Biblical a fortiori arguments that I could not see by myself; so, he might well surprise me again. But until I become acquainted with his thesis, I am very skeptical that the discrepancy is explicable in a convincing manner and therefore assume offhand that his wordy discourse on this issue is mere apologetics.

To conclude, Hirschensohn's contribution to the field of Biblical *qal vachomer* enumeration is quite impressive, even if he only focused on the Torah. If he had also independently researched the Nakh with equal zeal, he would no doubt have found many new cases in it.

12. Avi SION in *Judaic Logic*

I published my book *Judaic Logic* (JL) in Geneva in 1995[267]. This book developed a list of **23 instances** of *qal vachomer* in the Tanakh (besides the 11 instances given in Midrash GR). The 23 instances are: **1 Samuel 14:29-30, 17:37, 21:6; 2 Samuel 4:10-11, 12:18, 16:11; 1 Kings 8:27; 2 Kings 5:13, 10:4; Jonah 4:10-11; Psalms 78:20, 94:9 (a & b), 94:10; Proverbs 15:11, 19:7, 19:10, 21:27; Job 4:18-19, 15:15-16, 25:5-6; Daniel 2:9; 2 Chronicles 6:18**.

We can at present categorize these findings as follows. Of the 23 cases identified, *5 cases are 'historic firsts'*, namely: 1 Samuel 17:37[268], 21:6; Psalms 94:9 (2 cases), 94:10. Another 18 cases are 'independent' findings, namely: 1 Samuel 14:29-30; 2 Samuel 4:10-11, 12:18, 16:11; 1 Kings 8:27; 2 Kings 5:13, 10:4; Jonah 4:10-11[269]; Psalms 78:20; Proverbs 15:11, 19:7, 19:10, 21:27; Job 4:18-19, 15:15-16, 25:5-6; Daniel 2:9; 2 Chronicles 6:18. And the remaining 11 cases are 'derived', namely: the GR list of10 cases plus Gen. 4:24.

My list of 23 Biblical a fortiori arguments in JL was, I thought at the time I formulated it, *entirely* a 'historic first', because I developed it completely independently. That is, I did not compile these cases from any rabbinic or secular sources (for the simple reason that I knew of no such sources), but systematically researched and discovered them by myself. It is only in the course of the present study that I found out (sadly, I must admit) that 18 cases had already been discovered by others before me, and that I could only still claim 5 cases as new.

I describe in that book exactly how I proceeded. For a start (in JL, chapter 3), I investigated and established *the formal logic of* a fortiori argument wherever it occurs; this I later referred to when deciding whether a given possible a fortiori argument was to be declared valid or not. Secondly

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[267] The book was written, as I recall, in 1992-95. I first self-**p**ublished it in 1995, in small quantity for distribution to libraries; then published it through Editions Slatkine, Geneva, in 1997. I posted it in my website TheLogician.net about 2001. Later, I self-**p**ublished it through online publishers, first Lulu.com, then Amazon.com, and eventually others.

[268] This case was formulated by me and added to subsequent editions of JL in 2001, after being pointed out to me by a reader called Mark Leroux.

[269] This case was discovered fortuitously by me and added to subsequent editions of JL in 1998.

(in JL, chapter 5), I looked at *the language of* Biblical a fortiori arguments included in the GR list (e.g. *hen* (behold)/*ve ekh* (how then); *hine* (behold)/*ve af ki* (then also if), etc.), and on that basis I sought out other uses of similar language in the whole Tanakh by means of a printed concordance of the Bible.

This was patient manual work – there was no Internet at the time, and I knew of no computer readable editions of the Bible I could search through. I looked at some 500 verses that used such expressions and selected among them 21 cases that were visibly a fortiori in intent. The originality of the work done deserves recognition. Two further cases came to light later[270], making 23. Evidently (as I discovered later), I missed many cases, either through inattention or because they did not involve the key words and phrases I sought or because I could not see their a fortiori intent at the time.

A few words regarding **1 Samuel 17:37**. There David says to Saul concerning Goliath: "The Lord who saved me from the paw of the lion and the paw of the bear, He will save me from the hand of the Philistine." This case was in fact noticed and pointed out to me by a reader of JL named Mark Leroux in 2001, and added by me to subsequent editions of JL. I only therefore claim credit for interpreting it as saying: "If David had spiritual credit enough to be saved from innocent creatures (the lion and bear), then he has credit enough to be saved from evil ones (the Philistine)," and then publishing it.

However, Gabay recently (2023) informed me that Malbim[271], in a commentary *ad loc*, had already considered this verse as *qal vachomer*, having read it as saying: "Just as David triumphed over lions and bears through the power of his divine soul, he will similarly, and even more so, prevail over the Philistine warrior with this strength" (translation by Gabay). But, as I wrote back to him, this reading is not accurate, because it posits David as active subject and does not mention God's role, whereas the verse clearly posits God as the active subject and David as a mere passive recipient of His salvation. An interpretation must always reflect all the terms used in the verse being interpreted. Thus, although Malbim may be credited with having vaguely noticed the a fortiori *intent* of this verse, he cannot be said to have fully understood it.

Gabay considers Malbim's reading as at best implicit (*satum*). He proposes instead the following, not a fortiori, reading: "In the same way that God has protected me in the past from other dangers, he will protect me now as well." Now, as I replied to him, this reading is reasonable, in that it connects the two parts of the verse in the way of an *analogical* argument; but such argument is *logically weaker* than a fortiori argument. If Gabay's intent was to put the two halves of the verse on an equal footing, it could be done better through an *egalitarian a fortiori* argument: "*As much* (instead of 'more') spiritual credit is required for God to save someone from animals as for God to save someone from an evil man; and David had enough spiritual credit to be saved by God from animals, therefore he had enough of it to be saved by Him from Goliath."

This alternative *qal vachomer* interpretation is clearly quite credible, and in general an a fortiori reading is preferable to an analogical one because it offers more explanation for the bond between minor premise and conclusion. In the present case, the a fortiori reading adds David's 'spiritual credit' (implied by the surrounding narrative) as explanation of God's acts of salvation in both situations, thus denying them to have been mere happenstance. The egalitarian reading differs from my original reading in that in the latter the (tacit) major premise is that *more* spiritual credit is

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270 In 1998 and 2001, as already mentioned.

271 R. Meir Lob Ben Yehiel Michal, 1809-1879.

required to be saved from animals, who are relatively morally innocent creatures since they just follow their natural instincts, than to be saved from a willful enemy like Goliath, who being quite evil deserves to be opposed by God anyway. The given text could be interpreted either way: take your pick.[272]

13. R. Louis JACOBS

R. Louis JACOBS (b. 1920, in Manchester, UK, d. 2006)[273] wrote an essay entitled "The Qal Va-Homer Argument in the Old Testament," included as chapter 12 in his book *Rabbinic Thought in the Talmud*, published in London, UK, 2005[274].

In this essay, Jacobs lists **twenty-one instances** of *qal vachomer* in the Tanakh (pp. 111-113) (besides the 11 instances given in Midrash GR[275]). The 21 are: **Judges 14:16; 1 Samuel 14:29-30; 2 Samuel 12:18, 16:11; 1 Kings 8:27; 2 Kings 10:4; Isaiah 66:1 (a & b); Jeremiah 25:29, 45:4-5, 49:12; Ezekiel 33:24; Jonah 4:10-11; Proverbs 15:11, 19:7, 19:10, 21:27; Job 4:18-19, 9:13-14, 15:15-16, 25:5-6; Nehemiah 13:26-27**.

Only two of these cases are 'historic firsts', namely: Isaiah 66:1 (a & b). Another 7 are apparently 'independent finds', namely: Judges 14:16; 1 Samuel 14:29-30; 2 Samuel 12:18, 16:11; Ezekiel 33:24; Jonah 4:10-11; Proverbs 19:7. The remaining 18 can be assumed to be 'derivative'.

Isaiah 66:1. God: "The heaven is My throne, and the earth is My footstool; where (*eizeh*) is the house that ye may build unto Me? And where (*eizeh*) is the place that may be My resting-place?" {**-s**} There are clearly two antecedents and two consequents here; so. there are really *two* a fortiori arguments, intertwined, viz.: (a) If His heavenly throne is not big enough to house God, then an earthly house is not big enough to do so; and (b) if the entire earth is not big enough as a resting-place for God, then a delimited place is not big enough for that. I counted this verse as only one argument in AFL, and so did Jacobs in his said essay; but there are clearly two, though their intent is roughly the same. Note that their narrative is comparable to 1 Kings 8:27 and 2 Chronicles 6:18.

Since Jacobs mentions Ashkenazi (*Yefeh Toar*)[276], we can assume that he found the following 12 cases mentioned in his list in that work: 1 Kings 8:27; 2 Kings 10:4; Jeremiah 25:29, 45:4-5, 49:12; Proverbs 15:11, 19:10, 21:27; Job 4:18-19, 9:13-14, 15:15-16, 25:5-6. However, Jacobs does not have the Isaiah 20:6 case, which is listed in YT; this was probably an accidental omission, but may have been (for some unstated reason) intentional. Also, since Jacobs mentions Einhorn[277], we can assume he got the Nehemiah 13:26-27 case from him. It is certain, however, that Jacobs did not derive any other case from Einhorn, since the latter does not have a published list (though he claimed to have compiled a list in his private notebook). It is possible that Jacobs derived some cases from Rashi, maybe Ezekiel 33:24 and Proverbs 19:7; but since he does not tell us his sources, we cannot assert it other than speculatively.

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272 I devote so much space to this one case, not because it is particularly important, but because the discussion provides some valuable lessons in interpretation for eventual readers.

273 https://en.wikipedia.org/wiki/Louis_Jacobs. More on Jacobs at: https://louisjacobs.org/.

274 https://archive.org/details/rabbinicthoughti0000jaco.

275 He lists 10 instances on pp. 109-110; and he mentions Gen. 4:24 in fn. 3.

276 In fn. 6.

277 On p. 111.

After mentioning Einhorn, Jacobs writes[278] that Hirschensohn "adds the following examples (but these are extremely doubtful)" and he lists Genesis 3:22, 11:6, 17:17. These three cases are in fact (as we determined in chapter 11, above) valid and explicit; so, Jacobs was wrong in his rejection of them. One might think that Jacobs mentioning these three cases implies that he actually read the whole of Hirschensohn's book; but there are too many cases (8 of them, to be precise)[279] that the latter lists which Jacobs does *not* mention for such an assumption to be credible. So, while one might be tempted to assume five of the cases listed above as independent, namely: 1 Samuel 14:29-30; 2 Samuel 12:18, 16:11; Ezekiel 33:24; Proverbs 19:7, to have been derived from Hirschensohn (since he too has them), it is wiser not to do so.

It can safely be denied that Jacobs, for all his erudition, studied the relevant works of Katzenellenbogen (ZHK) and Strashun (MS) when he drew up his list. If we look the 7 cases above labeled as apparent 'independent finds', we see that 4 of them, viz. 1 Samuel 14:29-30, 2 Samuel 16:11, Ezekiel 33:24, and Proverbs 19:7, are found in ZHK, and thence in MS; while 3 of them, viz. Judges 14:16, 2 Samuel 12:18, and Jonah 4:10-11, are found in MS alone (i.e. but not ZHK). We might thus think that Jacobs could have learned of these cases from those authors. However, there are 1 case listed by ZHK alone[280], 4 cases by ZHK and thence MS[281], and 14 cases by MS alone[282], that Jacobs lacks in his listing – which tells us that Jacobs was not (or not fully) acquainted with these two sources. He might conceivably have rejected the 19 missing cases without saying why, but it seems very unlikely since they are numerous and clearly valid.

Note furthermore that of the 21 cases of Biblical a fortiori argument listed by Jacobs (besides the 11 traditional ones) in 2005, 12 were already analyzed and listed in my book *Judaic Logic* (JL) ten years earlier, but 9 were new to me when I came across his essay in 2013 and I analyzed and listed these in my book *A Fortiori Logic* (AFL). It is evident, however, that Jacobs did not refer to my earlier work (JL) at all, since he does not mention it. As an objective and honest researcher, he surely would have cited it if he had known of it and read it. In 2005, the initial limited edition (1995) of my book was available in certain university libraries, the more public Slatkine edition (Geneva, 1996) was on sale, and most significantly the whole book was posted (since 2001) on my website www.TheLogician.net. It is therefore surprising that Jacobs did not come across it. Apparently, he did not research the field very thoroughly.

In AFL, chapter 16, I examine Jacobs's work in this field in considerable detail. I demonstrate, for a start, that he did not master the formalities of a fortiori argument. He describes such argument as either *simple* (If A has x, then B certainly has x) or *complex* (If A, which lacks y, has x, then B which has y certainly has x); but this is a superficial formula, lacking many important formal features and therefore incapable of strict validation. Nevertheless, he did intuitively manage to compile a list of 21 valid cases, as already mentioned. The question is: how did he do it? In his essay on the subject, he writes (p. 111):

> "But the commentators to the Midrash and other scholars are puzzled by R. Ishmael's reference to only ten Scriptural cases. In fact, they point out, there are many more instances of an explicit *qal va-homer* in the Bible. Wolf Einhorn of Grodno observes that his

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278 In fn. 6 (citing pp. 39-60) and 7 (citing pp. 40-45).

279 Namely: Gen. 4:14, 6:3, 14:23, 39:8-9; Ex. 6:30; Deut. 32:39, Isa. 20:6, Ps. 78:20.

280 Namely, Deut. 32:39.

281 Namely: 2 Sam. 4:10-11, 12:21; Prov. 17:7; 2 Chron. 6:18.

282 Namely: 1 Sam. 14:39; 2 Sam. 11:11; 2 Kgs 5:12, 5:13, 18:23-24, 18:35; Isa. 1:3, 10:11, 20:6, 36:8-9, 36:20; Jerem. 2:11, 8:7; Ezek. 3:4-7.

> researches have yielded no fewer than forty instances and other commentators come up with similar results. Some of these must be rejected as far-fetched and dubious, but the following list contains all the definite references."

And he goes on to list 21 instances of Biblical a fortiori. Unfortunately, Jacobs exhibits a surprising lack of precision for a scholar of his caliber, failing to specify the source(s) of each of the 21 Biblical a fortiori arguments he goes on to list, and failing to explain why he selected these as "definite references" and rejected others as "far-fetched and dubious." I assumed in AFL, as a working hypothesis, not without misgivings, that all the cases Jacobs listed had been included in Wolf Einhorn's alleged list of "forty instances," because Einhorn's was the only name he mentioned in the paragraph just cited[283], and because he added that "other commentators" (left unnamed) had "come up with similar results." I thus ended up erroneously labeling all of Jacobs's cases with Wolf Einhorn's initials, WE. Of course, having now learned that Einhorn did not in fact publish a list of 40 cases, I am in the present paper relabeling Jacobs's 21 cases with his initials, LJ.

Lastly note, in his essay (on p. 111), Jacobs draws attention to the *baraita* of R. Eliezer ben R. Jose the Galilean, which distinguishes between explicit (*meforash*) and implicit (*satum*) *qal vachomer* arguments, and gives two Biblical examples of each; namely, for the explicit, Jeremiah 12:5 and Esther 9:12 (both these given in the *Genesis Rabbah* list of ten) and for the implicit, Psalms 15:4-5 (2 cases). I have analyzed the latter two cases in the Rashi chapter (6, above). I there agree, as does Jacobs in his essay, with R. Eliezer that they are at best implicit a fortiori argument.

14. Avi SION in *A Fortiori Logic*

I published my book *A Fortiori Logic* (AFL) in Geneva in 2013[284]. This book developed a list of **36 instances** of *qal vachomer* in the Tanakh (besides the 11 instances given in Midrash GR). The 36 instances are: **Judges 14:16; 1 Samuel 14:29-30, 17:37, 21:6; 2 Samuel 4:10-11, 12:18, 16:11; 1 Kings 8:27; 2 Kings 5:13, 10:4, 18:23-24; Isaiah 36:8-9, 66:1 (a & b); Jeremiah 25:29, 45:4-5, 49:12; Ezekiel 14:13-21, 33:24; Jonah 4:10-11; Psalms 78:20, 94:9 (a & b), 94:10; Proverbs 15:11, 19:7, 19:10, 21:27; Job 4:18-19, 9:13-14, 15:15-16, 25:5-6; Daniel 2:9, Nehemiah 13:26-27; 2 Chronicles 6:18, 32:15**.

We can at present categorize these findings as follows. *1 case is a 'historic first'*, namely: 2 Chronicles 32:15. This lone case being in addition to the 5 'historic firsts' listed by me in JL, namely: 1 Samuel 17:37, 21:6; Psalms 94:9 (2 cases), 94:10, making a total of 6 for me at time of writing. Another 3 cases are 'independent finds', namely: 2 Kings 18:23-24; Isaiah 36:8-9; Ezekiel 14:13-21. These 3 cases being in addition to the 18 'independent finds' listed by me in JL, namely: 1 Samuel 14:29-30; 2 Samuel 4:10-11, 12:18, 16:11; 1 Kings 8:27; 2 Kings 5:13, 10:4; Jonah 4:10-11; Psalms 78:20; Proverbs 15:11, 19:7, 19:10, 21:27; Job 4:18-19, 15:15-16, 25:5-6; Daniel 2:9; 2 Chronicles 6:18. And the remaining 20 cases are 'derived', namely: the GR list of 10 cases plus Gen. 4:24, and 9 cases that I found in Louis Jacobs's 2005 listing, namely: Judges 14:16; Isaiah 66:1 (a & b); Jeremiah 25:29, 45:4-5, 49:12; Ezekiel 33:24; Job 9:13-14; Nehemiah 13:26-27.

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[283] Although he did mention other authors in fn. 6, notably Ashkenazi and Hirschensohn.

[284] The book was written, as I recall, in 2010-13. I immediately posted it in my website TheLogician.net about 2001 and self-published it through online publishers, including Amazon.com, Lulu.com, and others.

Regarding Ezekiel 14:13-21, which I thought was a 'historic first' finding of mine, until I discovered during the present study that Rashi had beat me to it (thanks to Gabay, who drew my attention to the fact). I have analyzed it briefly in the Rashi chapter (6, above); but I will review it here in a bit more detail. It reads as follows:

Ezekiel 14:13-21: "Son of man, when a land sinneth against Me by trespassing grievously, and I stretch out My hand upon it… and send famine upon it… [Or] if I cause evil beasts to pass through the land… Or if I bring a sword upon that land… Or if I send a pestilence into that land… though Noah, Daniel, and Job, were in it, as I live, saith the Lord God, they shall deliver neither son nor daughter; they shall but deliver their own souls by their righteousness… How much more (*af ki*) when I send My four sore judgments against Jerusalem, the sword, and the famine, and the evil beasts, and the pestilence, to cut off from it man and beast" (brackets mine).

My paraphrase: The words are spoken by God. Their meaning is: More spiritual credit is required to prevent the execution *together* of several negative decrees than to prevent their execution *separately* (tacit major premise); whence it follows that if holy men (such as Noah, Daniel, and Job) lack sufficient spiritual credit to prevent the execution of each of the four negative decrees separately (minor premise), then they lack enough to stop all four of these decrees together (conclusion). This is a negative predicatal (**-p**) a fortiori argument.

I discovered this new case of *qal vachomer* in the Tanakh back in 2013, when I was close to completing AFL. What made its discovery difficult until then was the exceptional length of this argument, due to its complicated listing of four separate decrees and then drawing a conclusion about them together. I came across this case fortuitously and was quite surprised when I found it. I was proud of myself for realizing it. For this reason, I am sorry to now 'lose' it.

In his commentary, Rashi only mentions the last verse (v. 21, the one containing the expression *af ki*), without drawing attention to its antecedents (v. 13-20); so, I have wondered if he was aware of the relevance of preceding verses, or he declared the final verse as indicative of *qal vachomer* merely on the basis of its language. Nevertheless, I must *ex post facto* acknowledge him as an earlier discoverer of this case, because I do not see how v. 21 could be read out of context and interpreted in any way other than as the last leg of a long a fortiori argument. Rashi must have been at least somewhat aware of the premises to the conclusion in v. 21, even if he did not spell them out.

I am therefore obliged to classify Ezek. 14:13-21 in my list as an 'independent find' instead of as a 'historic first'. I nevertheless propose, in view of Rashi's incomplete presentation and my more thorough analysis of the case, to label it in the merged list in chapter 3 as R/AS.

15. The language of Biblical a fortiori discourse

In my book *Judaic Logic*, chapters 5 and 6, I studied Biblical a fortiori discourse empirically, without preconceptions, by examining the language used for it in the cases I had encountered thus far. I found that certain Hebrew verbal expressions, which I called key words and phrases, were used repeatedly, apparently as indicators of *qal vachomer* intent. However, there were different key words and phrases, not just one or one set of them; sometimes, too, there was no indicator at all. Moreover, while the verbal expressions were sometimes, or even often, indicative of a fortiori discourse, they were not used exclusively in such contexts. That is, they did not necessarily imply a fortiori argument, but merely suggested it as possibly present. Even so, the indicators I gradually identified helped me find new cases of Biblical a fortiori discourse; so, they were very useful.

I began my search with reference to the language used in the ten cases given traditionally as exemplary. These cases involved the expressions: *hen/ve-ekh* (2), *hen/af ki*, *hen/ve-af ki*, *hine/af ki*, *hine/ve-af ki*, *ki/ve-ekh*, *u/ve-ekh*, *ve/ha-lo*, and *meh*. *Hen* and *hine* are translated as 'behold', *ve-ekh* means 'how then', *af ki* means 'then when' or 'then if', *ha-lo* means 'is it not then that' (a rhetorical turn of phrase), and *meh* means 'what' or perhaps 'how many'. I searched for these expressions in a printed concordance, and then looked at about 500 Biblical references with a fortiori potential. I thus found a score of new (new to me) cases. Some of them involved other phraseology, which I discovered incidentally, such as *ki* (if, when, since, because), *lahen* (thus), *reu* (see), *im* (if), *gam* (also).

What transpired was that the terms used served as if/then operators. For instances, *hen* or *hine* has the semantic value of 'if', and *ve-ekh* or *af-ki* signifies 'then', so that combined together they announce respectively an **antecedent** clause and a **consequent** clause. The resulting if-then statement might often constitute an a fortiori argument, usually the minor premise and the conclusion of one; but sometimes it might not. Only *close scrutiny of the narrative, and careful linguistic and logical analysis, often aided by past rabbinic commentaries*, permit an ad hoc judgment as to whether or not a given text has or lacks a fortiori intent. This method has been used in the present study and gives our current expanded listing its credibility.

I recently discovered an excellent **online concordance of the Bible** at blueletterbible.org. This is a Christian site, but it includes the Hebrew Bible[285]. One can search in it for a Hebrew word (using Hebrew letters), and it will provide a list in English of all the chapters and verses where it appears[286]. One can also search for two or more words, and it will return where they appear together and separately. Moreover, not only is the relevant location in the Bible listed, but the textual content in Hebrew can be read on the same page without needing to go looking for it elsewhere. Also, statistical summaries are given, showing how many times the word string(s) occur in each book of the Tanakh, and in the whole Tanakh. This tool should, obviously, greatly facilitate the finding of new *qal vachomer* cases, if any.

Looking now at the 72 Biblical a fortiori valid and explicit cases listed in the preceding chapter, we can propose the following **list of key words and phrases**. (A dash '-' means no verbal indicator is used.)

Reference	Type	Antecedent	Consequent
Genesis 3:22-23	**+s**	*hen*	*ve-atah*
Genesis 4:14	**+s&**	*hen*	*ve-hayah*
Genesis 4:24	**-s&**	*ki*	*ve*
Genesis 6:3	**+s**	*beshagam*	*ve*
Genesis 11:6	**+s**	*hen*	*ve-atah*
Genesis 14:23	**-p**	*im*	*ve*

[285] N.B. when searching for an expression in this site, one must specify WLC as the book to search in. WLC refers to the Westminster Leningrad Codex.

[286] One should of course remain vigilant for Biblical chapter and verse references that differ in Christian editions of the OT, compared to the Jewish Bible. I cannot predict whether this issue will ever in fact arise in our research.

Reference	Type	Antecedent	Consequent
Genesis 17:17	**+s**	*ha-le*	*ve-im*
Genesis 18:12	**+s**	*hayta*	*ve*
Genesis 39:8-9	**+s**	*hen*	*ve-ekh*
Genesis 44:8	**+p**	*hen*	*ve-ekh*
Exodus 6:12	**-s**	*hen*	*ve-ekh*
Exodus 6:30	**-p**	*hen*	*ve-ekh*
Numbers 12:14	**+s**	*ve*	*ha-lo*
Deuteronomy 31:27	**+s**	*hen*	*ve-af ki*
Deuteronomy 32:39	**+p**	*ki*	*ve*
Judges 14:16	**-p**	*hine*	*ve*
1 Samuel 14:29-30	**+s&**	*reu (ki... ki)*	*af (ki... ki)*
1 Samuel 14:39	**-s**	*ki*	*ki*
1 Samuel 17:37	**+p**	-	-
1 Samuel 21:6	**+p**	*ki im*	*ve-af ki*
1 Samuel 23:3	**+p**	*hine*	*ve-af ki*
2 Samuel 4:10-11	**+s**	*ki... va*	*af ki... ve ha-lo*
2 Samuel 11:11	**-s**	-	*va*
2 Samuel 12:18	**+s&**	*hine*	*ve-ekh*
2 Samuel 12:21	**+s&**	*mah*	*ve-ka-asher*
2 Samuel 16:11	**+p**	*hine*	*ve-af ki*
1 Kings 8:27	**-s**	*hine*	*af ki*
2 Kings 5:12	**-s**	*ha-lo*	*ve*
2 Kings 5:13	**+s**	*ha-lo*	*ve-af ki*
2 Kings 10:4	**-s**	*hine*	*ve-ekh*
2 Kings 18:23-24	**-s&**	*im*	*ve-ekh*
2 Kings 18:35	**-s**	*asher*	*ki*
Isaiah 1:3	**+s**	-	-
Isaiah 10:11	**+s**	*ha-lo ka-asher*	*ken*
Isaiah 20:6	**-s**	*hine*	*ve-ekh*
Isaiah 36:8-9	**-s&**	*im*	*ve-ekh*

Reference	Type	Antecedent	Consequent
Isaiah 36:20	**-s**	*asher*	*ki*
Isaiah 66:1a	**-s**	-	*eizeh*
Isaiah 66:1b	**-s**	-	*eizeh*
Jeremiah 2:11	**+s**	-	*ve*
Jeremiah 8:7	**+s**	*gam*	*ve*
Jeremiah 12:5a	**-p**	*ki*	*ve-ekh*
Jeremiah 12:5b	**-p**	*u*	*ve-ekh*
Jeremiah 25:29	**-s**	*ki hine*	*ve*
Jeremiah 45:4-5	**-s**	*hine*	*ve*
Jeremiah 49:12	**+s**	*hine*	*ve*
Ezekiel 3:4-7	**+s**	*ki*	*u*
Ezekiel 14:13-21	**-p**	*ki*	*af ki*
Ezekiel 15:5	**-s**	*hine*	*af ki*
Ezekiel 33:24	**+s**	-	*va*
Jonah 4:10-11	**+s**	-	*va*
Psalms 25:8-9	**+p**	-	*al-ken*
Psalms 78:20	**+p**	*hen*	*gam*
Psalms 94:9a	**+p**	-	*ha-lo*
Psalms 94:9b	**+p**	*im*	*ha-lo*
Psalms 94:10	**+p**	-	*ha-lo*
Proverbs 11:31	**+s**	*hen*	*af ki*
Proverbs 15:11	**+p**	-	*af ki*
Proverbs 17:7	**+p&**	-	*af ki*
Proverbs 19:7a	**+p**	-	*af ki*
Proverbs 19:10	**+s**	-	*af ki*
Proverbs 21:27	**+s**	-	*af ki*
Job 4:18-19	**+p**	*hen*	*af*
Job 9:13-14	**-s**	-	*af ki*
Job 15:15-16	**+p**	*hen*	*af ki*
Job 25:5-6	**+p**	*hen*	*af ki*

Reference	Type	Antecedent	Consequent
Job 35:13-14	**+s**	*akh*	*af ki*
Esther 9:12	**+s&**	-	*meh*
Daniel 2:9	**+p**	*lahen*	*ve*
Nehemiah 13:26-27	**+s**	*gam*	*ve*
2 Chronicles 6:18	**-s**	*hine*	*af ki*
2 Chronicles 32:15	**-s**	*ki*	*af ki*
No. of valid explicit cases	72		

Our next task is to find out how often each linguistic expression is repeated in the above list of 72 cases. Expressions which are very rarely used are of little interest to us. What we need to look at are the expressions that are relatively often associated with a fortiori argument. Knowing them, we can look for and perhaps find new cases of *qal vachomer*.

It is worth first noting in passing the uniformities of language found in the different books of the Bible. Thus, notice the use of *hen/ve-ekh* in the adjacent cases from Gen. 39:8-9, 44:8 and Ex. 6:12, 6:30. From Deut. 31:27 to 2 Kgs 5:13, we see frequent use of *af ki*, *ve af ki*, *ve*, *ki*, in the consequent. In Jeremiah, the consequent is signaled by either *ve* or *ve-ekh*. In Proverbs, Job, and Chronicles, the consequent is almost always indicated by *af ki*. Such linguistic uniformities perhaps reflect different historical periods or different document authors. But we should perhaps not make too big a deal of them.

Let us first consider the frequency of each combination of antecedent and consequent in the 72 cases known so far: *hen/af* (1), *hen/af ki* (3), *hen/ve-af ki* (1), *hen/ve-atah* (1), *hen/ve-hayah* (1), *hen/ve-ekh* (5), *hen/gam* (1), *hine/ve* (3), *hine/ve-ekh* (3), *hine/af ki* (3), *hine/ve-af ki* (2), *lahen/ve* (1), *ki/u* (1), *ki/ve* (2), *ki/ki* (1), *ki/af ki* (2), *ki im/ve-af ki* (1), *ki/ve-ekh* (1), *ki hine/ve* (1), *ki... va/af ki... ha-lo* (1), *reu (ki... ki)/af (ki... ki)* (1), *akh/af ki* (1), *u/ve-ekh* (1), *ve /ha-lo* (1), *ha-lo/ve* (1), *ha-lo/ve-af ki* (1), *ha-le/ve-im* (1), *ha-lo ka-asher/ken* (1), *asher/ki* (1), *im/ve* (1), *im/ve-ekh* (2), *im/ha-lo* (1), *gam/ve* (3), *beshagam/ve* (1), *hayta/ve* (1), *mah/ve-ka-asher* (1), *-/meh* (1), *-/af ki* (6), *-/al-ken* (1), *-/eizeh* (2), *-/ha-lo* (2), *-/va* (3), *-/ve* (1), -/- (2). As can be seen, some combinations are more frequently used than others, but no combination can be claimed as *the only* verbal formula used in Biblical a fortiori argument.

If we look at the antecedents alone, we get the following statistics: *hen, hine, lahen* (25), *ki* (11), *im* (5), *ha-lo, ha-le* (4), no sign (18), others (10). If we look at the consequents alone, we get the following statistics: *af, af ki, ve-af ki* (23), *ve-ekh* (12), *ve, u, va,* etc. (23), *ha-lo* (4), others (10). Here again, we can well say that some expressions are more frequent than others, but we cannot claim any expression to be a consistent indicator of *qal vachomer* in the Tanakh. And of course, we have yet to see empirically whether any of the many expressions here used are *exclusively* used for a fortiori discourse. I very much doubt it offhand; but we shall see when we look into a concordance.

Regarding the 14 cases rejected, 7 because I judged them only ‘implicit’ (D), and 7 because I judged them ‘invalid’ (E), the following two tables show the key words/phrases, if any, used in them.

Reference	Type	Antecedent	Consequent
Genesis 6:9	**+p**	-	-
Genesis 17:20-21	**+s&**	*hine*	*ve*
Leviticus 10:19	**+s**	*hen*	*ve*
Habakkuk 2:4-5	**+s**	*hine*	*ve-af ki*
Psalms 15:4	**-s**	-	-
Psalms 15:5	**+s**	-	-
Job 28:17	**+s**	-	-
No. of cases judged implicit	**7**		

Reference	Type	Antecedent	Consequent
Genesis 3:1	**-s**	-	*af ki*
Genesis 27:37	**+p**	*hen*	*apo*
2 Samuel 10:16	**?**	*ve*	*ve*
Ezekiel 23:39-40	**+s**	*ve-hine*	*ve-af ki*
Proverbs 6:30	**?**	*ki*	-
Proverbs 10:17	**?**	-	*ve*
Proverbs 19:7b	**+s&**	-	-
No. of cases judged invalid	**7**		

As can be seen, some of the rejected cases, whether implicit or invalid, use the key words/phrases found in valid explicit cases; many of them do not involve any verbal indicator. This just goes to show that the key words and phrases concerned are not infallible indicators of a fortiori discourse. As for cases that involve no verbal indicators, there are such cases in the group of valid explicit cases, in that of implicit cases, and in that of invalid cases: so, this tells us nothing either.

Given the above listed key expressions, used so far in valid explicit cases, I would now look into the online concordance in search of more cases. I would (to begin with, at least) limit my investigation to a few of them, those most likely to produce new a fortiori results. The terms *hen* and *hine*, and maybe also *ha-lo* and *im*, found in many antecedents, and the terms *ve-ekh*, and *af*, *af ki*, *ve-af ki*[287], found in many consequents, seem to me good bets. The terms *ki* and *ve*, occurring

[287] Needless to say, *ve-af ki* is a subset of *af ki*, which is a subset of *af*. However, note, the concordance treats *ve-af* as one word, not as a conjunction of two.

alone, being found in both antecedents and consequents of a fortiori discourse, and in many other contexts, would require far more work and are, I'd say, less likely to yield interesting results[288].

The following table contains statistics on the occurrence of some promising key expressions. This is not intended as an exhaustive listing, but it just serves to show how big the task at hand is.

Hebrew expression	English transliter'n.	Occurs	Verses in Tanakh	Verses in Torah
הֵן	*hen*	111	106	45
הִנֵּה	*hine*	518	495	96
הֲלֹא	*ha-lo*	127	125	17
אִם	*im*	812	760	172
וְאֵיךְ	*ve-ekh*	18	17	5
אַף	*af*	177	166	33
אַף כִּי	*af ki*	42	41	8
וְאַף כִּי	*ve-af ki*	19	8	2

Of course, one can also look for combinations, and that may be the most fruitful method. For example, the combo *hen/ve-ekh* yields only three occurrences; namely, Gen. 44:8, Ex. 6:12, 6:30; *hen/ve-atah* occurs only in Gen. 3:22, 11:6; *hen/ve-af ki* occurs only in Deut. 31:27; *hen/af ki* or *hen/af* occurs only in Prov. 11:31; *hine/ve-af ki* occurs only in 1Sam. 23:3, 2 Sam. 16:11. All of these cases are, as it happens, already known to us, and accepted by me as both valid and explicit. But there could well have been new cases, so the search was informative.

On the other hand, *hine/ve-ekh* occurs four times in all; three of them are familiar, viz.: 2 Sam. 12:18, 2 Kgs 10:4, Isa, 20:6; but one is new to us: Gen. 26:9. We must ask: is the latter a new case of *qal vachomer*? It reads[289]: So, Abimelech called Isaac, and he said, "Behold (*hine*), she is your wife; so how (*ekh*) could you have said, 'She is my sister'?" And Isaac said to him, "Because I said, 'Lest I die because of her.'" Offhand, I do not see an a fortiori argument here; but maybe I could spin one. Similarly, *hine/af ki* occurs five times in all; four of them are familiar, viz.: 1 Kgs 8:27, 2 Chron. 6:18, 28:13, Ezek. 15:5; but one is new to us: Ex. 4:14. However, the latter differs in the separation and the order of the three words: *ki*, *af*, and *hen*, so that a *qal vachomer* is less likely in it.

I shall make no attempt, here, to look into the above two possible new cases, or any others like them, to find out whether any contains a valid and explicit *qal vachomer*, or an implicit one, or none at all. I leave these and further questions open because my intent here has only been to demonstrate that new cases *might* emerge from such research. I have decided not to push this

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[288] There are over 4000 occurrences of *ki*, and as for *ve* the concordance gives no statistic presumably because in Hebrew this word does not stand alone.

[289] The sentence of concern to us is the one uttered by Abimelech.

research further at the present time. It is truly outside the scope of the present work, which is a study and merger of certain past *qal vachomer* lists.

But it is clearly worth eventually looking into the occurrences listed by the concordance (see there), one by one, in search of new a fortiori cases. I may yet find the time to do it myself someday; but if I don't, I invite any interested person to do the job. The task is not daunting; it just requires time and patience. However, the evaluation stage requires some logical skill and experience: it is not easy or infallible.

It should be stressed that the occurrence of a verbal expression which is often indicative of a fortiori argument does not constitute sufficient proof that the Biblical passage in question does indeed involve a valid and explicit *qal vachomer*. Such expressions are mere indices, serving at best as an incentive to look more carefully at the given text and see what it might intend. As I have shown throughout the present work, each case needs to be analyzed in detail, with attention to the narrative context, and linguistic and logical considerations, as well as traditional interpretations, before a verdict can be made with any degree of certainty.

Needless to add, there may be cases of Biblical a fortiori argument that we have not yet discovered which might involve new indicative expressions. These might in turn help us find still more new *qal vachomer* passages.

Keep in mind, however, that there is no *mitzvah*, no religious obligation, to multiply the number of *qal vachomer* arguments in the Tanakh! Our goal here, as scholars, is merely to observe and record, objectively and without ulterior motives, what is manifest in the text. When in doubt, it is safer to let the case pass, rather than to get acquisitive and try to force feed it with an a fortiori interpretation.

In rabbinic writings, dating from the Mishnaic and Talmudic eras, and thereafter, unlike in the Tanakh, a fortiori arguments are indicated by means of reserved verbal expressions, viz. *qal vachomer* (lit. light and heavy), *kol sheken* (all the more so), and *al ahat kama ve-kama* (how much more so). These expressions are not found in Biblical era texts.

16. The point of it all

In the course of this research, we have come across some commentators (notably Rashi, but others too) who have in passing noticed some Biblical a fortiori arguments, but have not made a systematic effort to develop a list of such discourse. A few commentators, on the other hand, have tried to draw up lists of Biblical a fortiori arguments; we have found more than half a dozen of those. No doubt, there are commentators whose work we have not mentioned here.

In any event, what has become evident is that *none* of the commentators we came across have discovered, in either a scattered or systematic way, *all* existing Biblical a fortiori arguments known to us so far. Our present merged list of 72 *qal vachomer* in the Tanakh is thus (so far as we know) the first, and to date the most thorough, truly cumulative listing. There may well, of course, be yet more cases to be found; it is even very likely. Unfortunately, Gabay opted out of this project before its completion, leaving many questions I had put to him unanswered. All the same, his research efforts over some six months are to be saluted.

Also noteworthy is that there is no *integral* traditional list of Biblical *qal vachomer* dating from way back; no ancient 'tradition' exists in this respect. The Midrash *Genesis Rabbah* seemingly purported to be such a list, but obviously it fell far short, even if some rabbinical commentators

have tried (very unsuccessfully, in my view) to explain and excuse that deficiency. The idea that the oral tradition is omniscient and continuous, and is wholly and efficiently transmitted across the generations, is not confirmed in our specific area of interest (which does not mean that such transmission is not present in other fields).

What we observe objectively, as well, is that is that there is no *accumulation* of knowledge in this field over time. Surprisingly, each commentator we came across knew (or mentioned) only part of the work done by some (but not all) of his predecessors. Some were aware of some of the work of some of their predecessors, but even then they somehow failed to register all the work of these predecessors. Some were not at all aware of any of the work of certain of their predecessors. As a result of this broken continuity, there has been no cumulative knowledge base.

Most surprisingly, although some commentators after Rashi referred to his work occasionally, it seems that none of them thought to find and list all the cases flagged as *qal vachomer* by him! Evidently, anyway, although Rashi was apparently the first to flag many cases, he certainly did not flag all existing cases. He did not attempt to draw up an exhaustive list, but was content to here and there note in passing the *qal vachomer* intent of some verses. No doubt that was because his workload was already massive, and he had no time for such a relatively minor issue.

One may well ask: what is the point of it all? Who cares exactly how many a fortiori arguments there are in the Tanakh, and who found which first? In truth, these are not a very important issues in themselves; they are just minor topics in Judaic studies and Jewish intellectual history. From a religious viewpoint, of course, any kind and amount of Torah study is of some value; Torah study is one of the forms of Jewish meditation. Similarly, from a scientific perspective, such studies – finding out facts, sorting them, ordering them systematically – always constitute a good intellectual exercise for both the researchers and the readers, with unpredictable eventual benefits.

However, viewed in a larger context, more value can be ascribed to our present study. The larger context referred to is the scientific study of Judaism, including all its written documents (Tanakh, Mishna, Talmud, Midrash, etc.) and all its practices. I am referring to empirical studies, not dogmatic studies, of course. It is not enough to learn, for instance, the 'thirteen hermeneutic principles of R. Ishmael' or other such traditional accounts of 'Talmudic logic' – this is dogmatic study. One must analyze and evaluate them, and find out exactly how often they are in fact used in the Talmud, and also look for logic in the Talmud that is not mentioned in traditional accounts – this is empirical study.

The grand project of systematic scientific – empirical, analytical, and evaluative – research, through all texts and practices, is not limited to Judaism, of course, but is aimed at all religious traditions, as indeed at all human thought and endeavor. The scientific method is a marvelous, broadly applicable tool; and it is highly productive. Though this project is 'modern', it is not merely contemporary, but has been proceeding with growing intensity roughly since the Renaissance in Europe. With regard specifically to the Jewish experience, we can of course mention the 'Science of Judaism' (*Wissenschaft des Judentums*) movement which emerged in Germany in the early 19th Cent.

My own main area of interest is logic. My work in this field over the last three decades and more has been vast, including valuable original work in deductive and inductive logic (most of it still unknown to most academics and to most of the general public). I have made contributions in modal logic, the logic of causation, a fortiori logic, the logic of analogy, and many other important domains. I am interested in the reasoning powers of mankind, in theory and in practice. Naturally, as a practicing Jew, I also have looked at the implications of such general research on the specific

field of Judaic studies. I have focused much attention on the search for logic in the Torah, the Talmud. and other Jewish religious documents. Needless to say, I am neither the first to do so, nor the only one, nor (I hope) the last. But I do consider my work as more significant because it is based on wider original research in logic.

Thus, to return to the initial question, the present research on the number of Biblical a fortiori arguments and the history of their discovery is one small brick in a much larger project – determining the presence and development of logic in Jewish thought and life in the course of history (and beyond that, of course, in humanity's thought and life). The earliest sources of information are, of course, the Torah and the Nakh. Then we have the early 'rabbinic' era, with Mishna, Talmud, and associated documents. Followed by the Geonic period, the Rishonim (the earlier authorities), and the Acharonim (the later authorities), until the rabbinical and secular researchers of the present day. In this long and broad view of the matter, the value of the present study becomes evident.

My hope is that my work will inspire and guide others to do further research of the same sort.

Postscript. *I am confident that I correctly and fully processed the data that I received from Gabay, but I cannot guarantee that it was entirely accurate and exhaustive. I therefore invite any reader who finds any gap(s) or error(s) in the present study to kindly inform me by e-mail at avi-sion@thelogician.net.*

17. Tabulation of detailed results

The following snapshot not only shows (read vertically) the 72 valid explicit a fortiori cases that each author/book identified, as (apparently, to date) either the historic-first to do so (A), or independently but not the first (B), or derivatively (C), but also clarifies (read horizontally) the degree to which transmission of information to an author from his predecessors (A) occurred (C) or failed to occur (B or blank)[290]. This table, developed in a spreadsheet, has been very useful as an aid to accuracy and consistency in the composition of the present essay.

AS + YG / ATM	Type	AS/ AFL	LJ/ RTT	AS/ JL	HH/ BM	MS/ MY	ZWE /MT	ZHK /NO	SYA /YT	R/ CT	GR+
Genesis 3:22-23	+s				A						
Genesis 4:14	+s&				A						
Genesis 4:24	-s&	C	C	C	C	C	C	C	C	C	A
Genesis 6:3	+s				B					A	
Genesis 11:6	+s				A						
Genesis 14:23	-p				A						
Genesis 17:17	+s				A						
Genesis 18:12	+s				A						
Genesis 39:8-9	+s				A						
Genesis 44:8	+p	C	C	C	C	C	C	C	C	C	A

[290] Obviously, each row must have an A cell, and may only have one.

AS + YG / ATM	Type	AS/ AFL	LJ/ RTT	AS/ JL	HH/ BM	MS/ MY	ZWE /MT	ZHK /NO	SYA /YT	R/ CT	GR+
Exodus 6:12	-s	C	C	C	C	C	C	C	C	C	A
Exodus 6:30	-p				A						
Numbers 12:14	+s	C	C	C	C	C	C	C	C	C	A
Deuteronomy 31:27	+s	C	C	C	C	C	C	C	C	C	A
Deuteronomy 32:39	+p				C	C		A			
Judges 14:16	-p	C	B			A					
1 Samuel 14:29-30	+s&	B	B	B	C	C		A			
1 Samuel 14:39	-s					A					
1 Samuel 17:37	+p	A		A							
1 Samuel 21:6	+p	A		A							
1 Samuel 23:3	+p	C	C	C	C	C	C	C	C	C	A
2 Samuel 4:10-11	+s	B		B		C		A			
2 Samuel 11:11	-s					A					
2 Samuel 12:18	+s&	B	B	B	C	C		A			
2 Samuel 12:21	+s&					C		A			
2 Samuel 16:11	+p	B	B	B	C	C		A			
1 Kings 8:27	-s	B	C	B	C	C	C	B	A		
2 Kings 5:12	-s					A					
2 Kings 5:13	+s	B		B		B				A	
2 Kings 10:4	-s	B	C	B	C	C	C	B	A		
2 Kings 18:23-24	-s&	B				A					
2 Kings 18:35	-s					A					
Isaiah 1:3	+s					A					
Isaiah 10:11	+s					A					
Isaiah 20:6	-s				C	B	C		A		
Isaiah 36:8-9	-s&	B				A					
Isaiah 36:20	-s					A					
Isaiah 66:1a	-s	C	A								
Isaiah 66:1b	-s	C	A								
Jeremiah 2:11	+s					A					
Jeremiah 8:7	+s					A					
Jeremiah 12:5a	-p	C	C	C	C	C	C	C	C	C	A
Jeremiah 12:5b	-p	C	C	C	C	C	C	C	C	C	A
Jeremiah 25:29	-s	C	C		C	B	C		A		
Jeremiah 45:4-5	-s	C	C		C	B	C		A		
Jeremiah 49:12	+s	C	C		C	B	C		A		
Ezekiel 3:4-7	+s					A					
Ezekiel 14:13-21	-p	B								A	
Ezekiel 15:5	-s	C	C	C	C	C	no	C	C	C	A

AS + YG / ATM	Type	AS/ AFL	LJ/ RTT	AS/ JL	HH/ BM	MS/ MY	ZWE /MT	ZHK /NO	SYA /YT	R/ CT	GR+
Ezekiel 33:24	+s	C	B		C	C		B		A	
Jonah 4:10-11	+s	B	B	B		A					
Psalms 25:8-9	+p				C	C		A			
Psalms 78:20	+p	B		B	C	C		A			
Psalms 94:9a	+p	A		A							
Psalms 94:9b	+p	A		A							
Psalms 94:10	+p	A		A							
Proverbs 11:31	+s	C	C	C	C	C	C	C	C	C	A
Proverbs 15:11	+p	B	C	B	C	C	C	B	B	A	
Proverbs 17:7	+p&					C		B		A	
Proverbs 19:7a	+p	B	B	B	C	C		B		A	
Proverbs 19:10	+s	B	C	B	C	C	C	B	B	A	
Proverbs 21:27	+s	B	C	B	C	C	C	B	B	A	
Job 4:18-19	+p	B	C	B	C	C	C	B	A		
Job 9:13-14	-s	B	C		C	B	C		B	A	
Job 15:15-16	+p	B	C	B	C	C	C	B	B	A	
Job 25:5-6	+p	B	C	B	C	C	C	B	A		
Job 35:13-14	+s									A	
Esther 9:12	+s&	C	C	C	C	C	C	C	C	C	A
Daniel 2:9	+p	B		B						A	
Nehemiah 13:26-27	+s	C	C			B	B			A	
2 Chronicles 6:18	-s	B		B		C		A			
2 Chronicles 32:15	-s	A									

The above table concerns the 72 valid explicit cases found. The following table concerns the 7 valid implicit cases (D) and 7 invalid cases (E).

AS + YG / ATM	Type	AS/ AFL	LJ/ RTT	AS/ JL	HH/ BM	MS/ MY	ZWE /MT	ZHK /NO	SYA /YT	R/ CT	GR+
Genesis 3:1	-s				E						
Genesis 6:9	+p				D					D	D
Genesis 17:20-21	+s&				D						D
Genesis 27:37	+p				E						
Leviticus 10:19	+s				D			D			
2 Samuel 10:16	?				E						
Ezekiel 23:39-40	+s					E				E	
Habakkuk 2:4-5	+s										
Psalms 15:4	-s									D	
Psalms 15:5	+s									D	

AS + YG / ATM	Type	AS/ AFL	LJ/ RTT	AS/ JL	HH/ BM	MS/ MY	ZWE /MT	ZHK /NO	SYA /YT	R/ CT	GR+
Proverbs 6:30	?							E			
Proverbs 10:17	?				E						
Proverbs 19:7b	+s&				E						
Job 28:17	+s					D					

Explanation of the column headings and statistical summaries of the above raw data are all given in chapter 1.

18. Main references for this essay

The following are the works from which most of the information needed for the purposes of the present study was drawn. Note that the books mentioned can all be downloaded, or at least read, at the web addresses given below.

ASHKENAZI, R. Shmuel Yaffe. *Yefeh Toar* (*Beautiful of Form*). Composed in Bursa, Ottoman Turkey ca. 1560-80; published in Venice, 1597.

https://beta.hebrewbooks.org/42241

EINHORN, R. Zeev Wolf. *Midrash Tannaim* (*Exegeses of the Tannaim*). Published in Vilna, Russian Lithuania, 1839.

https://www.hebrewbooks.org/7058

*GENESIS (*or *BERESHITH) RABBAH*. Midrash composed ca. 300-500 CE. Authors unknown; R. Oshaya ben Hama was possibly one of them.

https://hebrewbooks.org/14385

HIRSCHENSOHN, R. Chaim. *Berure HaMidot* (*Clarifications of the Hermeneutic Principles*). Published in Jerusalem, Mandatory Palestine, 1928.

https://hebrewbooks.org/3094

JACOBS, R. Louis. "The Qal Va-Homer Argument in the Old Testament" in *Rabbinic Thought in the Talmud*. Published by Vallentine Mitchell in London, UK, 2005.

https://archive.org/details/rabbinicthoughti0000jaco

KATZENELLENBOGEN, R. Zvi Hirsch. *Netivot Olam* (*Ways of the World*). Published in Vilna, Russian Lithuania, 1822.

https://beta.hebrewbooks.org/24940

RASHI, R. Shlomo Yitzchaki. *Commentary on the Tanakh*. Composed in Troyes, France, before 1105; published Reggio di Calabria, Italy, 1475.

https://www.chabad.org/library/bible_cdo/aid/63255/jewish/The-Bible-with-Rashi.htm

SION, Avi. *Judaic Logic*: *A Formal Analysis of Biblical, Talmudic and Rabbinic Logic*. Self-published in Geneva, Switzerland, 1995.

https://www.academia.edu/51587434/JUDAIC_LOGIC_A_Formal_Analysis_of_Biblical_Talmudic_and_Rabbinic_Logic_entire_book_

SION, Avi. *A Fortiori Logic: Innovations, History and Assessments*. Self-published in Geneva, Switzerland, 2013.

https://www.academia.edu/51592132/A_FORTIORI_LOGIC_Innovations_History_and_Assessments_entire_book_

STRASHUN, R. Mattityahu. *Mattat Yah* (*Gift of God*). Composed in 1838-78; published in Vilna, Russian Lithuania, 1892.

https://beta.hebrewbooks.org/6310

There are very probably many more works, other than these, containing additional references to Biblical a fortiori discourse (not to mention non-Biblical discourse of this form), either by purposeful, ordered listing, or in a scattered, incidental manner. The extensive literature of the Mishnaic, Talmudic, and early post-Talmudic eras is very likely to be a rich lode of new information on this subject. Tosafot and other medieval commentators still need to be investigated, too. And of course, probably other authors not so far considered. So, there's still a lot of work to be done in this field! I hope that the methodology used in the present study serves as a model for future researchers.

7. CAUSAL PRESUPPOSITIONS OF PRAYER

My primary incursion into Theology has been the question of whether the existence of God can be proved or disproved – and my answer has consistently been that we can do neither decisively, but must take that proposition on faith, or refuse it on more or less the same uncertain ground. I have also done work on the issue of the causality of Creation, arguing that God's relation to His Creation would have to be volitional rather than causative, i.e. based on freewill and not on determinism (and even less on chance).

Here I wish to examine the metaphysics of prayer. What are the implications of praying, specifically as regards the causal relationship between God and His Creation? For when I pray (and I do), it is evident that I am *assuming* that God is involved day by day, minute by minute, in the workings of this world. And this, not just for little me, but for everyone and everything. If I even just say 'God be with me' as I leave my home, I am already assuming God's involvement.

1. Varieties of prayer

The term 'prayer' is quite broad, including many utterances about God or aimed at God[1]. Some prayers are **descriptive and/or prescriptive**. A notable example, with both these characters, is the *Shema Israel* prayer[2] in the Jewish prayer book, which is recited by pious Jews morning and evening. For examples, a descriptive element in it are the words "the Lord our God, the Lord is One"; a prescriptive element in it is "Hear Israel", and another further on is "And you shall love the Lord your God…"; and there are many more examples of both.

Many prayers of *praise* are enumerations of God's many powers and qualities; the laudatory aspect being implied by the very act of listing. In the Jewish context, the many and various blessings of God are laudatory prayers; so are many of the psalms. The term 'halleluyah' is a literally call to praise God. Prayers of praise acknowledge and celebrate God's great powers and moral qualities; and are means for expressing one's personal love and respect for Him and devotion to His cause. When such prayers are uttered, one is confirming one's faith in the things said in the traditional creed about God and about the behavior He reportedly expects of us.

Mostly, we think of 'prayer' as referring to **petition**, words through which we ask for something from God. The range of our requests is very large, covering all our normal material, mental and spiritual concerns. We may pray for oneself, for one's family and friends, for one's people or country, for all humanity, even for animals. We often pray for understanding and achievement on the spiritual, ethical, intellectual, and behavioral planes; for physical and mental health and longevity; for a fitting spouse and good children; for economic and financial sustenance; for professional capacities and success; for social respect and recognition; for political freedom and justice; for security and victory over evil people; and for harmony and peace.

Turning again for an example to the Jewish prayer book, if we look at the weekdays' *Amidah* (standing prayer, comprising nineteen petitions each ending in a benediction), we can see the great variety of objects that petitionary prayer may have. Briefly put, there is a prayer for support,

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[1] Needless to say, I use the English word God as referring to the Deity of the Jews, even though this word is not Hebrew and not one of His many names in the Jewish sources.

[2] Drawn from Deut. 6:4-9 and 11:13-21, and Num. 15:37-41.

protection and salvation; another for the gift of life to the dead; another hailing holiness; there are others for knowledge, for penitence, for pardon, for redemption, for healing of the sick, for material and emotional blessings, for ingathering of exiles, for righteousness and justice, for humbling of the arrogant, for mercy towards the righteous, for the rebuilding of Jerusalem, for messianic salvation, for answers to our prayers, for the return of God's presence in Zion, for gratitude, for enduring strength and peace, and much more[3].

All the prayers in the *Amidah*, note, include praise (blessing) of God, as well as overt or implied petitions. Some include prayers of *thanksgiving*. The latter, it should be noted, may refer to petitionary prayers that have already been answered, or may simply express gratitude for God's constant and manifold goodness towards us even without His waiting for our prayers.

An important form of prayer is *confession and penitence*. In confession we recognize our sins and transgressions, our moral and spiritual failures, our bad behavior; and in penitence we resolve to avoid such foolishness and change our ways for the better. A petition is involved in such prayer: a request for forgiveness. We may ask *questions* of God, hoping for answers. Prayer is communication with God, and sometimes this may take a reproachful turn. Prayers of *complaint* occur when someone has suffered greatly and does not see what the point of it all was. Here, the petition is for release from pain.

Note again that one may pray for oneself or on behalf of others. The same diverse categories of prayer as are found in formal prayer are also found, of course, in informal prayer. All this clearly assumes that God is very actively involved in the minutest details of the world-process long after the moment of Creation as such, including today. Alternatively, we could say that Creation is ongoing, with God fine-tuning His creation as interactively needed.

2. Divine providence

So, the act of prayer implies that, in discussing God's causal relation to the world, we must not just posit initial Creation, but must equally take into account *Providence*. And not just occasional punctual providence, but widespread and continual providence, covert if not overt. Without this assumption, our theology of Divine ætiology is obviously very deficient.

Note well that I am not here looking for or proposing some sort of *proof* of God's providence. I do not believe that any strict proof of it is possible. I am merely pointing out that as of the moment one prays one is logically *assuming* the existence of providence. It would be inconsistent to claim that God let the world proceed independently once He created it (as Deism and some earlier philosophies do), and at the same time indulge in any form of prayer to Him.

Needless to say, prayer is not a merely Jewish phenomenon, but is found in many other religious traditions, throughout history and to this day. In that case, the entity prayed to may or may not be God. Of course, the content and emphasis of prayers vary from one tradition to the next – indeed, from one person to the next, and even in different phases of one person's life.

The primary assumption of Jewish prayer is that God is a Person, a purely spiritual being *with consciousness, freewill and values* (somewhat like ours, but infinitely greater, fuller and more accurate), albeit being devoid of materiality or material constraints (unlike us). If anything, God has much more 'personality' than we do, since whereas we are often unconscious or uncertain of what we are doing, or weak of will, or lacking in purpose or direction, thus resembling impersonal entities, none of these deficiencies can ever be ascribed to God.

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[3] See David de Sola Pool, *Book of Prayers* (New York: Union of Sephardic Congregations, 1986).

Cognition, volition and valuation – in short, the possession of soul or spirit and its three essential functions – are what God has in common (though in widely different degrees) with us humans. His assumed abilities to hear prayer, evaluate the situation, and do something about it, signify continual interaction between Him and us. If God, say, had just issued some arbitrary commandments to us and then altogether withdrew, leaving His relationship with us at that simple level, without accepting feedback from us other than mere obedience or disobedience, our relation to Him would be much less complex than we commonly assume.

In Judaism, no one ought to, in principle, address prayers to anyone but God. We are supposed to have direct access at all times to God's full attention and compassion, with no need of intermediaries. One may obviously, however, ask a live person, ordinary or saintly, to pray to God on one's behalf, for whatever reason. Asking a *dead* person, ordinary or saintly, to pray to God on one's behalf seems more problematic, since doing so is an act loaded with heavy assumptions about the continued existence of that dead person as an individual (albeit as a disembodied soul), and more to the point, that that person can hear one's request and in turn engage in prayer to God. Still more problematic would be to petition a dead person for some active intervention *by* him or her besides mere prayer to God – here, one would be assigning some Godly powers to that person, thereby putting one's monotheism in doubt.

Many Jews do go and pray by the graves of holy ancestors – e.g. at the Cave of Machpelah, Rachel's Tomb, Joseph's Tomb, or King David's Tomb, and by the graves of famous rabbis, like R. Shimon Bar Yochai (in Meron, Israel) or R. Nachman of Breslov (in Uman, Ukraine) – but in principle they there simply recite prayers to God (e.g. Psalms): they do not usually petition the persons who are buried there (though maybe some individuals do so, I don't know)[4]. It is noteworthy that Jews never pray to Moses.

We Jews do, on the Sabbath eve, sing a song addressed to two visiting angels, invisible messengers of God, asking them to bless us on His behalf. This is, to my knowledge, the only Jewish prayer not addressed directly to God (though it could well have been so formulated)[5]. The idea of these angels comes from the Talmud, but the poem is a late composition (introduced by kabbalists circa 1700 CE). This song has rightly received some criticism from prominent rabbis (notably R. Jacob Emden, in his prayer book of 1745), but is now well established in all prayer books. Most people do not reflect on the metaphysical implications of their praying to anyone besides God.

In any case, our supplications are certainly not addressed to a mere impersonal entity or process or force, say like that implied by the concept of karma (which refers to a supposed 'appropriate reaction, sooner or later, to every action' mechanism of justice-without-mercy programmed in nature). It would be absurd to pray for help to a mere 'it', a deaf and blind, powerless, and indifferent object, devoid of personality – e.g. a stone statue or the Sun or even 'the Universe'.

Buddhists and Hindus (who both believe in karma) do not, of course, pray for help to karma, but rather respectively to buddhas[6] and bodhisattvas, or multiple gods; thereby ascribing to these

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[4] If they do petition dead persons, perhaps they only ask for intercession by prayer to God on their behalf, rather than for the performance of earthly miracles. I do not know if anyone has investigated this issue empirically.

[5] The poem could easily be fixed by changing the first phrase of each of its verses from the second person to the third person. That is, instead of 'peace be upon you' (*shalom aleichem*) putting 'peace be upon them' (*shalom aleihem*), and so forth.

[6] I think anyway that when ordinary Buddhists pray to 'the Buddha' they are unconsciously in effect praying to God. Their prayers in such cases are directed towards 'Heaven', rather than towards an enlightened man, i.e. the historical founder of Buddhism, or to a statue thereof. Even so, it does seem (to an outsider like me) that they regard the statues of buddhas and bodhisattvas as having some life and personality of their own. I refer here to popular Buddhism, as distinct from the more intellectual normative religion(s).

limited entities supernatural powers capable of changing the course of events (which is somewhat paradoxical, since one's karma is in principle inevitable and unavoidable until and unless one attains liberation/enlightenment). Sundry remaining pagans or idolaters do seemingly worship inanimate material objects, although most of them perhaps regard their idols as mere conduits to a more 'living' entity (e.g. a particular 'spirit' or a 'demon' or whatever).

Christians and Muslims do pray to the God of monotheism, according to major Jewish commentators. But Christians often tend to rather pray to Jesus (whom they openly view as a divine being, and not merely as a human intercessor)[7], and to sundry (in their eyes) saints alive or long dead. As for Muslims, they seem to consider their 'prophet' Muhammed as a quasi-divine being, since they regard any criticism of him as 'blasphemy'[8]. They do, I gather, address some words of salutation to him in their daily prayers[9]; but I do not know (have not yet tried investigating) whether they ever address any petitions to him, which would constitute deviation from monotheistic practice[10].

Prayer in front of images (in two or three dimensions) representing some god or saint is practiced not only by Hindus and Buddhists, but also by Catholic and Orthodox Christians; but (to their credit) not by Protestant Christians[11] or Muslims or Jews. The idea behind such prayer is presumably that the statue or illustration 'channels' the prayer to the person the image 'represents'. Or maybe, the idea is more simply that the image helps the worshipper to focus his attention on

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[7] Christians believe in a 'trinity', the father (of Jesus), the son (Jesus), and the holy ghost (apparently, a reification of the divine spirit active in the world), which they consider not as three distinct gods but as three aspects of one and the same godhead. In my personal view, this does not qualify as 'monotheism' in the sense Judaism attaches to the term. Christians, of course, regard their 'father' figure as corresponding to the Jewish concept of God, since that was historically the source of their notion. But they are not in fact identical ideas, but radically different ones, because Judaism certainly rejects the fantasies of God incarnating in human form or having a divine human offspring (such fantasies being distinctly pagan – as evidenced by their presence in pre-Judaic religions and in polytheist Hinduism). I do not intend this remark to be offensive, but only accurate. We often speak of the 'three monotheistic faiths', for the sake of peace between our three population groups (which is of course very desirable); but in an intellectual discussion we are duty-bound to be lucid and honest.

[8] To my mind, applying the term 'blasphemy' to criticism of Muhammed constitutes idolatry; all the more so since such accusation is sometimes coupled with heavy corporal, even capital, punishment. We often read of such harsh sentences being meted out even today in fanatic Islamic regimes like Pakistan, Iran or Saudi Arabia. I must say, I have little respect for any faith that resorts to or tolerates religious terrorism in this day and age.

[9] Muslims do address words (in the second person) to Muhammed in some daily prayers, e.g. when they say "Peace be on you, O Prophet." This implies they believe that he is alive somewhere (in 'heaven', presumably) and able to hear their millions of daily salutations from afar. But those particular words do not constitute a prayer of petition; they only add up to a salutation. I do not know if there are, in Muslim prayers, words addressed to Muhammed which constitute petitions to him.

[10] Regarding the 'monotheism' of Islam, I would like here to reiterate what I have said in previous works. Although it is known and indubitable that Muhammed's idea of Allah was derived from the Jewish teaching of God, it does not follow that these concepts exactly correspond. This is evident from the words attributed to Allah against Jews in Koran and Hadith (scripture and oral tradition), words that could not conceivably ever have been uttered by the God of the Jews (to a non-Jew, to boot). The two deities cannot truly be considered one and the same, since their alleged thoughts, words and deeds are not mutually consistent. It is not a matter of their having different divine names, but of the content and connotations of their names, i.e. the different religious beliefs encapsulated in them. There is some overlap in beliefs, but not all beliefs are held in common; and the differences are significant. Therefore, the monotheism (i.e. worship of a unique, single, overriding god) of Muslims is not identical with that of Jews – an effectively different '*theos*' is involved, though each is claimed to be one and only. The term 'monotheism' does not have a unitary, exclusive reference; it is rather vague regarding the god intended. Again, I do not intend this remark to be offensive, but only accurate.

[11] However, I presume that Protestants do have a mental image of Jesus when they pray to him.

the faraway person worshipped. In any case, God is too abstract a concept to be represented in material images[12].

It is logically obvious (to me at least) that monotheism is a more spiritual religious doctrine than polytheism, since the spiritual domain is of necessity unitary and unique rather than fractured and scattered. Likewise, worship of a purely spiritual God is obviously a higher level of spirituality than worship of one or more, partly or wholly material god(s). Presumably, those who worship Jesus or Buddha or Krishna, or the like, opt for a deity with a material body because they find it difficult to identify (solely or at all) with a purely spiritual God. That said, it can hardly be denied that people who adhere to such religions can nevertheless find in them much spiritual progress and solace. However, such 'spiritual materialism' can also have negative consequences.

As for atheism, which is more and more widespread today, it does not address, but simply bypasses, the metaphysical questions raised by our having a spiritual dimension, and so remains essentially materialist in outlook. Atheists can, of course, behave in ways which are de facto spiritually positive (often, due to being unconsciously influenced by religions); but atheism can also have significant negative spiritual effects. We can safely say that all religions, and atheism too, have, in various ways and to varying degrees, objectively spiritually negative aspects, though we may not all agree as to precisely what those might be.

It should be said that although all adherents to a certain religion ostensibly pray to the same godly person, they may not in fact have identical conceptions of that person; one might even go so far as to say that every individual necessarily has a somewhat different de facto conception. In Judaism, it is evident that different commentators have somewhat different conceptions of God; and there have been many different schools of thought across time. Notably, within the Talmud, among the medieval commentators, among the Kabbalists, in Hasidism, and in modern times, we can observe different viewpoints and sometimes marked controversies between commentators. Notwithstanding, these are all regarded as referring to one and the same God, the God of Judaism.

The point to stress here is that whatever one's conception, whether one prays to God or to some other god or to a saint, such praying implicitly ascribes certain powers, indeed superpowers, supernatural powers, to the deity or saint addressed. In petitionary prayer, the addressee is assumed able to hear the prayer and to respond to it at will. Otherwise, one would not bother praying. Praying is not the same as mere wishing or hoping, which are passive expressions; by praying for something we attempt to positively *affect* the course of events, albeit indirectly.

3. Divine interference in nature and human lives

And of course, this discussion takes us straight into the minefield of *theodicy* (discussions on the justice of God): why does God not help good people when He ought to do so (in our eyes), and also why does he apparently helps bad people when he ought not to do so (in our eyes)? For even if we grant the general principle that God abstains from interference in the affairs of his creatures (individual humans in particular, but also individual animals, and maybe even individual plants), it is often unclear why He would abstain in certain particular cases or situations for which (again

[12] Although Michelangelo's Sistine Chapel paintings by comes to mind, where God is depicted as a superb old man. Statues and illustrations, of course, serve the religion concerned, as an institution, by reminding all passers-by of its gods and saints; they are, as well as spiritual reminders, expressions of social and political power held by the institution and its guardians. A similar role is played, for religions that do not resort to statues and illustrations, by abstract symbols like the cross, the crescent or the six-pointed star or by selected words and phrases.

in our eyes) abstinence brings no benefit whatever to His creation but on the contrary perhaps causes great damage to it.

Notably, it is absurd to claim that God did not kill the likes of Hitler or Stalin or Mao early on in their careers, because He wanted to give these evil individuals time to either damn themselves fully or to grow spiritually and choose good over evil! The obvious question to ask here is: what about their millions of *victims*, the overwhelming majority of who were innocent of any crime deserving the horrific suffering they endured in one way or another – why were *they* not given a lifetime to grow spiritually and do good? Why were their many and urgent prayers and screams not heard? The same can be said regarding other, lesser murderers and sundry monsters, of course.

Clearly, the thesis that God allows evil to grow and act in order to make room for human freewill, while it sounds reasonable initially if thus vaguely formulated, does not stand up to scrutiny if considered in more detail. The issue is: whose freewill is being more protected, the good people's or the bad people's? Just how is the world's spiritual development improved when relatively or absolutely evil people are given, for any lapse of time, power of life and death over relatively or absolutely good people? We have no plausible answers to such radical questions.

Some thinkers claim that after the initial act of creation God has abstained from interfering in human affairs so as to ensure our freedom of the will and moral responsibility. But there is clearly a contradiction of sorts between this particular theistic (notably Deist) claim, which would involve at best mechanical karma (which is empirically not evident), and the widespread belief among most monotheists that we can pray to God and expect Him to interfere in the detailed manner already indicated, out of justice and mercy.

Prayer necessarily implies continual and intentional Divine interference in the world process, or at least in the lives of humans. God's interference is logically implied by petitionary prayers, which look forward to possible events, before the fact; and equally by prayers of thanksgiving and even of praise, which look back on past or current events, after the fact.

The big question is, of course, how such interference might be compatible with Natural Law. But, regarding this issue, the thesis that God's intervention occurs, as it were, *behind the scenes*, in ways that are not perceived inductively when we empirically observe and conceptually formulate (using the scientific method) apparent natural laws, is easy to deal with logically. This statement can be understood if we keep in mind that all our assumed 'natural laws' are ultimately based on generalization of a relatively limited number of observations – never on complete enumeration of all occurrences of the phenomena concerned in the whole universe and throughout time.

It is not theoretically inconceivable that God might abstain from breaking an apparent law while it is publicly being scrutinized by scientists; and yet go ahead and break it when no one is watching! This could be said of any natural law, even (say) one as certain to contemporary science as the law of conservation of matter and energy. In any case, if perchance we spotted a breach of the law, we would simply consider such breach as a perfectly natural phenomenon – and particularize the previous formulation of the law, making it less general, more conditional. But in such event, it could be said that God deliberately revealed the irregularity because He wanted us to narrow the presumed law's scope[13]. So, there is really no possible logical objection to conceiving God as

[13] For all we know, the seemingly lawless, indeterministic behavior of subatomic particle-wave phenomena might be due – not to natural spontaneity as current science assumes – but to intentional, purposeful choices and acts of will by God, i.e. by God acting on the minutest details of Nature's unfolding. There is simply no way for us to tell the difference!

engaged in 'hidden' breaches of natural law, possibly without ever being 'found out' (if He chose not to be).

This is even truer when we consider medical phenomena, which are not subject to such hard and fast rules, but rather to probabilities, due to multiple conditions of causation and acts of volition being involved in them. Thus, if a person is suffering from a very problematic disease and prays to God for a cure, or someone prays on his or her behalf; and that person quite unexpectedly suddenly recovers (as happens occasionally), it is not unthinkable that a miracle did indeed occur, i.e. that God actually interfered in some way (for instance, by secretly weakening or killing off attacking viruses) and cured the seemingly hopeless case. There is no way to prove this was indeed what happened; but on the other hand, there is no incontrovertible basis for skepticism. The same uncertainty applies to preventive supplications, asking God for protection from eventual dangers in general or specifically.

If one prays for good or bad and the requested event takes place, one cannot conclude that one's prayer (let alone all prayer) was effective, as this would constitute *post hoc ergo propter hoc* fallacious argument (and for all prayer, added generalization). If one prays for good or bad and the opposite occurred, one cannot infer that prayer in general is useless, as it is conceivable that God decided not to grant this particular petition but might still choose to satisfy others.[14]

Furthermore, many of our prayers have more to do with mental/spiritual issues than with material/physiological ones. We may pray for mental health, for insight and wisdom, for courage and strength, for happiness, for familial, social, economic and political success, and so forth. Such prayers are obviously requests for Divine intervention into our own souls or minds, and/or those of others. These issues are not directly or exclusively related to the laws of material nature that physicists, chemists, biologists, astronomers, and the practitioners of other relatively exact sciences, are concerned with.

God is here, clearly, assumed to be able to affect at will our inner, psychological situation, as much as the bodily (including the nervous system) and material environment of our lives. Such interference might take many forms: it might consist in *blocking* a person's intended act of will (affecting the internal causative environment of volition), or making a person's act of willing or of abstaining from some will *more or less difficult* as appropriate (influencing volition through consciousness of some positive or negative consideration), or simply making the person's volition *successful or unsuccessful* by controlling the external causal (causative or volitional) environment.

The claim here is that God can indeed invisibly manipulate our brains and body and surrounds, and even (through influence rather than determinism) our soul's efforts – *without* thereby denying the principle of freedom of the will (human or other). There is little expectation that science might *detect* such manipulations, today or ever in the future. The claim is therefore necessarily speculative, an issue of faith – though certainly abstractly conceivable and not logically impossible.

Thus, our resort to prayer implies that God is potentially actively involved in our individual and collective lives, and perhaps equally that of other animal species and even of plants (as when people pray for the wellbeing of their livestock or crops), in a very detailed way and at many levels. What sort of interference in nature would that have to be? Obviously, God might intervene on His

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[14] This conundrum is illustrated in the Biblical book of Jonah (4:1-2). Jonah was ordered by God to call on Nineveh to repent or face destruction; it seems that he tried to avoid doing so because he thought that if Nineveh repented he would be made ridiculous since his warning of destruction would not have come true. But that was invalid reasoning in that we cannot estimate *ex post facto* what was initially decreed to occur (destruction in this case) merely by looking at what did ultimately occur (repentance and non-destruction).

own initiative, independently of prayer; but where prayer was involved, God can be construed as having heard the prayer and made a decision to intervene (or not), based on ad hoc justice and mercy considerations, and His wider plans and goals for the person or people concerned, and even for the whole world.

Thus, the central assumption of praying is that God *hears* all prayer and may freely choose to consciously respond positively to it by means of pointed *conscious acts of will* on His part. The assumption is that He relates causally to the world by intentional volition on a day-to-day basis, not merely at the first act of creation.[15]

Some commentators claim that the assumption of petitionary prayer is that we have the power to change God's mind by its means. This is a silly objection, because we do not force God to change His mind – He changes His own mind by freewill. Some rebut that God never changes His mind and therefore that prayer is useless. There is no basis for such claim, because there is no a priori impossibility that God might keep an open mind and adapt His decisions to changing circumstances. After all, we believe that He has given us freewill, and therefore accepts a measure of moment-by-moment uncertainty in the unfolding of the world[16].

Without doubt, God does not need our prayers to know what is going on at a given time and to make decisions; but when we petition Him, we presumably voluntarily add *an additional factor* which may well (or may not) tip the scales on our behalf. That added factor may simply be the recognition by us, in the act of petitioning, that God is in charge of things and is the One to appeal to in all circumstances. By praying for valuable and virtuous outcomes, we show God that we are, or wish to be, on His side, the side of good, and we acknowledge His justice and mercy. That may be all that is needed to tip the scales in our favor with regard to the issue at hand.

4. The spiritual side of existence

While there is no technical reason to doubt the possibility of Divine providence, we can still of course ask why God would intervene at all, knowing that we are tiny inhabitants of a tiny planet in a tiny solar system, which is one of billions in a tiny galaxy, which is in turn one of billions in the known universe, which might in turn be but one of billions of universes in a yet unknown infinite multiverse! Given that we are individually and collectively truly minute entities, mere specks of dust – why would a God who created such a massive world have any interest in us? We would not adopt microbes as pets; why would God adopt us? When the world was thought to be relatively small, we could well imagine that we humans have some importance in the scheme of things; but now we know better, it seems far less likely.

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[15] If it is claimed that God sits outside the physical universe, and that He too is subject to the law that nothing can travel faster than the speed of light (unless something resembling 'quantum entanglement' is involved), then prayers to Him and reactions from Him would take billions of years to travel back and forth! Obviously, we must suppose that God can both hear our prayers and react to them *instantly*, in which case He is probably (as He is traditionally claimed to be) omnipresent. Or, perhaps more precisely, reflecting the apparent relation of our souls and bodies, the spiritual realm must be thought of as contiguous with (albeit outside) the physical realm, like the two sides of a sheet of paper.

[16] This said leaving apart the issue of Divine knowledge across time, of past, present, and future events, which can be said to occur on a higher level. I have addressed this issue elsewhere, explaining that just as we experience the present as an extended moment (albeit small) of time, rather than as a mere instant (i.e. point) of time, so presumably for God the present is a moment so greatly extended that it englobes all past and present and future events in one sweep. Thus, God can conceivably see our volitional choices beyond time without affecting our freedom.

But the answer to that objection could be that the vastness of material creation is relatively unimportant compared to the less visible spiritual sphere of existence. In this parallel 'dimension' or 'domain' of existence, which is perhaps God's own 'substance', whose 'light' all human souls are but tiny 'sparks' of[17], our lives may well have great significance in God's eyes, inciting Him to engage in micromanagement of the material and mental aspects of our existences for our spiritual benefit. The thing to keep in mind when considering this deeper issue is that the world is clearly not the merely material entity that modern science dogmatically assumes. It is still quite materialist and has *not even begun* to deal with the mysteries of consciousness and volition in any credible detailed manner.

For a start, most philosophers and scientists have not yet realized that there is an inconsistency in their theory that what we perceive in sense-perception (especially of sights and sounds) are *mental images of* external objects rather than the external objects (or aspects thereof) *themselves*. If all we ever perceived were mental images, there would be no thought of or basis for claiming external objects, and therefore no way to ever test the validity or not of our perceptions. Clearly, the fact of perception is much more mysterious than it seems at first blush, and we are still very far from understanding it theoretically.

Likewise, our current theories of the material universe do not take into consideration, let alone explain in sufficiently deep and convincing ways, the amazing facts of consciousness and volition. There are currently many efforts to empirically demonstrate that living matter mechanically evolves from inanimate matter under certain circumstances; but even if that is proven, it will still be necessary to find mathematical formulas which predict and explain such derivation, and those are still very far off. There are certainly to date no mathematical formulas that lead from living matter to consciousness and volition. Consciousness and volition cannot to date be scientifically predicted and explained from known laws of physics; they are not covered by those laws. Modern biology prefers to avoid these subjects, as does physics theory, because they are still much too abstruse for us.

Indeed, even with regard to inanimate matter and physical life, while modern science has made very impressive progress in *describing* the evolution of matter and then life from the Big Bang to this day on our Earth, it has not so far succeeded in *explaining why* existents and the laws of nature controlling them are as they are and not otherwise. I am not just referring to the obvious questions of how the initial substance which exploded came to at all be and how it came to suddenly explode, but moreover to all the details of material evolution that followed. How come the original energy or matter *had it programmed within itself* to evolve in the complicated ways that it did, forming light, more and more complex elementary particles, then molecules, and ultimately life and then consciousness and volition? Why were the formative forces involved (gravity, electromagnetism, etc.) as we have found them to be and not otherwise? Modern science just takes these natural events as givens – but they are still very mysterious and likely to stay that way forever.

All this said in passing, to remind us that atheism is far from triumphantly established and that we have good reason to continue wondering at the miracle of existence! Note well, however, that this does not mean that the creation narrative found in the Torah (and other such religious tracts), the timing and sequence of events there proposed, taken literally, has any credibility left today. It is

[17] I put all these terms in inverted commas to signify that they are only intended as rough analogies and not to be taken literally. To speak of spirituality, we are forced to appeal somewhat to material notions of substance or place; but that is mere imagery, not implying that such notions are really applicable.

already certain that things proceeded very differently in fact. Sad, but true[18]. Fundamentalists refuse to admit it or try hard to ignore it, but they only fool themselves because the evidence in favor of the current scientific viewpoint is overwhelming.

Notwithstanding, belief in God as Creator and as Providence is not logically affected by the scientific debunking of the Biblical scenario, even if some ideological atheists claim otherwise. Such belief is still rationally sustainable, for the simple reason that, being a spiritual hypothesis, it can adapt to any material conditions. It is not falsifiable by any discovery relating to matter. Provided we do not attach our belief in God to any given creation thesis, but always accept the latest scientific verdict as the best bet, we can always retort 'well, that's the way God chose to do things'!

5. The utility and value of prayer

The prime *purpose* of petitionary prayer is to call for God's attention on something of interest to us. We presume God is already, always and everywhere, aware of everything that is going on – yet we call on Him to deal with some particular subject of personal or communal interest to us. This signifies that the act of prayer carries some weight in God's evaluation of the situation at hand – so that He usually or frequently acts differently when someone has prayed than He would have otherwise. Perhaps more urgently or slowly, or more favorably (more justly or more mercifully), as appropriate. Thus, prayer is granted value and efficacy by God – presumably because the one praying thereby gains merit and establishes a closer relation to Him.

Since petitionary prayer depends on Divine approval, it is not invariably efficacious; whence it follows that the efficacy of prayer cannot be proved (nor disproved) scientifically but must be taken on faith. Even so, prayer always has a potentially beneficial psychological effect, giving us hope and courage in difficult or dangerous situations. Sometimes, just believing that God is there, ready to give us help and support in case of need, suffices to buttress our understanding and courage, and give us hope, improving our chances of success. Someone like me, who has faith in and recourse to prayer, cannot comprehend how people who don't pray manage to get through life's difficulties.

Regarding the causal efficacy of petitionary prayers, nothing can be proved, or disproved, by empirical and statistical means. When we pray, we know in advance that there is no certainty that

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[18] It would be nice if religious belief was still today as in the past a simple affair; but things have become far more complex, and the believer has to work his or her way through the intellectual difficulties now involved. The important thing is to remain scrupulously honest at all times: there is no virtue in faking solutions to problems or pretending there are no problems to solve. The most absurd and dishonest general argument often used by apologists (in print and orally) is to say that since scientific theories are open to debate and yield only probable conclusions, and are constantly changing, they are no better indeed less reliable than faith-based claims. (Imagine if such a standard was used in a court of law, and the judge preferred an established prejudice devoid of proof to the evolving results of detailed field investigations and careful reasoning – inevitably, an innocent man would be condemned or a guilty one would be cleared.) Science is an inductive discipline, based on precise empirical studies and stringent logical arguments, all peer reviewed (this is ideally true, although it must be acknowledged that sometimes unscientific ideologies with political motives are peddled as science). Serious science certainly does not consist of pure leaps of faith like religious claims do. It necessarily evolves over time as research uncovers new factual data and proposes new hypotheses for their explanation. That is so because man is not omniscient and never will be. He must work hard to find the nearest thing to truth that he can at any given stage of research. At each stage, the proposed scientific view is in principle more intellectually reliable than those at all the preceding stages (including prescientific doctrines). This dynamic adaptation to new information and ideas is the very virtue and value of science, which distinguishes its claims from static religious dogma.

our prayer will be favorably received and answered as we wish; still, we pray and hope for that. It is obvious to us that God has His own agenda, and He may opt either way. Perhaps He always responds positively, to our benefit, but not necessarily in the precise way that we imagined and desired.

Nevertheless, those of us who pray can testify subjectively to the miraculous efficacy of prayer. There have been times in my personal experience when I earnestly tried and tried again and again to overcome some failing or weakness within me, yet I could not muster the courage or will needed. Then, through prayer, I suddenly found myself relieved of the fear or bad habit worrying me. I knew full well that it wasn't my own doing, the independent power of my will, that solved the problem, since I had tried repeatedly, unsuccessfully to control myself. Just uttering prayers did not change my mind, either, since my prayers were not immediately answered. It was obvious to me, when release finally came, that I owed it to God's gracious help. I think most people who pray have had similar experiences.

All that has been said above applies equally, of course, to formal and informal prayer. In Judaism, formal prayers are those given in the standard prayer books, which were composed by rabbis over many centuries using material found in the Bible and other traditional literature. These prayers purport to give voice to every occasion, situation, and need, albeit in a general way; and they may concern the community as a whole, as well as the individual. Informal prayers may likewise be communal or individual, but they are composed ad hoc spontaneously.

One might think that formal prayers are less valuable than informal ones, as they are uttered regularly, in some cases two or three times daily, and can easily become rote; but in truth, while one may say much of them unconsciously by force of habit, very often in the recitation some word or phrase or sentence or paragraph stands out with special force. Moreover, the formal prayers cover much ground, so that almost no normal need or kosher desideratum is forgotten; as a result, informal prayers are rarely required (which does not mean they are not valuable).

Of course, a spontaneous prayer, be it a cry for help or an expression of gratitude, is often more passionate and deeply heartfelt than a ritual one. But sometimes the opposite is true: saying ritual prayers in the company of other people can sometimes greatly enhance one's sense of contact with God and stimulate strong emotional reactions. This is no doubt why Jews preferably pray in groups of ten or more.

While the main purpose of prayer is communicating with God – by way of all kinds of supplication (including confession and begging for forgiveness), or praise and thanksgiving (acknowledging God's great qualities and creative and helpful acts) – prayer has many valuable *side-effects*.

In particular, it is worth focusing on the side-benefits of formal prayer. First, the contents of our set prayers are *a daily teaching and reminder* of Judaic values and disvalues, virtues and vices – telling us what is good or bad, what to do or avoid – and of Jewish history and hopes. Second, just uttering the prayers constitutes a statement of belief in God and the things they say. Third, having to recite certain prayers daily or weekly or seasonally, at specified times of the day, constitutes a beneficial *discipline*, structuring our days and tying us to our religion. Fourth, this is also *meditation*, since it demands our attention to the words uttered and concentration on their meanings.[19]

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[19] Jewish prayer is normally in Hebrew. For those who lack, or are not fluent in, Hebrew, there are prayer books with translation into other languages. God is presumed to (obviously) understand all languages, so there is not a big problem in that respect. Nevertheless, Hebrew is preferred because it is the language the prayers were composed in and are recited in in synagogues, and it sounds very nice. For these reasons, many people who do not understand Hebrew still prefer to pray in that language if they are able to read it.

While such prayer tends to become rote to varying extent; it is never in truth devoid of intentionality (Heb. *kavanah*). Rather, it is similar to what occurs during silent meditation: we *weave in and out* of attentiveness, thinking of other things for a while and then returning to the meanings of the words uttered for a while. We may focus on some parts of our prayer for a while, and then get distracted by unrelated memories, issues or plans (or external events) for a stretch. Our degree of awareness of the prayers varies, going from high to low intensity and back again, depending on our energy level at the time.

Singing out loud with other congregants in the synagogue, and indeed all active participation in the many rituals, is a good way to momentarily forget one's problems or desires and to focus more fully on praying.

Sometimes, we may seem to be engaged in nothing more than lip service. But it would be unfair to so characterize our prayers if we recite them daily, weekly, or however regularly, when we already know and adhere to what they mean. We may be paying less attention to what we say right now than we ought to ideally, but still we do somewhat remember, and habitually subscribe to, the words uttered. This may be a weak form of *kavanah*, but it is still *kavanah*. This is like when we say 'Amen' to a prayer uttered by the cantor in the synagogue even though we did not actually hear what he said: we are confident of subscribing to whatever he said.

Thus, we always get at least some benefit from the formal prayers recited, even if not all the benefit we might have gotten with ideal degrees of mental effort. Even so, it is of course mandatory to try our best to pray with maximum concentration, so as to get the full benefit of the exercise. But sometimes, admittedly, we are simply too tired to be able to deliver; we are only human.

Anyway, all prayer is meaningful and valuable as an act of worship, as a statement of our belief in God and His providence and of our inspired choice of devotion to Him. Even when we pray without concentration – cursorily, speedily, while thinking of a multitude of other things, or because we are short of time – our prayer still embodies a significant sacrifice of our time, i.e. the gift of a part of our life we could have used in other ways. Prayer thus constitutes, at a minimum, a bit of self-sacrifice. Sometimes, this is all we want to do – merely to connect to God.

Another important side-benefit of prayer is that it makes us *more God-conscious* in our everyday lives, and thus enhances our spirituality and spiritual proximity to Him. The more we pray, formal or informal supplications and blessings, the more aware are we of God's presence in the world, and thus the closer we are to Him. That is why it is recommended by some great teachers to engage in a frequent if not continual 'dialogue' with God, at every opportunity asking for His help and support, apologizing for one's errors or misdeeds, thanking Him for His gifts, and so on – as if chatting with a human person one is living with here and now. To be sure, God does not ordinarily (unless you are a prophet, which is extremely unlikely nowadays) talk back in so many words; but one may with faith observe concrete results that may be regarded as His replies.

And of course, God-consciousness, i.e. the constant awareness through faith that God indeed exists and is actively involved in the world we reside in, encourages us to do good and abstain from bad, in thought, speech and action. In God's assumed presence, we are on our best behavior; and our mood is more optimistic and joyful. Inversely, if we sin we can (if we have any conscience left) feel the dismal loss and distancing from God which follows the sin. These experiences constitute additional valuable practical consequences of regular prayer.

Yet another wonderful side-benefit of prayer is due to the *sincerity* prayer demands. Since, we believe, God knows everything about us, including all our innermost emotions and thoughts, and

all our most secret deeds, we cannot when we address Him pretend to be what we are not[20]. This is especially true of spontaneous personal prayer; but also, to a large extent, of fixed ritual prayers if we recite them with awareness of the meanings of their words. Since God knows precisely what is in our hearts, and all details of our daily conduct, even more clearly than we do, we cannot be hypocritical but must be scrupulously honest with Him. We cannot fool Him, even when we are able to fool ourselves or others; we must speak truly. Therefore, if only incidentally, prayer tends to increase our inner consistency and self-knowledge, which in turn improves our mental serenity.

It should be pointed out, however, with regard to introspection, that there is some conflict of purpose between verbal prayer and silent meditation. Their psychology differs. In the former, particularly in prayer of penitence, the mind may be stirred-up by verbose self-examination and self-reproach, sometimes in a frenzied manner; whereas, in the latter, based on inner silence, the mind is rather allowed to settle down, and naturally reach clarity and peace.

Excessive speech, in thought or orally, can sometimes take us far from true self-knowledge, when it attempts to force on us mental insight from the outside, as it were, by means of cliché ideas and labels. On the other hand, admittedly, silent meditation, whose usual effect is to make the mind more transparent to inner scrutiny and more honest, can occasionally dull the mind or make us blind to our faults through ego pride (although, even in such cases, if one continues meditating the shortcomings eventually fall away).

In fact, regular practice of silent meditation greatly enhances regular prayer because it calms and clarifies our mind and strengthens our immunity to internal and external distractions, increasing our capacity of attention and concentration so that we can more consciously and powerfully direct our thoughts and words towards God. Instead of merely reciting prayers, we get to mentally aim them heavenward. Silent meditation is practiced sitting down, in an erect and still (yet relaxed) posture, during at least half an hour at a time, at least once a day, and allowing the mind to gently and naturally settle down. If this is done not long before prayer, the effect on it is tangible. But even if done regularly at other times of the day or night, it makes a remarkable difference.

[20] It should be said in passing that nothing is to be gained by resorting to alcohol, marijuana, or other drugs for the purpose of prayer. On the contrary, use of these substances makes any prayer insincere, since it is not the person one really is who is then praying but an artificial, modified version of oneself.

8. SUNDRY NOTES ON LOGIC

1. Logical thought

It is important to realize that logic is very much a human cognitive practice based on ad hoc conceptual reflection, and intelligent insight and understanding. Logic is not a mechanical process, as it is often made to appear nowadays due to widespread use of simplistic symbolic logic in education and in applications in computer and robot (AI) programming. Logic involves human consciousness and volition. One's consciousness of apparent facts, being currently experienced and/or remembered, and of theoretical context, including logic theory, influences one's volition of logical inferences.

Consciousness and volition are faculties of the soul. Machines, however sophisticated, do not (and I dare say, never will) have souls, consciousness, freewill, or values. Machines function in a deterministic framework, either moved by man-made programs or by chance events. Moreover, logic is not essentially verbal, as the proponents of symbolic logic seem to think. Logical insight occurs at a pre-verbal level, before it is put into words. Words and symbols come into play thereafter, improving the efficiency, breadth, and accuracy, of logical thought.

The science of logic is a later product of logical thought, not its source. It consists of valuable thoughts about logical issues in general (formal) terms, designed to facilitate and solve-in-advance problems that may arise in the course of particular thought processes. But it is not a substitute for the art of logic in actual practice, which depends on the acuity and skill of the practitioner. For this reason, no amount of theory can replace practice. Putative theoreticians of logic can albeit their theoretical baggage be unskilled; while habitual practitioners of logic with little theoretical baggage can still be very skillful.

Most people, male or female, from a young age to an advanced age, are able most of the time to think logically instinctively, without having studied formal logic, just as they can do arithmetic without knowing algebra or mathematics theory. Most of people's reasoning is correct; but not all. Study of formal logic greatly improves one's reasoning powers, and reduces the chances of error. Some logical inferences require a special effort, being very complicated; and in such cases, having a theoretical baggage can be very useful. Learning formal logic allows one to reason more consciously and accurately.

When we say that Man (i.e. any man and woman) has a faculty of reason, or rational faculty, we mean that he has the capability of reasoning, i.e. of engaging in logical thought. It is not a separate mental entity within man, called Reason, that reasons, it is man himself, the soul of man, that does so; he is the cognizing Subject of all logical insight and intellectual activity. Aristotle, the founder of formal logic studies, designated man as a rational animal; he meant it as a compliment. Certainly man, more than any other animal species, has this great natural power, the ability to think logically – but it is up to him to use it and to do so correctly. This requires conscientious study and practice.

Logic, of course, means both inductive and deductive logic. Logic is abstract thought, but it is directed at the totality of our experience: our raw bodily sensations, our sense perceptions, our memories, our imaginations, our bodily feelings and mental emotions, our intuitions of self, our conceptual cognitions, our volitions and the influences affecting them, and our valuations. It

impacts on our physical life, our mental life, our psychology, our emotions, our intellectual framework, our social life, and our spiritual life.

Reason is not the enemy of passions, but is often useful to restrain their excess. It should guide us intelligently, but not rule us dictatorially. We should never forget we are humans – not machines, but living, conscious beings with free will, moved by values and disvalues. Nevertheless, emotions do often warp our thinking, if only by slowing or blocking it. To think clearly, one must often first allow one's mind to calm down and clear out. Meditation is a good tool for that purpose.

The purpose of logic is to help us sort out our experiences and abstractions, and tell truth from falsehood. All potential knowledge, whether intuitive, perceptual, or conceptual, however processed, is subjected by logic to many technical tests before we are convinced by it and believe it to be true. Logic also tells us (roughly in everyday thought, more precisely in more scientific thought) to what degree we may indulge in such belief in each given case at a given time. The critical evaluation of truth or falsehood is always contextual – dependent on the totality of experience, abstract insight, and inference – and it constantly varies in time, ideally evolving towards more and more reliable information.

Logical perspicacity and ability depend on both intelligence and prior knowledge. A person's intelligence determines the breadth, depth, and height, of his or her considerations and proactivity in pursuit of knowledge. A narrow-minded person or a badly informed person will inevitably perceive the logical implications of a situation less accurately than one more free-thinking and better informed. A stupid or ignorant person has little hope of being persuaded by rational arguments; he or she has little intellectual scope, functioning within rigid limits. An open-minded person has mental space to move in, and can more easily adapt to new cognitive input.

2. The first three laws of thought

Aristotle's three laws of thought are well known, but unfortunately not always well understood. They are usually taken in the simplest possible way as logical laws for purposes of deduction; but they are much more broadly and deeply logical laws for purposes of induction.

The **law of identity** is often stated as A is A (where A is a symbol for any given thing). This implies, since the term "not-A" (the negation of A) is a special case of the term "A", that not-A is not-A. These propositions mean that what is A is A, and not not-A; and what is not-A is not-A, and not A. That is, the law of identity naturally leads to the **law of non-contradiction**, whereby nothing is *both* A and not-A. Furthermore, there are only two logical possibilities for any thing: either A or not-A. This is the **law of the excluded middle**, whereby nothing is *neither* A nor not-A. This law tells us that the negation of A (i.e. not-A) is intended as exhaustive of everything other than A. To postulate something besides A and not-A is to claim that not-A is not exhaustive, which is self-contradictory.

Application of these three laws requires discernment and intelligence; they cannot be used simply mechanically. These laws apply to a given thing, at a given place and time, in a given respect and condition. They do not exclude the possibilities that the same thing may be both A and not-A, or both not A and not not-A, in different parts or aspects of it, or in different circumstances; or that what is here and now A may *become* not-A, or that what is here and now not-A may become A; or that, of the various instances of a kind, some may be A and some not-A; and so forth. Additionally, the point of view of the observer needs to be considered; the same object may well look different viewed from different angles. Also, of course, it should be kept in mind that one and the same thing, or kind of thing, may have been given different names in different contexts.

These three laws of thought apply to existing and real things. They do not foretell whether a given thing is A or not-A; they do not exclude the possibility that we may not now, or maybe even ever, *know* whether that thing is A or not-A; ignorance always remains possible. But it cannot logically be said that we cannot ever know whether anything is indeed A or not-A, i.e. that we are *altogether and necessarily* ignorant, for such a statement is itself a claim to knowledge and therefore self-contradictory. These laws do not exclude that non-existent or unreal things may *seem* to someone to be both A and not-A, or neither A nor not-A. Indeed, if in the course of our thinking we come across something that seems to be both A and not-A, or neither A nor not-A, we know for sure from this very event that that thing (as it appears at that time) is non-existent and unreal.

It should be noted that 'A' is often or usually at first undefined, and then gradually clarified and better defined. So, in seeking to apply the laws of thought, we should not refer to a rigid model where a thing labeled 'A' is predefined, with a clear, certain, and permanent definition. Much of our knowing activity consists in getting to know over time just what 'A' means. In some cases, of course, the label 'A' is applied definitively to something, and stays stuck to that denotation. But in many if not most cases, the label 'A' is used more connotatively, as a handle with which focus on something we have not yet clearly succeeded in discerning from its context and therefore cannot yet define. In such situations, the three laws are clearly inductive tools, since they help us to logically delimit such vague terms and ensure they never fall into absurdity, i.e. into self-contradiction or into non-exhaustiveness.

Furthermore, it cannot logically be claimed, even hypothetically, that these three laws might be breached in things of which we are still, or forever, ignorant; e.g. in things too small or too far away or in transcendental things. We cannot make any assumptions about things lying outside or beyond our ken, for that would constitute a claim to knowledge and therefore be self-contradictory. We must in such situations assume that what we have not yet observed or cannot observe follows the rules of things that we can and have already observed. For in the latter case, we are not inventing anything new, whereas in the former case we would be doing so. This reasoning is in fact the foundation of induction. So, we can confidently claim and firmly insist that the three laws of thought are universal.

A veritable feat in the eyes of modern logicians is the discovery of some new (double) paradox. This is not merely a legitimate concern with consistency in knowledge, but an expression of the spirit of the times. The underlying aim is the perverse desire to invalidate human knowledge – to show its inevitable futility and craziness[311].

Infinite regression and **circularity** (*petitio principii*, begging the question) are techniques of criticism also often used in logic and philosophy. This means they are fundamental. Why, then, are they not counted as 'laws of thought'? Probably simply because they are not always conclusive, although they do have some impact. Infinity is not per se impossible; one can, for instance, claim that the universe is eternal without being inconsistent. Similarly, circular argument is not per se logically wrong: every proposition implies itself; the problem with circularity is that it is insufficient for proof, i.e.: more than that is needed to convince us. Thus, criticisms of infinite regression or of circularity do not have the same finality as, say, breaches of the law of noncontradiction or the law of the excluded middle. They recommend us to look for more reasons, but they do not imply that what we have put forward is logically unacceptable.

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311 See *Ruminations,* Chapter 5.7-9; *Logical and Spiritual Reflections,* Chapter 1.8-9.

3. Two additional laws of thought

The first three laws of thought, which were formulated by Aristotle, are that we ***admit facts as they are*** (the law of identity), ***in a consistent manner*** (the law of non-contradiction) ***and without leaving out relevant data pro or con*** (the law of the excluded middle). To complete these axioms of logic, and make them fully effective in practice, we must add two more. The **fourth**, which I have called **the principle of induction**; and a **fifth**, which I call **the principle of deduction**.

These five laws are nothing new, being used in practice by mankind since time immemorial; only our naming them in order to spotlight them and discuss them is a novelty. They qualify as 'laws of thought' because they are self-evident, and necessary to and implied in all rational thought.

The principle of deduction is a law of logic that no information may be claimed as a deductive conclusion which is not already *given*, explicitly or implicitly, verbally or tacitly, in the premise(s). The premises must obviously fully justify the conclusion, if it is to be characterized as deduced. This fundamental rule is true for all forms of deductive (as against inductive) arguments, which helps us avoid fallacious reasoning. It may be viewed as an aspect of the law of identity, since it enjoins us to acknowledge the information we have, as it is, without fanciful additions.

It may also be considered as the fifth law of thought, to underscore the contrast between it and the principle of induction, which is the fourth law of thought. The principle of induction may, in its most general form, be stated as: what in a given context of information appears to be true, may be taken to be effectively true, unless or until new information is found that puts in doubt the initial appearance. In the latter event, the changed context of information may generate a new appearance as to what is true; or it may result in some uncertainty until additional data comes into play.

Deduction must never be confused with induction. Although deduction is one of the tools of induction in a broad sense, it is a much more restrictive tool than others. Deduction refers specifically to inferences with 100% probability; whereas induction in a narrow sense refers to inferences with less than 100% probability.

Inductive reasoning is not subject to the same degree of restriction as deduction. Induction is precisely the effort to extrapolate from given information and predict things not deductively implied in it. In inductive reasoning, the conclusion can indeed contain more information than the premises make available; for instance, when we generalize from some cases to all cases, the conclusion is inductively valid *provided and so long as* no cases are found that belie it. In deductive reasoning, on the other hand, the conclusion must be formally implied by the given premise(s), and no extrapolation from the given data is logically permitted. In induction, the conclusion is tentative, subject to change if additional information is found, *even if* such new data does not contradict the initial premise(s)[312]. In deduction, on the other hand, the conclusion is sure and immutable, so long as no new data contradicts the initial premise(s).

As regards the terms, whereas in induction the conclusion may contain terms, denotations or connotations that are not manifest in the premise(s), in deduction the terms, denotations and connotations in the conclusion must be uniform with those given in the premise(s). If a term used in the conclusion of a deductive argument (such as syllogism or a fortiori) differs *however slightly* in meaning or in scope from its meaning or scope in a premise, the conclusion is deductively invalid. No equivocation or ambiguity is allowed. No creativity or extrapolation is allowed. If the

[312] For example, having generalized from "some X are Y" to "all X are Y" – if it is thereafter discovered that "some X are not Y," the premise "some X are Y" is not contradicted, but the conclusion "all X are Y" is indeed contradicted and must be abandoned.

terms are not exactly identical throughout the argument, it might still have some inductive value, but as regards its deductive value it has none.

Any deductive argument whose conclusion can be formally validated is necessarily in accord with the principle of deduction. In truth, there is no need to refer to the principle of deduction in order to validate the conclusion – the conclusion is validated by formal means, and the principle of deduction is just an ex post facto observation, a statement of something found in common to all valid arguments. Although useful as a philosophical abstraction and as a teaching tool, it is not necessary for validation purposes.

Nevertheless, if a conclusion was found not to be in accord with the principle of deduction, it could of course be forthwith declared invalid. For the principle of deduction is also reasonable by itself: we obviously cannot produce new information by purely rational means; we must needs get that information from somewhere else, either by deduction from some already established premise(s) or by induction from some empirical data or, perhaps, by more mystical means like revelation, prophecy or meditative insight. So obvious is this caveat that we do not really need to express it as a maxim, though there is no harm in doing so.

The principle of deduction is that the putative conclusion of any deductive argument whatsoever must in its entirety follow necessarily from (i.e. be logically implied by) the given premise(s), and therefore cannot contain any claims not supported in the said premise(s). If a putative conclusion contains *additional* information and *yet seems true*, that information must be proved or corroborated from some *other* deductive or inductive source(s). Inference in accord with this principle is truly deductive. Inference not in accord with this principle may still be inductively valid, but is certainly not deductively valid.

In truth, the principle of deduction is a redundancy. That the conclusion cannot go beyond what is given in the premises is obviously true of all deductive argument, without any need to state it as a special principle; it is the very definition of deduction, as against induction or fallacious thought, and so the subtext of any deductive act. Clearly, the principle of deduction is not an artificial, arbitrary or conventional limitation, but a natural, rational one.

4. On the concept of difference

The concepts of sameness and difference are so fundamental to our way of knowledge that they cannot be defined by means of other concepts. It is through apprehension of sameness or difference between things that we can distinguish them from each other, define them, classify them, and thus form concepts. Such apprehension may be concrete or abstract, and is naturally taken for granted. Observing two concrete objects, whether material, mental or spiritual, it suffices that they seem to us ‘alike’ or ‘not alike’ for us to classify them (in the respect concerned) as same or different. Likewise, abstractions can be apprehended as same or different (in some specified respect).

We can define same and different relatively, as each other’s negation. The word ‘same’ can be applied to extremely *identical* things (which are ultimately ‘one and the same’) or to merely *similar* things (which are same in some respects, though not in others). The word ‘different’ is applicable to all *dissimilar* things (no two things are so radically different that they have nothing in common, if only the fact of co-existing in the same world or the fact of being both thought of by us); but we can consider diametrical *opposites* (in the strict sense of contradictories, i.e. any thing and its absence) as the extreme case of difference.

Two things may be (and most likely are) the same in some respects and different in other respects. What do we mean by ‘respects’? We mean, for example, that two things may have the same color

(say, blue), but different shape (e.g. one being round and the other square; in such case, they are the same 'in respect of' color, but different 'in respect of' shape. Thus 'respect' here refers to some aspect of the things concerned that we choose to focus our attention on. Some things are identical in all respects except space and/or time. Thus, if something changes place without any changes within or around it (if such a thing is possible), it is only different in respect of time and space. If it has not moved (if such a thing is possible), it automatically differs in time as time passes. But a single thing cannot differ in place (other than take up some volume of space at once) without the passing of time.

Diametrical opposites (meaning here, strictly, any given thing's presence and absence) are necessarily different, but not all differents are opposites. 'Different' is a wider and thus milder concept than 'opposite'. Differents may be compatible or incompatible (i.e. able or not able to coexist in a given framework or in the world as a whole); and they may be inexhaustive or exhaustive (i.e. able to be both absent at once, or not, in a given framework or in the world as a whole). Opposites (in the limited sense of contradictories, to repeat) are necessarily both incompatible (this is the law of noncontradiction) and exhaustive (this is the law of the excluded middle).

Consider now a predication, say 'A is B' (where B is some term other than A). This tells us that the individual subject A (or 'this A') has the predicate B, i.e. it is a B-having (or B-doing) thing. This does not mean that A-ness is identical with B-ness, but it only means that these terms are compatible and that in the case of (this) A having B-ness is part of its overall identity (at least at the time concerned). Thus, we cannot argue syllogistically and say that 'since A is B and B is not A, it follows that A is not A'! Clearly, this would be committing the fallacy of four terms, since the middle term in the minor premise (A is B) is really 'has B-ness' whereas that in the major premise (B is not A) is 'B-ness' itself.

Thus, it is important, if we want to avoid antinomies, to distinguish between a predicated adjective or verb (e.g. our planet is round or rotating) and the quality or action it refers to (roundness or rotation). Saying 'A is B' is not the same as saying 'A is B-ness' or even 'A-ness is B-ness'. This is not a merely verbal issue. The terms A and B refer to different things, yet there is no contradiction in predicating B of A. Ontologically, this means that the identity of one thing (A) may well, and usually does, include many other things (B, C, D, etc.). Identities are, with few exceptions, complex composites; they are not single and uniform ('monads').

The concept of difference should not be confused with that of contradiction. Differents are not necessarily opposites.

The defining characteristic (say, B) of something (A) is one that is necessary (always found in that thing) and exclusive (never found in anything else; it is what makes that thing *distinctive*). But this does not mean that each thing (A) has only one necessary and exclusive attribute – it may have two or more of them (B, C, D, etc.); these are all also essential to the identity of A. Additionally, that thing (A) may, like all others, have many non-essential (temporary/conditional and non-exclusive) attributes.

Different things may share, in varying degrees, factors of identity, and at the same time be distinct in other ways; for example, the Sun and the Moon are both round (but not roundness), though their sizes differ; and on the other hand, the Sun is hot (not hotness), whereas the Moon is cold (not coldness). The factors of identity may be positive – or negative; for example, neither Sun nor Moon is alive and both lack consciousness. These things are obvious, but worth reminding.

The *negation* of any term is thus a rather complex event. It necessarily implies the negation of all its distinctive characteristics, but does not necessarily imply the negation of all its other attributes.

Something (A) may be considered absent even if some of its factors of identity remain present; the essential ones must be absent, but the non-essential ones need not be so.

5. On Aristotelian syllogism

An example of supercilious disinformation by modern commentators that I found recently was a statement by Prof. Tim Crane of Cambridge University (UK)[313], who wrote, that for Aristotle a syllogism may not contain a specific name but only a general term (p. 15). I quote him:

> "Aristotle thought that any argument where the conclusion follows from the premises can be represented as a syllogism. However, this is now known to be false: a syllogism in Aristotle's sense can contain only general terms – that is, not specific names for individual things, such as 'Aristotle', but only words that apply to many things, like 'white'. Ironically, perhaps the most famous example of 'syllogism' of all time – 'All men are mortal; Socrates is a man; therefore Socrates is mortal' – is not a syllogism in Aristotle's sense at all, since it contains the name 'Socrates'."

This is inaccurate. The historians of logic William and Martha Kneale[314] reject this view and point to an example of such syllogism in Aristotle's *Prior Analytics* 2:27, viz. 'Pittacus is generous, since ambitious men are generous, and Pittacus is ambitious' (1st fig. singular minor term). Actually, I found another example in the same passage: 'Wise men [i.e. at least some of them] are good, since Pittacus is not only good but wise' (3rd fig. singular middle term). I suspect many more examples could be found if we made the effort."

I offered a list of singular syllogisms in my work *Future Logic* (Chapter 9.4). In the first figure, the valid moods are: 'All M are P and this S is M; so, this S is P' and 'No M is P and this S is M; so, this S is not P'. In the second, they are: 'All P are M and this S is not M; so, this S is not P' and 'No P is M and this S is M; so, this S is not P'. In the third, the main ones are: 'This M is P and this M is S; so, some S are P' and 'This M is not P and this M is S; so, some S are not P'.

In truth, as I see it, and I think Aristotle ultimately had this in mind, singular syllogism is the very basis of plural syllogism. The subsumptive and distributive intent of both general and particular propositions is precisely that they signify a collection (full or partial, respectively) of singular propositions.

As Aristotle well showed, all syllogisms can be validated by means of positive syllogism in the first figure, by direct reduction to one (resorting to immediate inferences, mainly obversion and conversion) or by indirect reduction to one (resorting to *ad absurdum*, i.e. showing that denial of the putative conclusion would entail denial of one of its premises). However, what of positive syllogism in the first figure – how is that to be validated? Aristotle for this purpose resorted to *exposition*, i.e. to tracing the inclusion of an individual case in a broader term (a genus) on the basis of its inclusion in a narrower term (a species). This demonstrated that what is true of any one individual is obviously true of any set of them (some or all).

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313 In the volume *Philosophy* (London: Duncan Baird, 2009). I choose as example this professor at Cambridge and other UK universities, not because he has any particular importance, but precisely because he is a run-of-the-mill teacher in today's university.

314 In their work *The Development of Logic*, on p. 67.

Thus, although Aristotle's account of syllogism is largely centered on plural (i.e. general or particular) syllogisms, its ultimate foundation is really the singular syllogism. Without singular syllogism, plural syllogism cannot be validated – indeed, it becomes meaningless.[315]

Another issue relating to syllogism which is commonly used to belittle and mock Aristotelian syllogism is *substitution*. First, let me quote some of what I wrote on this topic in *Future Logic* (Chapter 19.1) without any reference to 'modern logic'.

> "Substitution is a widely used, yet little noticed logical process, which is open to formal treatment of sorts. It consists in replacing a term with another which has the same units, but views them in a somewhat different perspective. The entity referred to remains the same, only its label changes (*qua* what it is referred to); the substitution is thus justifiable. We may substitute a generic term for a species, if we keep the same quantity, or a species for an individual. For instance: 'X has (some number of) Y, All Y are Z, so X has (that many) Z', or 'X has this Y, this Y is Z, so X has a Z'. Example: 'Man has a mind, a mind is an organ, so man has an organ (at least one)'. Note that this is not a normal, first-figure, classificatory syllogism. Here, the major premise must be, or be made, affirmative and classificatory; but the minor premise and conclusion are possessive (in this case). Needless to say, verbs other than 'to have' are open to substitution, too. Example: 'Bill hit Joan: Bill hit a woman'."

All this is simple enough, yet modern logicians boast of their 'discovery' of this thought process thanks to their advanced symbolic methods! Lest anyone doubt Aristotle's prior awareness of the process of substitution, here is one citation from his work (*Topics*, Book 1, Chapter 7) which settles the matter:

> "For all these uses mean to signify numerical unity. That what I have just said is true may be best seen where one form of appellation is substituted for another. For often when we give the order to call one of the people who are sitting down, indicating him by name, we change our description, whenever the person to whom we give the order happens not to understand us; he will, we think, understand better from some accidental feature; so we bid him call to us 'the man who is sitting' or 'who is conversing over there' – clearly supposing ourselves to be indicating the same object by its name and by its accident."

This is of course just one special case (viz. substituting an individual's name or description), but it clearly lays the foundation and shows Aristotle was aware of the thought process. I have no doubt further research would reveal many more references to substitution in Greek and other pre-modern logic.

Attacks on Aristotelian syllogism are rather common in the literature of modern logic. This is usually carried out in a scoffing tone, intended to convey the intellectual superiority of the writer compared to poor old silly Aristotle. I give some examples of this malady in some of my past works[316].

6. On figures of opposition

A figure of opposition is a diagram designed to show the 'opposition' between pairs of propositional forms, i.e. whether they are contradictory, contrary, subcontrary, subalternative, or

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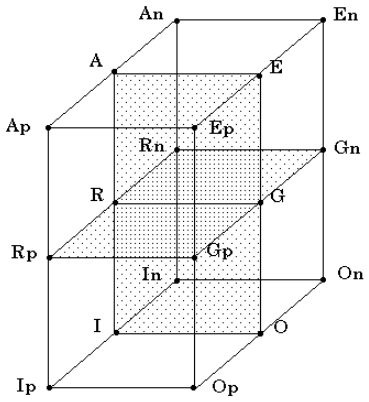
315 Note well that plural syllogism is not collective. Collective syllogism is more akin to singular syllogism.

316 See for instances: *Future Logic,* Chapters 9.4, 19.1; *Phenomenology,* Chapters 7.2, 7.4-5; *Ruminations,* Chapter 4.5.

otherwise related. The archetype of such diagrams is the ancient 'square of opposition', which relates Aristotle's four actual categorical propositions (the plural forms, traditionally labeled **A, I, E,** and **O**).

There are several 'figures of opposition' in my book *Future Logic* (FL), some of which involve novel features. In Chapter 6, I introduce a rectangle of opposition, expanding the traditional square of opposition in order to relate the two singular forms (which I label **R** and **G**) to the said four plural forms. In Chapter 14, I make use of a three-dimensional figure of opposition in order to similarly relate the eighteen forms of categorical propositions, including the said six actual forms (no suffix), plus six corresponding necessary forms (suffix **n**) and six corresponding possible forms (suffix **p**). I reproduce that diagram here (the oppositions involved are listed in a table given there[317]):



In order to broaden the study of categorical propositions to include modal ones (**n** and **p**) as well as actual ones, I had to introduce two new concepts of opposition, namely 'implicance' and 'unconnectedness'; the former referring to propositions that mutually imply each other, and the latter to propositions neutral to each other, i.e. which are neither contradictory, nor contrary, nor subcontrary, nor subalternative (one implying but not implied by the other), nor implicant. The resulting list of six concepts of opposition was exhaustive: any pair of propositions had to be related in one (and of course, only one) of these ways.

As it happens, in the set of 18 categorical forms there considered, no two different forms were implicants, although of course implicance was still exemplified by the fact that (in the cross-tabulation shown there) every form implied and was implied by itself. But there were numerous pairs of forms which were 'unconnected' to each other, as above defined; for examples: A and Rn, A and In, A and Ep, A and Gp. The same concepts of opposition can, of course, be used in any other context, wherever they are deemed useful[318].

This was not fanciful innovation for the sake of innovation on my part; it was a necessary act, in order to have the conceptual means to express all possible oppositional relations. When dealing only with the six actual categoricals, the traditional four oppositional relations sufficed; but when the menagerie of categorical propositions was expanded to include modal forms, the said two additional concepts of opposition became necessary.

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317 See Table 14.1.

318 I use them again in my study of *The Logic of Causation*.

I say this because, to my utter surprise and amusement, some people[319] later wondered why I had done that! Thus, a certain Jean-Yves Béziau[320], followed by a certain Alessio Moretti[321]. In FL, I wrote:

> By the 'opposition' of two propositions, is meant: the exact logical relation existing between them — whether the truth or falsehood of either affects, or not, the truth or falsehood of the other. In this context, note, the expression 'opposition' is a technical term not necessarily connoting conflict. We commonly say of two statements that they are 'opposite', in the sense of incompatible. But here, the meaning is wider; it refers to any mental confrontation, any logical face-off, between distinguishable propositions. In this sense, even forms which imply each other may be viewed as 'opposed' by virtue of their contradistinction, though to a much lesser degree than contradictories. Thus, the various relations of opposition make up a continuum.

And I defined unconnectedness as

> Unconnectedness (or neutrality): two propositions are 'opposed' in this way, if neither formally implies the other, and they are not incompatible, and they are not exhaustive. Note that this definition does not exclude that unconnecteds may, under certain conditions, become connected (or remain unconnected under all conditions).

Béziau objected to this, saying:

> "In fact, according to this definition, two atomic propositions are unconnected, and must be considered as opposites, like "Snow is white" and "The sky is blue", or "John likes cheese" and "John likes wine". Obviously we are going too far and confusing here negation with distinction, maybe coming back to Plato's theory in the Sophist where negation is identified with otherness. The standard definition of *opposite* runs as follows: A person or thing that is *as different as possible* from someone or something else: The colors 'black' and 'white' are opposites" (Longman dictionary of contemporary English, Italics are [Béziau's])."

This was clearly a ridiculous objection his part. The word 'opposition' has traditionally been used in a wide sense, to refer not only to conflictual relations (incompatibility, contradiction, contrariety) but equally to non-conflictual relations (compatibility, subcontrariety, subalternation). I only expanded it further adding two more of the latter kind (implicance, unconnectedness). There was absolutely no confusion between negation and difference on my part; quite the contrary, since implicants and unconnecteds, as indeed subcontraries and subalternatives, though different do not negate each other.

Actually, Béziau was not criticizing only my use of the term 'opposition' but attacking the inclusion even of the traditional concept of subalternation under this term[322]. Apparently, he could not grasp a non-literal reading of the word 'opposition' in the context of logic theory! It is perfectly legitimate to expand the connotation of a word contextually as needed (rather than invent an entirely new one). A term can even mean one thing and its opposite (e.g. the term 'to sanction')[323].

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319 I mention these people anonymously in a footnote in *The Logic of Causation*, Chapter 19.

320 In a paper called "New Light on The Square of Oppositions and Its Nameless Corner" (2008, I think), which can be read here: http://wwwa.unine.ch/unilog/jyb/sep.pdf. Pp. 8-9.

321 In "The Geometry of Logical Opposition" and "The Geometry of Oppositions and The Opposition of Logic To It" (both 2009)". Pp. 175 and 15, respectively. The first one can be read here: https://www.academia.edu/11372480/The_Geometry_of_Logical_Opposition.

322 Apparently, the compatibility implied by subcontrariety did not annoy him.

323 This is called enantiosemy.

Moretti seconded Béziau, imagining my contribution as "an attempt to re-think Aristotle's theory." Aristotle never explicitly states that two propositions can *only* be related in the way of contradiction, contrariety, subcontrariety, or subalternation; he does not formulate a 'theory' limiting opposition to these four categories, but merely pragmatically uses these categories. To use two additional categories, when they become useful, is not to 're-think his theory'.

In any case, given any two propositions P and Q, there is logically bound to be a number of possible relations between them. It is not an arbitrary matter to predict and list all these possible relations. It does not matter what we call them: they are objectively there and, because useful, needing to be named. There is no escape from taking into consideration *all* the following permutations; they are facts of logic! The permutations are shown below:

P and Q are contradictory = if P, then notQ + if notP, then Q;

P and Q are contrary = if P, then notQ + if notP, not-then Q;

P and Q are subcontrary = if P, not-then notQ + if notP, then Q;

P and Q are implicant = if P, then Q + if notP, then notQ;

P subalternates Q = if P, then Q + if notP, not-then notQ;

P is subalternated by Q = if notP, then notQ + if P, not-then Q;

P and Q are unconnected = if P, not-then Q + if notP, not-then notQ.

Anyway, glancing at these essays taught me that there are many people fascinated by and writing about 'figures of opposition'[324]. For me, figures of opposition are useful tools; they help us to order relations between propositions in our minds in a practical way. They transmit information in pictorial format, but they have no importance in themselves. They are just vehicles – it is the information they encapsulate which matters. They are not a big deal in my eyes, nothing to write home about.

7. On the inductive role of deduction

One of the most damaging attacks on Aristotelian syllogism has been the claim that it teaches nothing, being merely circular argument, in which the premises cannot be known true without the conclusion being first known true. If so, then syllogistic argument is a parody of reasoning. I refuted this fallacious claim in my book *Phenomenology*, under the heading *Syllogism Adds to Knowledge*[325]. That account of the inductive role of syllogism cannot be fully grasped and accepted by people who deny validity to generalization, as I explain in the same work under the heading *Generalization is Justifiable*[326].

What such reflections teach us is the inductive role of deductive argument. Of course, deduction generally plays an important role in induction, since it is used to infer possible implications of our empirical findings and to make testable predictions from our theoretical constructs. But the point made here is that induction is often formally inherent in specific deductive processes.

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[324] I contacted Béziau, and he informed me that he had organized a congress on this subject in 2007 (http://www.square-of-opposition.org/), after which a special issue of the journal Logica Universalis was published (https://link.springer.com/journal/11787/volumes-and-issues/2-1?page=1).

[325] Chapter 7.4.

[326] Chapter 7.2.

The inductive utility of deduction can also be illustrated with reference to dilemmatic argument. I have in the past, in my work *Future Logic*[327], made a distinction between normal hypotheticals and abnormal hypotheticals. In the normal case, 'if P, then Q' has the tacit conjunct 'if notP, not-then Q'; whereas in the abnormal case, 'if P, then Q' has the tacit conjunct 'if notP, then Q', and so amounts to saying 'Q' (or not-notQ) categorically. The default inductive assumption concerning a hypothetical proposition is that it is normal. Only when it is proved abnormal do we so look upon it.

This can be seen with reference to simple constructive dilemma:

If P1 then Q and If P2 then Q
but P1 and/or P2
therefore Q

When we discover the first horn (if P1, then Q) we think it is normal; ditto for the second horn (if P2, then Q); then we find that 'P1 and/or P2' (the minor premise) and this pushes us to the conclusion that 'Q'. When we thus discover that 'Q' is true independently of P1 or P2, we must admit that the initial horns were, in fact, abnormal (and not as initially assumed, normal). Thus, we eventually *change* the status of the given hypotheticals (the major premise) in our minds. This is the discovery process involved in such dilemmatic argument. We do not usually know in advance that the conclusion 'Q' is necessary independently of P1 or P2 –rather, we discover this through the coming together of the premises in our minds.

The same can be said regarding paradoxical argument. In that special case, P1 and P2 are respectively a proposition P and its negation notP. The major premise is then 'Whether P or notP, then Q; the minor is 'either P or notP'; and the conclusion is 'Q'.

We see in the case of such simple dilemmatic arguments the marked difference between deductive and inductive logic. From a deductive logic point of view, the simple dilemma is a static argument with the horns of the major premise as tacitly abnormal hypotheticals and the minor premise a normal disjunction, the conclusion being necessary. But this traditional picture of the simple dilemma is not the true representation. It is more precisely an inductive argument, starting with apparently normal horns, and ending with the discovery that the horns are in fact abnormal. The major premise confronted with the minor premise produces a change in our view of the major premise (from normal to abnormal). Note this well: deductive logic involves static relations; inductive logic involves time, process, and change.

Moreover, deductively, the premises yield a necessary conclusion. But inductively, of course, the conclusion is no more probable than the net probability of the premises that gave rise to it (assuming no other sources of confirmation for it exist). If the premises are certain, so is the conclusion indeed. But if the premises are found somewhat open to doubt, then the conclusion is also not as necessary as it first seemed. Thus, the issue is in flux, according to one's context of knowledge. This demonstrates the far superior realism of the inductive understanding of reasoning.

8. On strict or material implication

The following comments are intended put some novel order in the concept of conditioning, and thus to highlight certain failures of understanding displayed by many logicians since the late 19th Century.

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[327] Chapter 31.3.

Of course, implication in various guises was used in human discourse long before it was discussed by philosophers. But so far as we know, according to logic historians, the notion of implication was first elucidated by Diodorus (Cronus of Megara, d. circa 307 BCE), who defined it as a sequence of events (or concepts or propositions) such that the first (the antecedent) is always followed by the second (the consequent). Soon after, his disciple Philo (known as the Megarian, though his origin is not sure) advocated a simpler definition of implication, which eschewed the specification 'always'. For Diodorus, implication (the hypothetical form 'If P, then Q') was a *modal* relation, meaning that the conjunction 'P and notQ' *never* occurs; whereas for Philo, it sufficed to simply deny that conjunction (effectively, *at a given point in time*), saying 'not-(P and notQ)'. As closer scrutiny makes clear, Philo was a bad student who should have listened more carefully to his teacher, Diodorus.

Nevertheless, modern logicians (at least those mathematically inclined) considered implication as referring to Philo's form of implication (Philonian implication, later called material implication). This was the situation at least until Clarence I. Lewis revived Diodorus' form of implication (now called strict implication), in 1918 and more forcefully in 1932. Even after this, many logicians have continued to formulate logic theories or teachings with reference to implication as mere *negation* of conjunction instead of as *impossibility* of conjunction (of P and notQ, given 'If P, then Q').

They do not yet realize that without appeal to strict implication, we would be hard put to express the difference between disproof ('proving the contradictory') and non-sequitur ('showing the conclusion does not follow from the premises'). Such refinement of discourse is impossible using material implication.

If we compare the 'truth-tables' for strict and material implication, they would seem superficially the same as regards the positive aspect 'if P, then Q'. The truth of P implies that of Q and the falsehood of Q implies that of P. But when we look at the negative aspect, their difference becomes glaring. In strict implication, the negation of 'if P, then Q' is the negative hypothetical 'if P, *not-then* Q' (meaning 'P is not necessarily followed by Q', not to be confused with 'if P, then notQ'), which (by itself) leaves the respective truths and falsehoods of P and Q open and does not allow us to infer from P the truth or falsehood of Q or from Q the truth or falsehood of P. Whereas in material implication, the negation of 'if P, then Q' is simply 'P and notQ', i.e. it tells us *categorically* that P is true and Q is false!

Moreover, we should ask the question: how can we possibly come to know a material proposition of the *negative* form 'not-(P and notQ)'? Either we know it directly through some logical insight – in which case it really means 'the conjunction (P and notQ) is impossible', i.e. it is merely a derivative of the strict implication 'if P then Q'[328]. Or we know it because we have *already* established one of its material *positive* alternatives, i.e. one of the conjunctions 'P and Q' or 'notP and Q' or 'notP and notQ'. Thus, there is no way in fact for us get to know the negative conjunctive proposition 'not-(P and notQ)' *directly*; we have no independent access to such information.

However, this is only half the story. Already in antiquity to some extent (if only implicitly), and especially since the Scholastics (who seem to have coined the terms), a distinction was made between 'de dicto' and 'de re' modalities. The former related to the states of our knowledge (including speculations and hypotheses) about things, whereas the latter concerned the things

[328] A strict hypothetical proposition 'if P, then Q' may of course be unconditional or conditional, In the latter case, it is effectively nested in another strict hypothetical proposition, say 'if R, then (if P, then Q)'. But the underlying condition R must ultimately be specified, it cannot be vaguely referred to 'the present conditions' – otherwise, the 'if P, then Q' clause cannot really be claimed to be known as a fact.

themselves. That is, more specifically in the present context, the former concerned the logical mode of modality (epistemology), while the latter was about natural, temporal, extensional, spatial, or similar ontological modes (notably that related to volition, the personal mode).

Many modern logicians after Lewis, whether out of ignorance or naivety, or a stubborn desire to simplify complex issues, have tended to conflate de dicto conditioning with strict implication and de re conditioning with material implication. That is evident in the terminology used – strict means formal, in accordance with logical discourse (i.e. de dicto), and material means contentual, relating to things (i.e. de re); and it is evident in the kind of examples they give to illustrate material implication (e.g. 'if it rains, the match will be called off'). But of course, this is quite wrong. Both strict and material implication are (stronger and weaker) forms of logical conditioning. Material implication cannot adequately do the job of sundry 'de re' forms of conditioning[329].

The poverty of modern assumptions in this respect is evident as soon as we try using them to interpret or explain commonly used expressions like 'when and if', for example. If 'when' and 'if' both belonged to the logical mode of modality, their conjunction would be incomprehensible. Whereas, when we understand the difference between modes of conditioning and between connection and basis of conditioning, such conjunction becomes clear. The 'when' tells us that a natural or temporal modality of connection (a de re connection) between the theses applies, while the 'if' tells us that the base of the antecedent is uncertain, i.e. is merely a logical possibility (and not a de re one). The formal possibilities of material implication cannot be compared to the precision and complexity of discourse made possible by 'de re' conditioning. That our practice is more in accord with the latter is easily demonstrated by examples.

Much of this confusion has remained hidden from public view, and to the logicians themselves, due to excessive reliance on symbolic logic.[330]

9. About collectional propositions

Are 'collectional' propositions with *logical* modality conceivable? Such propositions would have the form: "It is logically possible, necessary, impossible, not-necessary that *all/some* S are P (or not-P) *simultaneously* at a given point or in a given period of time". Put in this way, the answer to the question seems obviously: yes. Ordinarily, we intend *natural* modality when we use such 'collectional propositions' – but since logical modality is continuous with natural modality (i.e. logical necessity implies natural necessity and natural possibility implies logical possibility), we must extend the collectional form to include logical modalities. But it remains essentially *qua* natural modality that the form functions.

This becomes evident when we ask the question: what if instead of the bracketed actual proposition 'all/some S at once are P (or not-P)' we tried using a proposition with another natural modality, i.e. potentiality or natural necessity, viz. 'all/some S at once can-be/must-be P (or not-P)'? Would such propositions be meaningful collectional forms? Clearly not, because such modal propositions (involving can or must) are *not time bound* like their actual equivalent.

An exception to this rule might, however, be proposed, when taking into consideration the phenomena of 'acquisition or loss of powers' which imply change from a 'cannot' to a 'can' or

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329 This is made evident in my *Future Logic*, part IV, on de re conditioning.

330 For more details on this topic, see my *Future Logic*, part III, on logical conditioning. Note in particular Chapter 24.3, regarding the strict-material implication distinction. See also: *Ruminations*, Chapter 4.8, on the strict-material implication distinction; and *Volition and Allied Causal Concepts*, Chapter 6.5, regarding 'personal conditioning'.

vice-versa. In the latter cases one might well ask whether all or some of the individual subjects subsumed can, must, cannot, can-not acquire or lose the powers concerned at once in the same time-frame. In this context, too, one might admit logical modality in place of the usual natural modality with regard to the collectional aspect, even though the bracketed propositions involve non-actual natural modalities.[331]

10. On the logic of change

In my 1990 work *Future Logic* (FL), Chapter 17, I deal with some aspects of the logic of change[332]. Reviewing the work done there on this subject, I see no errors in my treatment. The system presented there is consistent; but I now realize that an equally consistent alternative system could and perhaps should have been proposed.

Briefly put, I there distinguish (under Aristotelian influence) between two kinds of change: *alteration* (getting to be) and *mutation* (becoming), the former relatively superficial and the latter more radical; this is in contrast to *attribution* (being), which is static. On this basis, I distinguish three forms of categorical proposition: the attributive (e.g. This egg is soft, or is hard), where neither subject nor predicate is changing; the alterative (e.g. This egg has gotten to be hard, or hardened, where the subject remains 'this egg' but the predicate changes from 'soft' to 'hard'), and the mutative (e.g. This soft egg has become a hard egg, where the initial subject 'this soft egg' is replaced by another subject 'this hard egg').

A fourth form is then needed which also reports change without specifying whether alteration or mutation is involved. This may be called *transition* ('ending up as', or 'changing to'). A transitive proposition is thus vaguer (e.g. 'this egg will end up hard'); it is effectively a disjunction of an alteration and a mutation, i.e. it states what the two have in common. Thus, 'ends up as' (or 'changes to') is equivalent to saying 'either gets to be or becomes' (the alternatives of this disjunction being merely claimed as conceivable in the context of knowledge available, without claiming them as both in fact capable of occurring). The word transitive can also be applied, by extension, to alterative and mutative propositions, since they are species of the generic form of change.[333]

,In order to analyze these forms accurately, it is wise to initially refer to *singular* propositions, those involving the indicative 'this'. Once we understand the issues in relation to the singular forms, it is easy to develop the same logical principles for the plural forms, i.e. those with the quantifier 'all' or 'some'. Thus, in the proposition 'this S ends up as P', for example, the precise, underlying subject of our discussion is something (whether real or imaginary) that is physically or mentally pointed at, signified by the indicative 'this'. Its initial state is the subject S – 'this S' signifies a predication, it means 'this thing which is now S'; and its final state is the predicate P – 'this P' also signifies a predication, it means 'this thing which is at some (later) time P'.

Now, in FL, I define the three said forms for change in a very broad manner, specifying only part of the start and finish of the processes referred to. Thus, the form 'This S gets to be P' only requires that the starting state for the underlying subject (this thing) to be S, without committing as to whether it is also P or notP; but the ending state for this same thing must be 'both S and P'. On the

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331 On this last topic see my *Future Logic*, Chapter 34.3.

332 I do not there deal with *evolutionary* change, note well; this is a quite distinct form of change, applicable to species rather than individuals, which I deal with in my *Volition and Allied Causal Concepts*, Chapter 14.1.

333 Needless to say, in everyday discourse we sometimes say 'becomes' when we mean 'gets to be', or 'gets to be' when we mean 'becomes', and so forth. The meanings are here fixed for logical treatment.

other hand, the form ‘This S becomes P’ requires the starting state to be S and notP, and the ending state to be notS and P. Consequently, the form ‘This S ends up as P’ requires that this thing begin as S (whether also P or notP) and end as P (whether also S or notS). Now this works out well, as already said, in that a consistent system can be built on this basis to express change.

Where a state is not specified by the form, it can be readily be specified by adding an appropriate attributive proposition. Thus, we can add the attributive statement ‘This S is now notP’ to ‘This S gets to be P’ if we wish to emphasize that a change (from SnotP to SP) has occurred. If we instead add the statement ‘This S is now P’, we apparently have no change occurring (since we go from SP to SP). However, neither of these combinations excludes that changes occur *in the interim*. That is, the proposition ‘This S gets to be P’ allows for all sorts of changes between the starting and ending states specified, even though they remain invisible. If we want to document them, we of course can do so using additional attributive or transitive propositions.

The same holds true for the other two forms, viz. ‘This S becomes P’ and ‘This S ends up as P’: we can specify any unspecified starting and ending and interim states through additional propositions. The four forms can in this manner cover all possible situations.

The only objection that might be raised – and this is the point of my present comments – is that it is possible for the alterative form ‘This S gets to be P’, and likewise consequently the vaguer transitive form ‘This S ends up as P’, to be true, when in fact no change at all has occurred. If the starting state is SP and the final state is SP, and nothing has happened in the interim, this thing has (in respect of S and P) remained static – yet we are using a proposition intended for change!

This need not bother us so long as we are conscious of it. But if we are bothered by it, and wish to be more literal, we would have to redefine the form ‘This S gets to be P’ as implying a change from S-notP to S-P (which still does not preclude interim changes, of course). This would then contrast to the form ‘This S becomes P’ implying a change from S-notP to notS-P (again without precluding interim changes). Consequently, the generic form ‘This S ends up as P’ would also imply a start at S-notP, but for the finish only specify P, leaving the issue of S or notS open.

In this way, an alternative system might be proposed for the logic of change, with somewhat different rules for opposition, eduction and syllogism. Such a system, though more literal, would be more complicated. I have no wish to develop this alternative system here, but only to point out its possibility.

11. On a fortiori reasoning

In my 1995 book *Judaic Logic*, in Chapter 3, I present an original and detailed list and validation of eight a fortiori arguments. This includes four ‘copulative’ moods, of which two ‘subjectal’ and two ‘predicatal’, and four ‘implicational’ moods, of which two ‘antecedental’ and two ‘consequental’, each of the said pairs comprising a positive and a negative mood. Each mood is clearly spelled out, with its four terms or theses (the major P, the minor Q, the middle R, and the subsidiary S). For examples (two copulative forms):

> The positive subjectal mood: P is more R than Q (is R), and, Q is R enough to be S; therefore, all the more, P is R enough to be S.
>
> The positive predicatal mood: More R is required to be P than to be Q, and, S is R enough to be P; therefore, all the more, S is R enough to be Q.

After painstaking logical and linguistic analysis of the eight argument forms, I *validate* them by reducing them to established argument forms. I interpret the premises of positive moods by means of hypothetical, comparative, and inclusive, propositions, which I then process syllogistically to

obtain the putative conclusion. For negative moods, I resort to reductio ad absurdum to corresponding positive moods. Note that I do not suggest that the a fortiori arguments themselves are syllogism, but only that syllogistic reasoning is partly involved in their validation (other kinds of reasoning being also partly involved).

These validation procedures demonstrate without a shadow of doubt the *deductive* efficacy of the eight a fortiori argument forms I presented. Such argument is clearly not merely inductive, but able to yield conclusions of 100% certainty given the stated premises. As with all deductive inference, one can of course object to a material conclusion by raising doubt regarding one or the other of the material premises, or all of them, since all material propositions are based on induction; but given reliable material premises, the material conclusion from them through a fortiori argument is formally indisputable. To repeat, my *Judaic Logic* work certifies indisputably that a fortiori argument is deductive reasoning.

Yet one Stefan Goltzberg, in his doctoral thesis dating from 2010-11[334], had the gall to suggest otherwise. I discovered this document only recently (in 2023) and was appalled by the dishonesty of its author. Quoting me as saying: "To be precise, I did not prove the various irregular a-fortiori to be invalid, but rather did not find any proof that they are valid," he writes (my translation from the French original):

> "Avi Sion['s]... conclusion is not that the *qal vahomer* is invalid, but he claims to have been unable to demonstrate the validity of all forms of this reasoning."

This is a misrepresentation of what I said, deliberately ignoring the context of my statement, in order, presumably, to buttress his own doctrinal agenda! Why write that my conclusion "is not that the *qal vahomer* is invalid," when my conclusion is unequivocally that a fortiori argument is valid? The statement he cites, "I did not prove... etc." is in Chapter 3.3 of my book; it explicitly concerns a very limited set of irregular arguments *that might be thought to be a fortiori but in fact are not*. It certainly does not express my views on regular a fortiori argument, as Goltzberg insinuates. The context of my statement was as follows:

> To sum up the research: implicational a-fortiori, whether antecedental or consequental, involving the negative relationships, were found invalid, using the above mentioned and other methods… To be precise, I did not prove the various irregular a-fortiori arguments to be invalid, but rather did not find any proof that they are valid… These findings allow us to conclude that, although the analogy between regular copulative and implicational arguments is very close, there are irregular cases where their properties diverge, and copulatives are found valid while analogous implicationals are found invalid. They are significant findings, in that: they technically justify our initial separation of copulative and implicational a-fortiori into two distinct classes; they confirm, surprisingly, that our initial list of valid moods is pretty exhaustive (discounting obvious derivatives and variant subsets); and they confirm the general lesson of the science of logic that processes which prima facie might seem feasible, often turn out, upon closer inspection, to be illegitimate.

Clearly, I was referring *specifically* to "implicational a fortiori arguments involving negative relationships" (which are listed and examined earlier in the same chapter) and there is no suggestion that their invalidity throws doubt on the validity of a fortiori argument *in general*. The arguments referred to are in fact very rarely, if ever, attempted by people (most are not that stupid);

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[334] In "Théorie bidimensionnelle de l'argumentation: définition, présomption, argument *a fortiori*," pp. 244-5. Doctoral thesis presented to the Université Libre de Bruxelles, Faculté de Philosophie et Lettres, in the academic years 2010-11.

I only consider them in order to anticipate their possible use and ensure the exhaustiveness of my formal treatment. They are a side issue in my thesis, a mere footnote. The main issue in it are the genuine, regular forms which I thoroughly analyze and rigorously validate – and these, Goltzberg does not mention, let alone list and examine. It is absurd for a writer to focus on the side issue and ignore the main issue. It is bound to mislead readers, causing them to overlook the more significant and important argument forms dealt with.

Moreover, although I found no direct way to prove the specified argument forms invalid, I judged them invalid by indirect inference from the fact that I tried to but could not prove their validity. This was perfectly legitimate reasoning, based on the inductive principle that if the premises cannot be demonstrated to imply some putative conclusion, it must be regarded as a *non sequitur*. An argument is 'proved' logically invalid if the putative conclusion contradicts the premises in some way; if no implication and no contradiction is found, the argument is not 'proved' invalid, but it can still be (and indeed must be) declared logically invalid.

It is as if someone said that Aristotle's "conclusion is not that the syllogism is invalid, but he claims to have been unable to demonstrate the validity of all forms of this reasoning," because he found that some conceivable attempts at syllogistic reasoning are incapable of validation! The fact that, for example, the faulty syllogism 'All M are P and no S is M, then no S is P' is incapable of validation and therefore invalid obviously does not throw doubt on the validity of a long list of other, perfectly reliable syllogisms, including for instance the mood 'All M are P and all S are M, then all S are P'. On the contrary, *there would be no need for validation efforts if there were no invalid forms contrasting with valid forms*.

Although Goltzberg's statement, if read strictly, just means: "there are some forms of such reasoning that cannot be validated," the oblique way he has phrased it ("unable... all"), and the fact that he focuses attention solely on irregular and not on regular forms, effectively suggests that: "no form of such reasoning can be validated." I find it hard to believe this ambiguity was accidental; I am inclined to believe it was intentional, its purpose being apparently to belittle my contribution to the study of a fortiori argument. Note that he devotes less than one page to my work, even though it constitutes the first ever in-depth and thorough logical analysis, and definitive certification, of a fortiori argument. Apparently, he feared that acknowledging this intellectual feat would eclipse his superficial narrative.

Goltzberg's long (293p) dissertation is, nevertheless, an interesting one; it is not my purpose here to belittle it. If I had had access to it when I wrote my 2013 book *A Fortiori Logic*, I would have reviewed it there in detail, as I did many other texts. Critically, to be sure; but with conscientious honesty and fairness.

In fact, I have already reproved Goltzberg at length for his inaccuracy, not to say dishonesty, in my *A Fortiori Logic* (chapter 26). However, I had not at that time read his doctoral thesis. My critique there was based on other, earlier essays of his, notably "The A Fortiori Argument In The Talmud" (14p.), published in 2010[335]. In that essay, he said almost exactly what I reproach him for here. I wrote to him at the time (Aug. 2010) and told him my concerns. He refused to admit them (see our conversation in my *A Fortiori Logic*).

And evidently, I now realize after seeing the dissertation he submitted soon after[336] for his doctorate, he did not take my criticism to heart, and inserted almost verbatim the same erroneous

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[335] In: *Judaic Logic*, ed. Andrew Schumann. Piscataway, N.J.: Gorgias, 2010. This can be read online at: stefangoltzberg.files.wordpress.com/2010/11/stefan-goltzberg-2010-the-a-fortiori-argument-in-the-talmud.pdf.

[336] I assume not before.

statements in his dissertation. Moreover, he did not even have the decency to send me a copy of what he had written concerning my work in his dissertation! This was additional moral failure on his part, in my estimate.

12. About modern logic

Modern logicians have, since George Boole in the mid-19th Century, made a relentless effort to emulate the symbolic capabilities of mathematics and its resort to axiomatization. But in my view, that was a wrong direction to take – not out of some attachment to a traditional approach to logic, but because ordinary language logic is by far superior to symbolic logic.[337]

Human beings think first in ordinary language before they convert these thoughts to symbolic language. Conversely, when we read a symbolic statement, we must in our head convert it to ordinary language for it to mean something to us: we do not understand it directly. Symbols have meaning for us only insofar as we have assigned them meaning through ordinary language. Therefore, they do not add any meaning – if anything, they tend to obscure the original meaning and to oversimplify it. They are sometimes useful, but sometimes they are serious obstacles. In particular, the use of symbols alienates lay people from logic theory, whereas one of the main tasks of logicians is to improve the logic capacities of non-logicians.

Symbols are only useful to abbreviate long ordinary language formulas or processes, or to repeat and mechanize what has already been painstakingly thought beforehand in ordinary language. But they cannot help us discover much of anything significant; they are not shortcuts to new knowledge. Real discoveries in the science of logic depend on ordinary language – on awareness and capture of actual human thought processes and their meanings. I would never have managed to formalize the logic of causation or a fortiori argument or analogical argument if I had been mentally imprisoned in symbolic logic. Such radical discoveries and innovations are inconceivable using artificial symbols.

Moreover, it is very misleading to try and make of logical science a fixed 'deductive system', which starts with a number of 'axioms' and mechanically infers and justifies all the processes of logic from them. Such an approach may function in mathematics, but it is inapplicable to logical science. Why? Because unlike mathematics (which deals in numbers), logical science (which deals with much more complex concepts) is not an a priori given, but is to a large extent an empirical discipline, insofar as it is based on careful observation of actual human discourse (which, of course, takes place in ordinary language). It is only after such observation that the logician can engage in formalization and then (if the discourse is found valid) in validation.

Logic is an ongoing enterprise, a product of creative insights as well as deductive reasoning. It is essentially synthetic rather than analytic. There is, in fact, no such thing as a purely analytic science, arising without any use whatever of our senses. Even mathematics is not purely analytic, much less logical science. If we had no experiential contact whatever with the world, with any physical or mental world other than a mere point, we could not construct a mathematics or a logical science.

When in logic or mathematics we speak in formal terms (using symbols like x or y for numbers or for other terms or for theses) or in material terms (with specific words, like '1' or '2' or '=', or like 'dogs' or 'is' or 'if-then'), we rely on experience. We rely, for a start, on the experience of symbols or words in our minds, or on paper, or on a computer screen. More broadly, we call upon our

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337 See my comments on this topic in *Future Logic*, Chapter 64; and in later works.

concrete experience of sense-perceptions, memories, and imaginations, and to the more subtle, abstract 'experiences' of our rational insights. Without the mass of introspective and outward-looking experience we bring to bear, we would be unable to develop any mathematics or science of logic.

Logicians and mathematicians (such as the 'logical positivists') who think that their discourse is 100% 'a priori' (independent of experience) are idiots, because they fail to observe and take into consideration their own intellectual functioning. They blithely ignore the concrete and abstract experiences which make their discourse at all possible.

The ideas of symbolization and axiomatization are pillars of the modern approach to logic theory. So much so that modern logicians pretentiously qualify classical logic discourse as 'informal' and only its own productions as 'formal'. In truth, what makes logical science 'formal' is the use of symbols *for terms or theses*, not the use of symbols *for the relations between* terms or between theses. Symbolizing the relations is the only innovation of modern logic; however, this innovation was not only unnecessary, but a hindrance to progress. From its beginnings, modern logic tried to make this discipline more scientific in appearance by resorting to relational symbols and by looking for a development from basic building blocks. That these measures might cause confusion and skew our understanding of the nature and function of logic was never considered. I have written many essays on this topic[338].

Modern logic has developed and evolved as it has essentially due to the philosophical naivety (if not perversity) of many of its proponents. At the basis of every formal error made we can spot epistemological and/or ontological errors, regarding concepts and their formation. I draw attention to some of the errors that should have been avoided in some of my past works[339].

The modern revision of logic theory has cut very deep, down to the level of 'metalogic'. This is of course perfectly desirable and legitimate, provided it is done in a convincing manner. But the viewpoints of modern logicians in this field are not very convincing[340].

The pretensions of modern logicians are numerous. But modern logic is not a monolith. It is the work of many people who do not necessarily agree with each other on all topics. For this reason, it can only be analyzed and refuted bit by bit[341].

It would take me a few lifetimes, or a large research team many years of work, to critique every book and essay involving 'modern logic' claims in detail. Unfortunately, so much has been written with such claims over the past century and a half that it is virtually impossible to walk back all of them. Generally, based on experience, I calculate that I must write an essay twice as long as any given essay to fully criticize it, point by point. So, all one can hope for is that new generations of logic enthusiasts will emerge, and the fake logic of so-called modern logic will in time be left by the wayside as so much make-believe.

13. Our 'brave new world'

Symbolic logic has, of course, been found useful and widely used in computer programming. This is due to its virtue of brevity compared to the wordiness of ordinary language logic. However, such

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338 The following are my earliest general comments: *Future Logic*, Chapter 64.1-2; *Phenomenology*, Chapter 7.8; *Ruminations*, Chapter 4.4.

339 See *Future Logic,* Chapter 4.3; *Phenomenology,* Chapters 3.4, 4.3, 7.6-7; *Ruminations,* Chapters 3.1, 4.2-3.

340 See *Future Logic,* Chapter 66.1-2.

341 See *Ruminations,* Chapters 2.1, 4.1, 6.1-10.

utility should not be overstated. Symbolic logic cannot be used to discover new aspects of logic – it is not a practical theoretical tool. To uncover and understand logical thoughts used by humans, we must observe human thinking and discourse, which occurs in ordinary language, and carefully analyze and thereby validate (if indeed valid) what is being thought and said, before indulging in symbolization. This is not something any computer can do by itself. Errors are possible and common in such theoretical efforts; there is nothing (or very little) mechanical about them. It is only after they are completed satisfactorily that the results can be applied in computer programming.

To date, most of the symbolic logic used in computer programming has been very simple stuff: inclusion in or exclusion from classes (syllogism), implication and its negation (if/then and if/not-then statements). More complex logic (causal logic, a fortiori logic, analogical logic, and so on) has not so far been widely used. This is evident from the fact that most people, including university professors, have not yet really come to grips with such more complex logics (although some have tried). However, things are about to change radically due to the demand for precise algorithms for the development of so-called 'artificial intelligence' (AI). Since AI programmers have the ambition to mimic all human thought processes, they are now obliged to look more extensively and carefully into the more complex processes of human thought.

I must say that I am not very optimistic about the 'brave new world' of AI, which is why I have hesitated to communicate with people involved in that endeavor. I kind of hope that they discover my work, because it answers many of the questions they will be asking; but at the same time, I hope they do not do so in my lifetime, because I fear the possible and maybe probable negative consequences of AI. This development will no doubt have a lot of positive effects on human life, but it is also full of dangers for us all. This is, of course, widely discussed in the media and in academia, so no need for me to say more about it.

A recent paper by one Yanyi Pu can be cited as an example of the present trend in AI studies[342]. It looks interesting, showing many of the complexities and difficulties involved in AI programming. Much of what it says is over my head, I must admit. However, I notice that, although it mentions (on p. 10) my book *A Fortiori Logic* (2013), it does not actually acknowledge or discuss the numerous theoretical findings in it! My impression is that the author did not even look into my book, let alone read it, even though it contains an extensive and detailed analysis of a fortiori reasoning. This seems evident to me by the fact that, although he or she does mention some important features of a fortiori argument – such as its use of a middle term, its reference to sufficiency, and its positive and negative polarities – he or she does not consider the important distinctions between subjectal and predicatal, and between copulative and implicational, forms of it, nor apparently deal with the issues of validation. Without in-depth consideration of these features and distinctions, and others still, one cannot claim to understand a fortiori argument well enough to program realistic and accurate AI automatization of such reasoning processes.

I sometimes wonder if maybe, ironically, it is only when AI will have developed sufficiently to search all world literature on any subject, and chronologically order all discoveries, and accurately compare and assess all theories of logic, that there will be 'intelligence' enough to realize the originality and importance of my work in the field. Until then, it seems, I should not expect it to receive the attention and respect it deserves.

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342 "Generation of Explanations for Logic Reasoning" (June 2023). Submitted to the University of Cambridge, Department of Computer Science and Technology, in partial fulfilment of the requirements for the Master of Philosophy in Advanced Computer Science. https://arxiv.org/pdf/2311.13455.pdf.

14. A final word

Crucial to the art and science of logic is independent critical thought. Logic is not essentially a mechanical calculus, not a mechanical process. Its practice depends on careful observation and true understanding and insight. A Zen koan can perhaps be used to illustrate this idea.

It is the account of the enlightenment of Te Shan (Jap. Tokusan) thanks to Lung T'an (Jap. Ryutan). This is described in case #28 of *The Gateless Gate*[343] and in the commentary to case #4 of *The Blue Cliff Record*. Te Shan (China, 781-867) was a very intellectual student of Buddhism. He went and met the Zen master Lung T'an, who granted him a long interview.

When Te Shan was on his way out, Lung T'an handed him a lamp because it was dark outside. Just as Te Shan took the lamp, Lung T'an blew it out. This caused Te Shan to be "vastly and greatly enlightened." After that, Te Shan burned all the learned commentaries he had brought with him and of course became a Zen adept.

A possible explanation of Te Shan's enlightenment is that the blowing out of the lamp by Lung T'an was symbolic of the need to face the world without intellectual crutches. He suddenly realized that he did not need the 'light' provided by intellectual commentaries, but ought rather to go out into the 'dark' without fear. Contact with ultimate reality is to be achieved (primarily) wordlessly, and not through (excessive) analysis and speech.

[343] Boston, Mass.: Wisdom Publications, 2004.

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